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THE NAVIER-STOKES FLOW AROUND A ROTATING OBSTACLE WITH TIME-DEPENDENT BODY FORCE

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Abstract. We study the motion of a viscous incompressible fluid filling the whole three-dimensional space exterior to a rigid body, that is rotating with constant angular velocity ω, under the action of external force f. By using a frame attached to the body, the equations are reduced to (1.1) in a fixed exterior domain D. Given $f = \operatorname{div} F$ with $F \in BUC(\mathbb{R}; L_{3/2,\infty}(D))$, we consider this problem in $D\times\mathbb{R}$ and prove that there exists a unique solution $u \in BUC(\mathbb{R}; L_{3,\infty}(D))$ when F and $|\omega|$ are sufficiently small. If, in particular, the external force for the original problem is independent of t, then f is periodic with period $2\pi/|\omega|$. In this situation, as a corollary of our result, we obtain a periodic solution with the same period. Stability of our solution is also discussed.

1. Introduction. This note is a continuation of the recent studies [5], [19] on the Navier-Stokes fluid around a obstacle (rigid body) $\mathbb{R}^3 \setminus D$, where D is an exterior domain with smooth boundary ∂D. We are particularly interested in the situation that the obstacle is rotating about the x_3-axis with constant angular velocity $\omega = (0, 0, a)^T$, $a \neq 0$. In the reference frame attached to the obstacle, the unknown velocity $u = (u_1, u_2, u_3)^T$ and pressure p should obey ([2], [10], [17])

$$\begin{aligned} \partial_t u + u \cdot \nabla u &= \Delta u + (\omega \times x) \cdot \nabla u - \omega \times u - \nabla p + f, \\ \operatorname{div} u &= 0 \end{aligned} \tag{1.1}$$

subject to the boundary conditions

$$u|_{\partial D} = \omega \times x, \qquad u \to 0 \quad \text{as } |x| \to \infty \tag{1.2}$$

where $\times$ denotes the usual exterior product in $\mathbb{R}^3$. In [5] Farwig and the present author constructed a unique steady solution $u \in L_{3,\infty}(D)$, weak-L_3 space, provided that the

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body force $f(x) = \operatorname{div} F(x)$ is independent of t and that both $F \in L_{3/2,\infty}(D)$ and $|\omega|$ are small enough. This class of solution is consistent with the pointwise estimate $|u(x)| \le C/|x|$ derived first by Galdi [11] when $f = 0$ (or f satisfies some pointwise decay estimates). In [19] Shibata and the present author have recently proved the stability of the small steady solution obtained above with respect to small initial disturbance in $L_{3,\infty}(D)$ together with some definite decay rates of the disturbance as $t \to \infty$. Another stability theorem was established by Galdi and Silvestre [12] within the framework of L_2 theory.

In this paper we continue to study the rotating body problem; to be precise, we discuss the existence and stability of the solution $u \in BUC(\mathbb{R}; L_{3,\infty}(D))$ to (1.1) in $D \times \mathbb{R}$ with time-dependent body force $f(x,t) = \operatorname{div} F(x,t)$ subject to (1.2) when both $F \in BUC(\mathbb{R}; L_{3/2,\infty}(D))$ and $|\omega|$ are small enough, where BUC denotes the class of bounded and uniformly continuous functions. Since the steady solution may be regarded as a time-independent case of our solution, the theorems in this paper cover the previous results [5], [19] mentioned above as a special case. In particular, another proof of the existence of the steady solution in the class $L_{3,\infty}(D)$ via semigroup is provided, while the proof of [5] relied upon some tools from harmonic analysis (see [6], [18]) together with the method (construction of parametrix) developed by [23]. And also, as a corollary of our existence theorem, we obtain a time-periodic solution on account of the uniqueness of the solution in the class $BUC(\mathbb{R}; L_{3,\infty}(D))$; that is, when $f(x,t)$ is periodic, so is our solution $u(x,t)$ with the same period. As a simple example, $f(x,t)$ is actually periodic with period $2\pi/|a|$ when the body force $g(y)$ in the original frame is independent of t, since f and g must satisfy the relation $f(x,t) = O(at)^T g(O(at)x)$ with

$$O(t) = \begin{pmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Galdi and Silvestre [13] constructed a time-periodic solution in much more difficult situation that, for instance, the rotating axis is oscillating periodically. But the uniqueness and stability of their solutions are not clear, differently from the solution in this paper, because the solution in [13] does not possess good summability at infinity. The stability problem in their physically interesting setting will be the focus in the future.

One should emphasize that the drift term $(\omega \times x) \cdot \nabla u$ causes some difficulties due to its hyperbolic effect. In fact, Farwig and Neustupa [8] proved that the essential spectrum of the operator $\Delta u + (\omega \times x) \cdot \nabla u - \omega \times u - \nabla p$ in L_2 is given by $\{\lambda = \mu + ika; \mu \le 0, k \in \mathbb{Z}\}$; it is the same in the L_q-setting as well, see [7]. Hence the semigroup $\{T(t)\}_{t \ge 0}$ generated by the operator above ([16] in L_2, [14] in L_q) is no longer analytic unlike the usual Stokes semigroup ([15], [25]). Nevertheless, our semigroup $T(t)$ enjoys some remarkable smoothing actions, see [17], [14], and that is the point for the construction of a local solution. Furthermore, in [19] decay estimates of parabolic type, so-called L_p-L_q estimates, of the semigroup

$$\|\nabla^j T(t) f\|_q \le C t^{-j/2 - (3/p - 3/q)/2} \|f\|_p \quad (t > 0) \tag{1.3}$$

have been deduced, where

$$1 < p \le q \le \infty \ (p \ne \infty) \quad \text{for } j = 0; \quad 1 < p \le q \le 3 \quad \text{for } j = 1, \tag{1.4}$$

and $\|\cdot\|_q$ denotes the norm of $L_q(D)$. The main tool in this paper is the similar estimate to (1.3) in the Lorentz space $L_{q,1}(D)$, whose dual is $L_{q/(q-1),\infty}(D)$, and we follow in principle the argument developed by Yamazaki [26]. In particular, his interpolation technique implies a sharp estimate of the integral form, see (3.2), which is crucial in this paper.

This paper is organized as follows. In section 2, after introducing notation, we state our main theorems. Section 3 is devoted to the proof.

2. Results. Before stating our results, we introduce notation. By $BUC(\mathbb{R};E)$ we denote the class of bounded and uniformly continuous functions with values in a Banach space E. For $1<q<\infty$ and $1\le r\le\infty$, let $L_{q,r}(D)$ be the Lorentz space with norm $\|\cdot\|_{q,r}$. We especially need $L_{q,1}(D)$, $L_{q,q}(D)\equiv L_q(D)$ with norm $\|\cdot\|_q$ and $L_{q,\infty}(D)$. The last one is well known as weak-L_q space, and a measurable function f is in $L_{q,\infty}(D)$ if and only if

$$\sup_{\sigma>0}\sigma|\{x\in D;|f(x)|>\sigma\}|^{1/q}<\infty,$$

where $|\cdot|$ denotes the Lebesgue measure. Note $L_{q,1}(D)^*=L_{q/(q-1),\infty}(D)$ and

$$L_{q,r}(D)=(L_{q_0}(D),L_{q_1}(D))_{\theta,r}$$

with

$$1<q_0<q<q_1<\infty,\quad \frac{1}{q}=\frac{1-\theta}{q_0}+\frac{\theta}{q_1},\quad 1\le r\le\infty, \tag{2.1}$$

where $(\cdot,\cdot)_{\theta,r}$ is the real interpolation functor. For more details about the Lorentz space, see [1]. In this paper we use the same symbols for denoting the spaces of vector and scalar functions as long as there is no confusion.

Let $C_{0,\sigma}^\infty(D)$ be the class of all vector fields u which are of class C^∞ and have compact supports as well as $\operatorname{div} u=0$. For $1<q<\infty$, the solenoidal space $J_q(D)$ is defined by the completion of $C_{0,\sigma}^\infty(D)$ in the norm $\|\cdot\|_q$. Then the space $L_q(D)$ of vector fields admits the Helmholtz decomposition ([9], [22], [24])

$$L_q(D)=J_q(D)\oplus\{\nabla p\in L_q(D);p\in L_{q,loc}(\overline{D})\}.$$

Using the projection P from $L_q(D)$ onto $J_q(D)$, we define the Stokes operator

$$D_q(A)=\{u\in W_q^2(D)\cap J_q(D);u|_{\partial D}=0\},\qquad Au=-P\Delta u, \tag{2.2}$$

where $W_q^m(D)$ is the usual L_q-Sobolev space of m-th order ($m\ge 0$). Then $-A$ is the generator of a bounded analytic semigroup of class (C_0) on $J_q(D)$ for each $q\in(1,\infty)$ ([15], [25], [4]), so that the fractional powers A^α are well defined, see (3) of Theorem 2.1. The Stokes operator with rotation effect that plays an important role here is given by

$$\begin{cases} D_q(\mathcal{L}_a)=\{u\in D_q(A);(\omega\times x)\cdot\nabla u\in L_q(D)\},\\ \mathcal{L}_a u=-P[\Delta u+(\omega\times x)\cdot\nabla u-\omega\times u],\end{cases} \tag{2.3}$$

where $\omega=(0,0,a)^T$. Geissert, Heck and Hieber [14] first proved that $-\mathcal{L}_a$ generates a (C_0)-semigroup $\{T_a(t)\}_{t\ge 0}$ on $J_q(D)$ for each $q\in(1,\infty)$. The uniform boundedness of this semigroup in t has been shown by [19] as a part of (1.3): $j=0$, $p=q$. By real interpolation, $T_a(t)$ can be extended as the semigroup in various solenoidal Lorentz spaces

$$J_{q,r}(D) = (J_{q_0}(D), J_{q_1}(D))_{\theta,r}$$

where q and r satisfy (2.1). Note that it is not of class (C_0) in the space

$$J_{q,\infty}(D) = J_{q/(q-1),1}(D)^*$$

since the domain of the generator is not dense in this space, while for $r < \infty$ the class $C_{0,\sigma}^\infty(D)$ is dense in $J_{q,r}(D)$ and, therefore, $T_a(t)$ is a (C_0)-semigroup in those spaces.

Let us rewrite the problem (1.1)–(1.2) with $f = \operatorname{div} F$ in terms of the semigroup. To do so, one needs an auxiliary function

$$b(x) = \frac{-1}{2} \operatorname{rot}(\zeta(x)|x|^2\omega)$$

where $\zeta \in C_0^\infty(\mathbb{R}^3; [0,1])$ is a cut-off function that fulfills $\zeta = 1$ near the boundary ∂D. Then we have $\operatorname{div} b = 0$, $b|_{\partial D} = \omega \times x$ and

$$\|b\|_{q,\infty} = \alpha_q |\omega| \tag{2.4}$$

for some $\alpha_q > 0$, where $1 < q < \infty$. Set $\widetilde{u} = u - b$, which together with pressure p satisfies

$$\begin{aligned} \partial_t \widetilde{u} + \widetilde{u} \cdot \nabla \widetilde{u} + b \cdot \nabla \widetilde{u} + \widetilde{u} \cdot \nabla b &= \Delta \widetilde{u} + (\omega \times x) \cdot \nabla \widetilde{u} - \omega \times \widetilde{u} - \nabla p + \operatorname{div} F + \operatorname{div} H, \\ \operatorname{div} \widetilde{u} &= 0, \end{aligned} \tag{2.5}$$

in $D \times \mathbb{R}$ subject to

$$\widetilde{u}|_{\partial D} = 0, \qquad \widetilde{u} \to 0 \quad \text{as } |x| \to \infty, \tag{2.6}$$

where

$$H(x) = -b \otimes b + \nabla b + b \otimes (\omega \times x) - (\omega \times x) \otimes b$$

so that

$$\operatorname{div} H = -b \cdot \nabla b + \Delta b + (\omega \times x) \cdot \nabla b - \omega \times b$$

and

$$\|H\|_{q,\infty} = \beta_q (|\omega|^2 + |\omega|) \tag{2.7}$$

for some $\beta_q > 0$, where $1 < q < \infty$. The boundary value problem (2.5)–(2.6) is then reduced to

$$\partial_t \widetilde{u} + \mathcal{L}_a \widetilde{u} = -P \operatorname{div}(G[\widetilde{u}, F] - H) \qquad (t \in \mathbb{R}) \tag{2.8}$$

in a suitable function space, where

$$G[\widetilde{u}, F](t) := \widetilde{u}(t) \otimes \widetilde{u}(t) + \widetilde{u}(t) \otimes b + b \otimes \widetilde{u}(t) - F(t). \tag{2.9}$$

Following Kozono and Nakao [20], we will convert (2.8) into an integral equation. Let $\widetilde{u}$ be the solution of (2.8) and let $-\infty < s < t < \infty$. Then we have (at least formally)

$$\begin{aligned} \partial_\tau \{T_a(t-\tau)\widetilde{u}(\tau)\} &= T_a(t-\tau)\{\partial_\tau \widetilde{u}(\tau) + \mathcal{L}_a \widetilde{u}(\tau)\} \\ &= -T_a(t-\tau) P \operatorname{div}(G[\widetilde{u}, F](\tau) - H) \end{aligned}$$

for $s < \tau < t$. Integrating this from $\tau = s$ to $\tau = t$, we get

$$\widetilde{u}(t) = T_a(t-s)\widetilde{u}(s) - \int_s^t T_a(t-\tau) P \operatorname{div}(G[\widetilde{u}, F](\tau) - H)\, d\tau. \tag{2.10}$$

We now intend to find a solution $u(t)$ which remains bounded as $t \to -\infty$ in a suitable norm. Combining this with L_p-L_q estimates (1.3) and letting $s \to -\infty$, see (3.18), we obtain

$$\widetilde{u}(t) = -\int_{-\infty}^{t} T_a(t-\tau) P \operatorname{div}\left(G[\widetilde{u}, F](\tau) - H\right) d\tau \qquad (t \in \mathbb{R}). \tag{2.11}$$

We seek a solution of (2.11) in the class $\widetilde{u} \in BUC(\mathbb{R}; L_{3,\infty}(D))$ as in [5]. The term $\operatorname{div}(\widetilde{u} \otimes \widetilde{u} - F)$, however, prevents us from direct analysis of (2.11). So, we employ a duality argument within a framework of Lorentz spaces developed by [26] with the aid of $T_a(t) = T_{-a}(t)^*$; here, note the skew-symmetry of $(\omega \times x) \cdot \nabla - \omega \times$. Instead of (2.11), we thus consider the following equation:

$$\langle \widetilde{u}(t), \phi \rangle = \int_{-\infty}^{t} \langle G[\widetilde{u}, F](\tau) - H, \nabla T_{-a}(t-\tau)\phi \rangle \, d\tau \tag{2.12}$$

for all $\phi \in C_{0,\sigma}^{\infty}(D)$.

THEOREM 2.1. *Let $f = \operatorname{div} F$ with $F \in BUC(\mathbb{R}; L_{3/2,\infty}(D))$.*

(1) There exists a constant $\delta > 0$ such that if

$$|\omega| + \sup_{t \in \mathbb{R}} \|F(t)\|_{3/2,\infty} \le \delta, \tag{2.13}$$

then (2.12) admits a unique solution $\widetilde{u} \in BUC(\mathbb{R}; J_{3,\infty}(D))$ subject to

$$\sup_{t \in \mathbb{R}} \|\widetilde{u}(t)\|_{3,\infty} \le c_0\left(|\omega| + \sup_{t \in \mathbb{R}} \|F(t)\|_{3/2,\infty}\right) \tag{2.14}$$

with some constant $c_0 > 0$ independent of ω and F.

(2) Let $3/2 < q < 3$ and suppose in addition that $F \in BUC(\mathbb{R}; L_{q,\infty}(D))$. There exists a constant $\widetilde{\delta} = \widetilde{\delta}(q) \in (0, \delta]$ such that if

$$|\omega| + \sup_{t \in \mathbb{R}} \|F(t)\|_{3/2,\infty} \le \widetilde{\delta}, \tag{2.15}$$

then the solution $\widetilde{u}$ obtained in (1) belongs to $BUC(\mathbb{R}; J_r(D))$ subject to

$$\sup_{t \in \mathbb{R}} \|\widetilde{u}(t)\|_r \le c_r\left(|\omega| + \sup_{t \in \mathbb{R}} \|F(t)\|_{3/2,\infty} + \sup_{t \in \mathbb{R}} \|F(t)\|_{q,\infty}\right) \tag{2.16}$$

with some constant $c_r > 0$ independent of ω and F for all $r \in (3, q_*)$, where $1/q_* = 1/q - 1/3$.

(3) In addition to the assumptions of (2), suppose that $f \in C(\mathbb{R}; L_q(D))$ for the same q as in (2). Then the solution $\widetilde{u}$ obtained in (2) belongs to $C(\mathbb{R}; D_r(A^{1/2}))$ and satisfies (2.11) in $J_r(D)$ for all $r \in (3, q_*)$, where A is the Stokes operator defined by (2.2).

REMARK 2.1. Even if f is Hölder continuous with values in $L_q(D)$ in (3) of Theorem 2.1, the solution $\widetilde{u}$ is in general never of class C^1 with values in $J_r(D)$ because the semigroup $T_a(t)$ is not analytic.

COROLLARY 2.1. *Let $f = \operatorname{div} F$ with $F \in BUC(\mathbb{R}; L_{3/2,\infty}(D))$. In addition to (2.13), suppose that F is periodic with period $l > 0$, i.e., $F(t + l) = F(t)$ in $L_{3/2,\infty}(D)$ for all $t \in \mathbb{R}$. Then the unique solution $\widetilde{u}$ obtained in (1) of Theorem 2.1 is also periodic with the same period.*

In particular, when F is independent of t, it can be regarded as a periodic function with arbitrary period. Thus the solution obtained in (1) of Theorem 2.1 is a steady flow in $L_{3,\infty}(D)$. The steady flow in this class has been already constructed by [5], [11]. We note, as observed in [18], that the class $\widetilde{u} \in L_3(D)$ with $\nabla\widetilde{u} \in L_{3/2}(D)$ is too restrictive; in fact, the Stokes flow does not belong to $L_3(D)$ in general even if $F \in C_0^\infty(D)$. We essentially need the slightly larger space $L_{3,\infty}(D)$, where the exponent 3 comes from the nonlinearity. We thus find that the right space to be used for our problem (2.12) is $BUC(\mathbb{R}; L_{3,\infty}(D))$ as in [26]. Another candidate may be $BC(\mathbb{R}; L_{3,\infty}(D))$, but this does not seem to work well in showing the continuity with respect to t.

Let $u\,(= \widetilde{u} + b) \in BUC(\mathbb{R}; L_{3,\infty}(D))$ be the solution of (1.1)–(1.2). Consider the evolution of perturbation from this solution u and the associated pressure p when the initial perturbation v_0 is prescribed. By (v, π) we denote the perturbation from (u, p); then (v, π) should obey

$$\begin{gathered} \partial_t v + v \cdot \nabla v + u \cdot \nabla v + v \cdot \nabla u = \Delta v + (\omega \times x) \cdot \nabla v - \omega \times v - \nabla \pi, \\ \operatorname{div} v = 0, \end{gathered} \tag{2.17}$$

in $D \times (0, \infty)$ subject to the boundary and initial conditions

$$v|_{\partial D} = 0, \qquad v \to 0 \quad \text{as } |x| \to \infty, \qquad v(\cdot, 0) = v_0. \tag{2.18}$$

The problem (2.17)–(2.18) is reduced to

$$v(t) = T_a(t)v_0 - \int_0^t T_a(t-\tau) P \operatorname{div}(v \otimes v + v \otimes u + u \otimes v)(\tau)\, d\tau \tag{2.19}$$

and, further, to

$$\langle v(t), \phi \rangle = \langle v_0, T_{-a}(t)\phi \rangle + \int_0^t \langle (v \otimes v + v \otimes u + u \otimes v)(\tau), \nabla T_{-a}(t-\tau)\phi \rangle\, d\tau \tag{2.20}$$

for all $\phi \in C_{0,\sigma}^\infty(D)$. Our stability theorem is as follows.

THEOREM 2.2. *Let $u \in BUC(\mathbb{R}; L_{3,\infty}(D))$ with $\operatorname{div} u = 0$ and let $v_0 \in J_{3,\infty}(D)$.*

(1) There exists a constant $\eta > 0$ such that if

$$\sup_{t>0} \|u(t)\|_{3,\infty} + \|v_0\|_{3,\infty} \le \eta,$$

then (2.20) admits a unique solution $v \in BC((0, \infty); J_{3,\infty}(D))$ subject to

$$\sup_{t>0} \|v(t)\|_{3,\infty} \le c_1 (\sup_{t>0} \|u(t)\|_{3,\infty} + \|v_0\|_{3,\infty}) \tag{2.21}$$

with some constant $c_1 > 0$ independent of u and v_0, and $v(t) \to v_0$ weakly in $J_{3,\infty}(D)$ as $t \to 0$. Here, BC denotes the class of bounded continuous functions.*

(2) Given $q \in (3, \infty)$, there exists a constant $\widetilde{\eta} = \widetilde{\eta}(q) \in (0, \eta]$ such that if

$$\sup_{t>0} \|u(t)\|_{3,\infty} + \|v_0\|_{3,\infty} \le \widetilde{\eta},$$

then the solution v obtained in (1) belongs to $C((0, \infty); J_r(D))$ and satisfies

$$\|v(t)\|_r = O(t^{-1/2+3/2r}) \qquad \text{as } t \to \infty \tag{2.22}$$

for all $r \in (3, q)$.

(3) Suppose in addition that $u \in C(\mathbb{R}; W^1_s(D))$ *for some* $s \in (3,\infty)$*. Then the solution* v *obtained in (2) belongs to* $C((0,\infty); D_r(A^{1/2}))$ *and satisfies* (2.19) *in* $J_r(D)$ *for all* $r \in (3,q)$*.*

Theorem 2.2 can be proved along the same lines as in [19]; so, we may omit the proof. Also, the idea of the proof of (3) is essentially the same as that of (3) of Theorem 2.1, which will be given in the next section.

3. Proof. In this section we prove Theorem 2.1. We begin with decay estimates of the semigroup in the Lorentz space, which have been shown in [19].

LEMMA 3.1. *Let* p *and* q *satisfy* (1.4) *for* $j = 0, 1$*, but* $q \neq \infty$*. Given* $a_0 > 0$*, there exists a constant* $C = C(a_0, p, q) > 0$ *such that*

$$\|\nabla^j T_a(t) f\|_{q,1} \le C t^{-j/2-(3/p-3/q)/2} \|f\|_{p,1} \qquad (t > 0) \tag{3.1}$$

for $f \in J_{p,1}(D)$ *provided* $|\omega| = |a| \le a_0$*.*

When $j = 1$ and $1/p - 1/q = 1/3$, the rate of (3.1) is just $1/t$. For this case one can apply the method of Yamazaki [26] to obtain

LEMMA 3.2. *Let* $1 < p < q \le 3$ *with* $1/p - 1/q = 1/3$*. Given* $a_0 > 0$*, there exists a constant* $k_p = k_p(a_0) > 0$ *such that*

$$\int_0^\infty \|\nabla T_a(t) f\|_{q,1}\, dt \le k_p \|f\|_{p,1} \tag{3.2}$$

for $f \in J_{p,1}(D)$ *provided* $|\omega| = |a| \le a_0$*.*

Proof. Following [26], we give the proof for completeness. Fix $q \in (1,3]$ and consider the sublinear operator

$$f \mapsto \|\nabla T_a(\cdot) f\|_{q,1} : J_{p,1}(D) \to L_{r,\infty}(\mathbb{R}_+)$$

where $p \in (1,q)$ and $1/r = 1/2 + (3/p - 3/q)/2$. This is actually bounded for all $p \in (1,q)$ on account of (3.1). We now suppose that $1/p - 1/q = 1/3$. We take p_0, p_1 in such a way that

$$1 < p_0 < p < p_1 < q, \qquad \frac{1}{p} = \frac{1-\theta}{p_0} + \frac{\theta}{p_1}.$$

Now, applying the real interpolation $(\cdot,\cdot)_{\theta,1}$ leads us to the conclusion. ∎

We first show the uniqueness part of Theorem 2.1.

PROPOSITION 3.1. *There are constants* $\delta_1 > 0$ *and* $\delta_2 > 0$ *so that the solution of* (2.12) *in the class* $\widetilde{u} \in BUC(\mathbb{R}; J_{3,\infty}(D))$ *with*

$$\sup_{t\in\mathbb{R}} \|\widetilde{u}(t)\|_{3,\infty} \le M \tag{3.3}$$

is unique if $|\omega| \le \delta_1$ *and if* $M \in (0, \delta_2)$*.*

Proof. Let u_1, u_2 be the solution of (2.12) with (3.3). Set $u = u_1 - u_2$, which fulfills

$$\langle u(t), \phi\rangle = \int_{-\infty}^t \langle (u\otimes u_1 + u_2 \otimes u + u \otimes b + b \otimes u)(\tau), \nabla T_{-a}(t-\tau)\phi\rangle\, d\tau$$

for all $\phi \in C_{0,\sigma}^\infty(D)$. By (3.2) together with (2.4) we find

$$\begin{aligned}|\langle u(t), \phi\rangle| &\le 2(M + \|b\|_{3,\infty}) \sup_{t\in\mathbb{R}} \|u(t)\|_{3,\infty} \int_{-\infty}^t \|\nabla T_{-a}(t-\tau)\phi\|_{3,1}\, d\tau \\ &\le 2k_{3/2}(M + \alpha_3|\omega|) \sup_{t\in\mathbb{R}} \|u(t)\|_{3,\infty} \|\phi\|_{3/2,1}.\end{aligned}$$

By duality we get

$$\sup_{t\in\mathbb{R}} \|u(t)\|_{3,\infty} \le 2k_{3/2}(M + \alpha_3|\omega|) \sup_{t\in\mathbb{R}} \|u(t)\|_{3,\infty}.$$

We set, say,

$$\delta_1 := \frac{1}{4\alpha_3 k_{3/2}}, \qquad \delta_2 := \frac{1}{4k_{3/2}}. \tag{3.4}$$

Then $|\omega| \le \delta_1$ together with $M < \delta_2$ implies $u = 0$. ■

Proof of (1) of Theorem 2.1. Set

$$E_M = \{\widetilde{u} \in BUC(\mathbb{R}; J_{3,\infty}(D));\ \sup_{t\in\mathbb{R}} \|\widetilde{u}(t)\|_{3,\infty} \le M\},$$

where M will be determined later, see (3.6). Given $\widetilde{u} \in E_M$, we define $\Psi\widetilde{u}$ by the relation

$$\langle (\Psi\widetilde{u})(t), \phi\rangle = \text{the RHD of (2.12)}, \qquad \forall \phi \in C_{0,\sigma}^\infty(D).$$

Similarly to the proof of Proposition 3.1, it follows from (3.2), (2.4) and (2.7) that

$$|\langle (\Psi\widetilde{u})(t), \phi\rangle| \le k_{3/2}\{\sup_{t\in\mathbb{R}} \|F(t)\|_{3/2,\infty} + \beta_{3/2}(|\omega|^2 + |\omega|) + 2\alpha_3|\omega|M + M^2\}\|\phi\|_{3/2,1}$$

for all $\phi \in C_{0,\sigma}^\infty(D)$. When $|\omega| \le \delta_1 \equiv 1/(4\alpha_3 k_{3/2})$, see (3.4), the duality gives $(\Psi\widetilde{u})(t) \in J_{3,\infty}(D)$ with

$$\sup_{t\in\mathbb{R}} \|(\Psi\widetilde{u})(t)\|_{3,\infty} \le k_{3/2}N + \frac{1}{2}M + k_{3/2}M^2$$

where

$$N := \sup_{t\in\mathbb{R}} \|F(t)\|_{3/2,\infty} + \beta_{3/2}(\delta_1 + 1)|\omega|.$$

Suppose

$$N < \frac{1}{16k_{3/2}^2} \tag{3.5}$$

and set

$$M := \frac{1 - \sqrt{1 - 16k_{3/2}^2 N}}{4k_{3/2}} < 4k_{3/2}N < \frac{1}{4k_{3/2}} \equiv \delta_2, \tag{3.6}$$

see (3.4); then we have $k_{3/2}M^2 - \frac{1}{2}M + k_{3/2}N = 0$ which yields

$$\sup_{t\in\mathbb{R}} \|(\Psi\widetilde{u})(t)\|_{3,\infty} \le M.$$

We also have

$$\langle (\Psi\widetilde{u})(t+h) - (\Psi\widetilde{u})(t), \phi\rangle = \int_0^\infty \langle G[\widetilde{u}, F](t+h-\tau) - G[\widetilde{u}, F](t-\tau), \nabla T_{-a}(\tau)\phi\rangle\, d\tau$$

where $G[\widetilde{u}, F](t)$ given by (2.9) is uniformly continuous with values in $L_{3/2,\infty}(D)$; given arbitrary $\varepsilon > 0$ there is a constant $\eta = \eta(\varepsilon) > 0$ such that whenever $|h| \le \eta$,

$$\sup_{t\in\mathbb{R}} \|G[\widetilde{u}, F](t+h) - G[\widetilde{u}, F](t)\|_{3/2,\infty} \le \varepsilon.$$

This together with (3.2) implies

$$|\langle (\Psi\widetilde{u})(t+h) - (\Psi\widetilde{u})(t), \phi\rangle| \le k_{3/2}\, \varepsilon \|\phi\|_{3/2,1}.$$

As a consequence,

$$\sup_{t\in\mathbb{R}} \|(\Psi\widetilde{u})(t+h) - (\Psi\widetilde{u})(t)\|_{3,\infty} \le k_{3/2}\, \varepsilon$$

holds provided that $|h| \le \eta$. We thus obtain $\Psi\widetilde{u} \in E_M$. Let u_1, $u_2 \in E_M$. Following exactly the same line as in the proof of Proposition 3.1, we get

$$\sup_{t\in\mathbb{R}} \|(\Psi u_1)(t) - (\Psi u_2)(t)\|_{3,\infty} \le 2k_{3/2}(M + \alpha_3|\omega|) \sup_{t\in\mathbb{R}} \|u_1(t) - u_2(t)\|_{3,\infty}.$$

In view of (3.6), the condition $|\omega| \le \delta_1 \equiv 1/(4\alpha_3 k_{3/2})$ yields $2k_{3/2}(M + \alpha_3|\omega|) < 1$. Hence, the mapping Ψ has a unique fixed point $\widetilde{u} \in E_M$ with (3.6) if both $|\omega| \le \delta_1$ and (3.5) hold. This completes the proof. ∎

Proof of Corollary 2.1. Let $\widetilde{u}$ be the solution obtained in (1) of Theorem 2.1 and set $v(t) := \widetilde{u}(t+l)$, where $l > 0$ is the period of F. We then have

$$\begin{aligned}\langle v(t), \phi\rangle &= \int_{-\infty}^{t+l} \langle G[\widetilde{u}, F](\tau) - H, \nabla T_{-a}(t + l - \tau)\phi\rangle\, d\tau \\ &= \int_{-\infty}^{t} \langle G[v, F](\tau) - H, \nabla T_{-a}(t - \tau)\phi\rangle\, d\tau.\end{aligned}$$

By Proposition 3.1 we obtain $v = \widetilde{u}$, which proves the assertion. ∎

Proof of (2) of Theorem 2.1. Fix $q \in (3/2, 3)$ and assume that F belongs to

$$BUC(\mathbb{R}; L_{3/2,\infty}(D) \cap L_{q,\infty}(D))$$

and satisfies (2.13). Let $\widetilde{u} \in BUC(\mathbb{R}; J_{3,\infty}(D))$ be the solution obtained in (1) of Theorem 2.1. We define an auxiliary mapping $Q_{\widetilde{u}}$ by the relation

$$\langle Q_{\widetilde{u}}[v](t), \phi\rangle = \int_{-\infty}^{t} \langle \widetilde{u}(\tau) \otimes v(\tau) + \widetilde{u}(\tau) \otimes b + b \otimes \widetilde{u}(\tau) - F(\tau) - H, \nabla T_{-a}(t-\tau)\phi\rangle\, d\tau$$

for all $\phi \in C_{0,\sigma}^\infty(D)$. Given $v \in BUC(\mathbb{R}; J_{3,\infty}(D) \cap J_{q_*,\infty}(D))$, we see from (3.2) that

$$\begin{aligned}|\langle Q_{\widetilde{u}}[v](t), \phi\rangle| \le (&\sup_{t\in\mathbb{R}} \|F(t)\|_{r,\infty} + \|H\|_{r,\infty} + 2\|b\|_{r_*,\infty} \sup_{t\in\mathbb{R}} \|\widetilde{u}(t)\|_{3,\infty} \\ &+ \sup_{t\in\mathbb{R}} \|\widetilde{u}(t)\|_{3,\infty} \sup_{t\in\mathbb{R}} \|v(t)\|_{r_*,\infty}) k_{r_*/(r_*-1)} \|\phi\|_{r_*/(r_*-1),1}\end{aligned}$$

for $r = 3/2$ and $r = q$, where $1/r_* = 1/r - 1/3$. In particular, when $r = q$, we have

$$\begin{aligned}\sup_{t\in\mathbb{R}} \|Q_{\widetilde{u}}[v](t)\|_{q_*,\infty} \le k_{q_*/(q_*-1)} \{&\sup_{t\in\mathbb{R}} \|F(t)\|_{q,\infty} + \beta_q(|\omega|^2 + |\omega|) \\ &+ 2\alpha_{q_*}|\omega| \sup_{t\in\mathbb{R}} \|\widetilde{u}(t)\|_{3,\infty} + \sup_{t\in\mathbb{R}} \|\widetilde{u}(t)\|_{3,\infty} \sup_{t\in\mathbb{R}} \|v(t)\|_{q_*,\infty}\}\end{aligned} \tag{3.7}$$

by (2.4) and (2.7). As in the proof of (1) of Theorem 2.1, $Q_{\widetilde{u}}[v](t)$ is uniformly continuous with values in $J_{q_*,\infty}(D)$ as well as in $J_{3,\infty}(D)$; thus, $Q_{\widetilde{u}}[v] \in BUC(\mathbb{R}; J_{3,\infty}(D) \cap J_{q_*,\infty}(D))$. Furthermore, for v_1, $v_2 \in BUC(\mathbb{R}; J_{3,\infty}(D) \cap J_{q_*,\infty}(D))$ we find

$$\sup_{t\in\mathbb{R}} \|Q_{\widetilde{u}}[v_1](t) - Q_{\widetilde{u}}[v_2](t)\|_{r_*,\infty} \le k_{r_*/(r_*-1)} \sup_{t\in\mathbb{R}} \|\widetilde{u}(t)\|_{3,\infty} \sup_{t\in\mathbb{R}} \|v_1(t) - v_2(t)\|_{r_*,\infty} \tag{3.8}$$

for $r = 3/2$ and $r = q$. Set

$$\widetilde{\delta} = \widetilde{\delta}(q) := \min\left\{\delta,\ \frac{1}{2c_0(k_{3/2} + k_{q_*/(q_*-1)})}\right\}$$

where δ and c_0 are as in (1). Then, in view of (2.14), still smaller condition (2.15) yields

$$\sup_{t\in\mathbb{R}} \|\widetilde{u}(t)\|_{3,\infty} \le c_0(|\omega| + \sup_{t\in\mathbb{R}} \|F(t)\|_{3/2,\infty}) \le \frac{1}{2(k_{3/2} + k_{q_*/(q_*-1)})} \tag{3.9}$$

which implies that the equation

$$Q_{\widetilde{u}}[v] = v \tag{3.10}$$

possesses a unique solution $v \in BUC(\mathbb{R}; J_{3,\infty}(D) \cap J_{q_*,\infty}(D))$. By (3.7) together with (3.9) this solution v enjoys

$$\sup_{t\in\mathbb{R}} \|v(t)\|_{q_*,\infty} \le 2k_{q_*/(q_*-1)}\{\sup_{t\in\mathbb{R}} \|F(t)\|_{q,\infty} + \beta_q(|\omega|^2 + |\omega|)\} + 2\alpha_{q_*}|\omega|. \tag{3.11}$$

Estimate (3.8) for $r = 3/2$ tells us, however, that the solution of (3.10) is unique only within $BUC(\mathbb{R}; J_{3,\infty}(D))$. Since $\widetilde{u}$ itself satisfies $Q_{\widetilde{u}}[\widetilde{u}] = \widetilde{u}$, we see that v must coincide with $\widetilde{u}$. We thus conclude from (3.11), (2.14) and interpolation inequality that $\widetilde{u}$ belongs to $BUC(\mathbb{R}; J_{3,\infty}(D) \cap J_{q_*,\infty}(D)) \subset BUC(\mathbb{R}; J_r(D))$ with (2.16) for all $r \in (3, q_*)$. This completes the proof. ■

We finally show (3) of Theorem 2.1. The strategy is similar to that of Kozono and Yamazaki [21]. Let $\widetilde{u}$ be the solution obtained in (2) of this theorem. Fix $\tau_0 \in \mathbb{R}$ arbitrarily. Our task is to find an open interval $I_{\tau_0} \ni \tau_0$ so that $\widetilde{u}$ can be identified with a solution which enjoys further regularity on I_{τ_0}. In order to do so, one needs some auxiliary results on the local existence and uniqueness of solutions to the following initial value problems associated with (2.11) and (2.12), respectively:

$$v(t) = T_a(t-t_0)v_0 - \int_{t_0}^t T_a(t-\tau)P\{(v\cdot\nabla v + b\cdot\nabla v + v\cdot\nabla b - f)(\tau) - \operatorname{div} H\}\,d\tau \tag{3.12}$$

and its weak form

$$\langle v(t), \phi\rangle = \langle v_0, T_{-a}(t-t_0)\phi\rangle + \int_{t_0}^t \langle G[v,F](\tau) - H, \nabla T_{-a}(t-\tau)\phi\rangle\,d\tau \tag{3.13}$$

for all $\phi \in C^\infty_{0,\sigma}(D)$. Let $r \in (3, q_*)$. Since $\widetilde{u}$ satisfies (2.12) for all $\phi \in J_{r/(r-1)}(D)$ by continuity, one can replace ϕ by $T_{-a}(t-t_0)\phi$ in (2.12) with $t = t_0$ to obtain a formula of $\langle \widetilde{u}(t_0), T_{-a}(t-t_0)\phi\rangle$. From this we find that $\widetilde{u}$ satisfies (3.13) with $v_0 = \widetilde{u}(t_0)$ for all $-\infty < t_0 < t < \infty$.

Proposition 3.2. *Assume that $f \in C(\mathbb{R}; L_q(D))$ for some $q \in (3/2, 3)$. Let $r \in (3, q_*)$ and $v_0 \in J_r(D)$, where $1/q_* = 1/q - 1/3$. Fix $t_0 \in \mathbb{R}$. Then there exist $t_1 \in (t_0, \infty)$ and* *a unique function*

$$v \in C([t_0, t_1]; J_r(D)) \cap C((t_0, t_1]; D_r(A^{1/2})) \tag{3.14}$$

which satisfies (3.12) *for* $t_0 \le t \le t_1$ *in* $J_r(D)$. *Furthermore,*

$$t_1 - t_0 \ge \gamma(\|v_0\|_r) \tag{3.15}$$

with some continuous non-increasing function $\gamma(\cdot) = \gamma_{q,r}(\cdot) : (0, \infty) \to (0, \infty)$.

The reason why $D_r(A^{1/2})$ is employed here is to take the boundary condition $v|_{\partial D} = 0$ into account. We use the following estimate to show Proposition 3.2.

LEMMA 3.3. *Let* $1 < p \le q < \infty$. *Given* $a_0 > 0$, *there exists a constant* $C = C(a_0, p, q) > 0$ *such that*

$$\|A^{1/2} T_a(t) f\|_q \le C t^{-1/2-(3/p-3/q)/2} \|f\|_p \qquad (0 < t < 1) \tag{3.16}$$

for $f \in J_p(D)$ *provided* $|\omega| = |a| \le a_0$.

Proof. We first assume that $f \in C_{0,\sigma}^\infty(D) \subset D_q(\mathcal{L}_a)$. Then we have $T_a(t) f \in D_q(\mathcal{L}_a) \subset D_q(A)$ and

$$\|A^{1/2} T_a(t) f\|_q \le C \|\nabla T_a(t) f\|_q$$

where we have used Theorem 4.4 of Borchers and Miyakawa [3]. Note that (1.3) with $j = 1$ holds for $0 < t < 1$ as long as $1 < p \le q < \infty$, see [19]. This combined with the above provides (3.16) for smooth f. Standard approximation procedure implies that $T_a(t) f$ belongs to $D_q(A^{1/2})$ for general $f \in J_p(D)$ together with (3.16). ∎

Proof of Proposition 3.2. The class in which the solution is constructed is (3.14) subject to

$$\begin{aligned} &\sup_{t_0 \le t \le t_1} \|v(t)\|_r + \sup_{t_0 < t \le t_1} (t - t_0)^{1/2} \|A^{1/2} v(t)\|_r \\ &\le C(\|v_0\|_r + |\omega|^2 + |\omega| + \sup_{t_0 \le t \le t_0+1} \|f(t)\|_q) \end{aligned}$$

with some constant $C = C(q, r) > 0$, where t_1 is chosen such that (3.15) holds with (3.17) below. The proof is done by the contraction mapping principle with the aid of (3.16). Since it is standard, we may omit it. We note only that the condition $3 < r < q_*$ ensures the integrability in $D_r(A^{1/2})$ of the term which contains f. Finally, the function $\gamma(\cdot)$ can be taken as

$$\gamma(\rho) = \min\{C_0(\rho + |\omega|^2 + |\omega| + \sup_{t_0 \le t \le t_0+1} \|f(t)\|_q)^{-2r/(r-3)},\ C_1|\omega|^{-2},\ 1\} \tag{3.17}$$

with some constants $C_0 = C_0(q, r) > 0$, $C_1 = C_1(r) > 0$. ∎

The following lemma is concerned with the uniqueness for (3.13).

LEMMA 3.4. *Let* $3 < r < \infty$ *and* $v_0 \in J_r(D)$. *Then the solution of* (3.13) *on* $[t_0, t_1]$ *is unique within the class* $v \in C([t_0, t_1]; J_r(D))$.

Proof. Let both v_1 and v_2 belong to $C([t_0, t_1]; J_r(D))$ and satisfy (3.13). We put $v = v_1 - v_2$ and

$$K_0 := \max_{t_0 \le t \le t_1} (\|v_1(t)\|_r + \|v_2(t)\|_r).$$

Suppose that $v(t)=0$ for all $t\in[t_0,\sigma]$ and set

$$K(t;\sigma):=\max_{\sigma\le\tau\le t}\|v(\tau)\|_r.$$

Since v obeys

$$\langle v(t),\phi\rangle=\int_\sigma^t\langle(v\otimes v_1+v_2\otimes v+v\otimes b+b\otimes v)(\tau),\nabla T_{-a}(t-\tau)\phi\rangle\,d\tau,$$

we use (1.3) to get

$$|\langle v(t),\phi\rangle|\le C_r(K_0+2\|b\|_r)\frac{(t-\sigma)^{1/2-3/2r}}{1/2-3/2r}K(t;\sigma)\|\phi\|_{r/(r-1)}.$$

We set

$$t_*:=\left(\frac{1/2-3/2r}{2C_r(K_0+2\|b\|_r)}\right)^{2r/(r-3)}$$

to obtain $K(\sigma+t_*;\sigma)=0$. We repeat this procedure for $\sigma=t_0,\ t_0+t_*,\ t_0+2t_*,\dots$ to accomplish the proof. ■

We are now in a position to complete the proof of Theorem 2.1.

Proof of (3) of Theorem 2.1. Given $q\in(3/2,3)$, let $\widetilde{u}$ be the solution obtained in (2) of Theorem 2.1. Fix $r\in(3,q_*)$ arbitrarily and set $m:=$ the RHS of (2.16). Given any $\tau_0\in\mathbb{R}$, we take $t_0:=\tau_0-\gamma(m)/2$, where $\gamma(\cdot)$ is the function in Proposition 3.2. From this time t_0 we solve the equation (3.12) with $v_0=\widetilde{u}(t_0)\in J_r(D)$. By Proposition 3.2 we have a solution

$$v\in C(\overline{I_{\tau_0}};J_r(D))\cap C(I_{\tau_0};D_r(A^{1/2}))$$

with $I_{\tau_0}=(t_0,t_1)$ for some t_1. Since one can take t_1 so that

$$t_1-t_0\ge\gamma(\|\widetilde{u}(t_0)\|_r)\ge\gamma(m),$$

we find $\tau_0\in I_{\tau_0}$. Note that v is the solution of (3.13) as well. It thus follows from Lemma 3.4 that v must coincide with $\widetilde{u}$ on I_{τ_0}. As a consequence, $\widetilde{u}\in C(I_{\tau_0};D_r(A^{1/2}))$. Since $\tau_0\in\mathbb{R}$ is arbitrary, we are led to $\widetilde{u}\in C(\mathbb{R};D_r(A^{1/2}))$. Let $-\infty<s<t<\infty$. From the argument above it follows that for every $\sigma\in[s,t]$ there is an open interval $I_\sigma\ni\sigma$ so that $\widetilde{u}$ fulfills

$$\widetilde{u}(\zeta)=T_a(\zeta-\eta)\widetilde{u}(\eta)-\int_\eta^\zeta T_a(\zeta-\tau)P\,\mathrm{div}\,(G[\widetilde{u},F](\tau)-H)\,d\tau$$

for all η, $\zeta\in I_\sigma$ with $\eta<\zeta$. Since there are $\sigma_1,\cdots,\sigma_n\in[s,t]$ such that $[s,t]\subset\cup_{j=1}^n I_{\sigma_j}$, we see that $\widetilde{u}$ satisfies (2.10) in $J_r(D)$ for all $s<t$. We take $\widetilde{r}\in(3,r)$ to show that (2.16) together with (1.3) implies

$$\|T_a(t-s)\widetilde{u}(s)\|_r\le C(t-s)^{-(3/\widetilde{r}-3/r)/2}\|\widetilde{u}(s)\|_{\widetilde{r}}\to 0\quad(s\to-\infty).\tag{3.18}$$

Thus $\widetilde{u}$ satisfies (2.11) in $J_r(D)$. We have completed the proof. ■

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