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ORBITS IN STRIPS

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By a *strip* we mean a subset of the complex plane that is bounded by two parallel lines.

Let A be a bounded linear operator on a Hilbert space such that, for each fixed unit vector x, the scalar-valued orbit

$$\{\langle A^n x, x\rangle : n \in \mathbb{N}\} \tag{1}$$

is contained in some strip (depending on x). Does it follow that all orbits (1), for all unit vectors x, are contained in a common strip?

If the space is finite-dimensional, we can associate with $A = (a_{ij})$ the Gerschgorin set

$$\mathcal{G}(A) = \bigcup_i \Big\{ z \in \mathbb{C} : |a_{ii} - z| \le \sum_{j \ne i} |a_{ij}| \Big\}.$$

Suppose that the spectral radius of A is no more than 1, and that all $\mathcal{G}(A^n)$, for $n \in \mathbb{N}$, are contained in a fixed strip. Does it follow that the powers A^n are bounded, for all $n \in \mathbb{N}$?

The above questions are motivated by the paper [1].

References

[1] A. Gomilko, I. Wróbel, and J. Zemánek, *Numerical ranges in a strip*, in: Operator Theory 20, K. R. Davidson, D. Gaşpar, Ş. Strătilă, D. Timotin, and F.-H. Vasilescu (eds.), Proceedings of the 20th International Conference on Operator Theory, Timişoara (Romania), June 30-July 5, 2004, Theta, Bucharest, 2006, 111–121.

