

POWERS OF OPERATORS

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Let T be a bounded linear operator on a Banach space, with the *single-point spectrum* $\{1\}$. Suppose that

$$\|T^n - T^{n+1}\| \leq \frac{\text{const}}{n^{\frac{1}{2}+\epsilon}} \quad (1)$$

for a fixed $0 < \epsilon < \frac{1}{2}$ and all $n \in \mathbb{N}$. Does it follow that, actually,

$$\|T^n - T^{n+1}\| \leq \frac{\text{const}}{n}$$

for all $n \in \mathbb{N}$? (This implication is not true for $\epsilon = 0$, see e.g. [4, Theorem 2.5 with $p = 2$].)

It is possible that (1), with $0 \leq \epsilon \leq 1/2$, implies that

$$\|T^n\| \leq \text{const}$$

for all $n \in \mathbb{N}$? See [4, Theorems 2.5 and 2.2].

The above questions are motivated by the papers [2], [3], [4], [5], [7], [8], [9]. See also [1, Lemma 2.1], [6, Theorem 4.5.3] and [10, p. 373].

References

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