PDES, SUBMANIFOLDS AND
AFFINE DIFFERENTIAL GEOMETRY
BANACH CENTER PUBLICATIONS, VOLUME 69
INSTITUTE OF MATHEMATICS
POLISH ACADEMY OF SCIENCES
WARSZAWA 2005

CORRECTIONS TO MY PAPER "ON THE GEOMETRY OF CONVEX REFLECTORS"

(Banach Center Publ. 57 (2002), 155–169)

VLADIMIR I. OLIKER

Department of Mathematics and Computer Science, Emory University
Atlanta, GA 30322, U.S.A.
E-mail: oliker@mathcs.emory.edu

Professor Kurt Leichtweiss pointed out to me that the geometric description of the directrix hypersurface $D(R)$ of a reflector R given in the paragraph at the top of page 161 of my paper [1] is incorrect; more precisely, as it follows from the arguments below, the given description is valid only in a special case. I am grateful to Professor Leichtweiss for bringing this to my attention.

In order to correct the erroneous statement, the following two sentences in lines 3–6 from the top of the page 161 the first of which starts with "Consequently, the part of the directrix…" and the second one ends with "..tangent to f_{n-1} at $r(m)$." should be replaced by the following two paragraphs.

(…) Consequently, the part of the directrix over the point $r(m)$ is a portion of a circle with center at $r(m)-[r(m)]^\top$ lying in the 2-plane orthogonal to the $(n-1)-$plane $T_m(f_{n-1})$ tangent to the face f_{n-1} at $r(m)$; here, $[\cdot]^\top = \mathrm{proj}_{T_m(f_{n-1})}[\cdot]$. The radius of that circle is equal to $\rho(m)\sqrt{1-(m^\top)^2}$.

This assertion can be verified as follows. At the point $r(m)$ of f_{n-1} the unit outward normals to f_{n-1} lie in a 2-plane orthogonal to $T_m f_{n-1}$. For any supporting paraboloid $P_m(y)$ at $r(m)$ a reflected direction y is given by the reflection law (5) and, therefore, $y^\top = m^\top$. Then for any such y

$$r(m)-\rho(m)y = r(m)-[r(m)]^\top - [\rho(m)y]^\perp,$$

where $[\rho(m)y]^\perp$ is the part of $\rho(m)y$ orthogonal to $T_m(f_{n-1})$. Since the terminal point of the vector $r(m)-\rho(m)y$ lies on the sphere of radius $\rho(m)$ with center $r(m)$, we conclude that the corresponding part of the directrix $D(R)$ over $r(m)$ is a portion of a planar circle as described in the assertion. (…)

2000 *Mathematics Subject Classification*: 52A20, 53C45, 78A05.

Remark 1. It follows from the above arguments that the corresponding geometric description in lines 3–6 at the top of page 161 of [1] is true when m is orthogonal to $T_m f_{n-1}$.

Remark 2. The statement of Theorem 5 in [1] (the only result in the paper the proof of which depends on the above arguments) remains valid without any changes. An alternative proof of Theorem 5 is given in our recent paper [2], where we also give a characterization of focal functions of compact convex reflectors.

We also take this opportunity to correct two misprints in [1].

1. On page 159, the formula at the bottom of the page should read

$$-r(y) = \xi + \frac{c^2 - 1}{2(c + \langle \xi, y \rangle)} y.$$

2. On page 162, the formula between formulas (15) and (16) should read

$$-\partial_j u = (\nabla_{jk} p + p e_{jk}) e^{ki} \left[\frac{\partial_i p}{|\nabla p|^2 + p^2} u + \frac{1}{\sqrt{|\nabla p|^2 + p^2}} \, \partial_i y \right].$$

These misprints do not affect any of the arguments and conclusions, since subsequent expressions use the correct formulas. The correct forms of misprinted formulas are also obvious from the context.

References

[1] V. I. Oliker, *On the geometry of convex reflectors*, in: PDEs, Submanifolds and Affine Differential Geometry, Banach Center Publ. 57, Inst. Math., Polish Acad. Sci., 2002, 155–169.

[2] V. I. Oliker, *A Minkowski-style theorem for focal functions of compact convex reflectors*, preprint, 2004.