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LOGIC AND THE FOUNDATIONS OF MATHEMATICS IN LVOV (1900–1939)

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1. Introduction. Although Warsaw in 1918–1939 was the capital of logic and the foundations of mathematics in Poland, the development of related investigations in Lvov in 1900–1939 is interesting, at least for two reasons. Firstly, Stanisław Leśniewski, Jan Łukasiewicz, Wacław Sierpiński, Zygmunt Janiszewski and Stefan Mazurkiewicz, that is, the most important people who created mathematics in Warsaw studied or taught in Lvov. Secondly, logic and the foundations of mathematics (henceforth, I will omit 'of mathematics' and 'the foundations' will refer to the foundations of mathematics) had its own form in Lvov University, particularly in 1930–1939, that is, after Leon Chwistek became professor of mathematical logic. Hence, a full explanation and description of the Polish glory in logic and the foundations must take into account the history of philosophy and mathematics in Lvov and at its university. It is perhaps interesting to add that Lvov was not the strongest circle of mathematical logic and the foundations before 1918 in Poland. Doubtless, this position should be attributed to Cracow. In particular, Łukasiewicz's lectures in logic in Lvov remained on the level of the algebra of logic, whereas in Cracow Śleszyński taught ideas of Frege and the Principia Mathematica of Whitehead and Russell (see Woleński 1995, Woleński 1995a, Woleński 1995b, Woleński 2001, Woleński 2003, Woleński 2004 for a general account of the history of mathematical logic and the foundations in Poland; Woleński 1989 gives a detailed presentation of the history and achievements of the Lvov-Warsaw School; see also Skolimowski 1967; for the history of mathematics in Poland see Kuzawa 1966, Kuratowski 1980, Duda 2007; Kura-

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There is problem of whether one should use 'Lvov' or 'Lviv'. I choose the first possibility, because the paper concerns the period when the city was Polish.

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towski's book devotes much more attention to logic and the foundations than Kuzawa's. I will refer to many original works, but the included references are incomplete. See Jadacki 1980 for the fullest bibliography of logic in Poland until 1939).

2. Philosophy and mathematics in Lvov before 1900. The University of Lvov was established by King Jan Kazimierz in 1661, and once more by the Austrian Emperor Franz I in 1819 (see Woleński 1997 for a historical outline, in particular, as far as the matter concerns the history of philosophy). It was considered by Austrians to be a provincial university, mostly devoted to teaching clergymen and officials; German was the main language of instruction. Even Austrians historians are not able to point out something particularly interesting about this university as far as the matter concerns the development of science and humanities in it. Liberal reforms in the Austro-Hungarian Empire after 1866 (the battle of Sadova and the Prussian victory over Austria) also resulted in the re-Polonization of Lvov University. This fact changed its scientific shape very soon. Thus, the university became an important Polish academic centre about 1900. Józef Puzyna (mathematics) and Kazimierz Twardowski (philosophy) were professors of the utmost importance for the subsequent development of logic and the foundations in Lvov. Mathematics was also strong at Lvov Technical University, but, at least until 1918–1939, it had no links with logic and the foundations. The former introduced elements of set theory and topology in his textbook on the theory of analytic functions and also established Polish terminology (see Puzyna 1900; in particular he introduced the term 'mnogość' as the counterpart of Menge or set. On Puzyna, see Łomnicki and Ruziewicz 1921, Płoski 1988, and Sumyk 2005. He was also a very good teacher – Stanisław Ruziewicz became perhaps his most important student). Lvov also had the first author who published a study in mathematical logic in Poland, namely Stanisław Piątkiewicz, who wrote a study on the algebra of logic (see Piątkiewicz 1888; on Piątkiewicz, see Batóg 1971, Batóg, Murawski 1996).

3. Twardowski and the beginnings of the Lvov-Warsaw School. Twardowski's role in the development of logic and the foundations in Poland can be illustrated by the followings words (Tarski 1992, p. 20):

> Almost all researchers who pursue the philosophy of exact sciences in Poland are indirectly or directly the disciples of Twardowski, although his own work could hardly be counted within this domain.

In fact, Twardowski did not intend to create a logical school in Poland. His principal aim consisted in building a strong philosophical circle in Lvov. The beginnings were difficult as notes one of Twardowski's first students (Witwicki 1920, p. XI):

> He found the lecture halls almost empty. Several of his acquaintances [...] and several bolder strangers used to come in, partly out of courtesy, and partly out of curiosity, in order to see how the young professor looked and lectured. Gradually the hall filled and soon it could not accommodate all those wishing to listen; with the passage of time the lectures had to be transferred outside

> the university because no university hall could accommodate the listeners, who in the early morning hurried to secure themselves a place.

Being a student of Brentano, Twardowski wanted to implant in Poland the philosophical and methodological program of his teacher. In particular, this concerned how to do philosophy. According to Twardowski, philosophy should be scientific. Above all, it must consist of properly expressed and justified statements. Thus, philosophical language should be clear and free of ambiguities: there is no clear thought without clear words. Twardowski strongly recommended a moderate ambition of philosophers. They should avoid speculation and resign from constructing great systems, because they inevitably fall into world-views. This program favored logic, methodology, semiotic (this name was not used in Poland at the time) and concrete philosophical problems to be discussed and eventually solved, but it was definitely against speculation. Speaking in present terms, he propagated analytic philosophy.

Twardowski was a great organizer of Polish philosophical life. He established the first systematic philosophical seminar with a good library and a psychological laboratory, initiated the journal *Ruch Filozoficzny* (Philosophical Movement), providing informational about philosophical life both in Poland and abroad and created the Polish Philosophical Society in Lvov. This society also played an important role also for logic. Many important logical papers were delivered at the meetings of the Polish Philosophical Society in Lvov, including perhaps the two most important talks in the history of logic in Poland, namely Jan Łukasiewicz's lecture on many-valued logic (1920) and Alfred Tarski's lecture on the concept of truth. The abstracts of both talks appeared in *Ruch Filozoficzny* (see Łukasiewicz 1920, Tarski 1930–1931). The Lvov Scientific Society published *Sprawozdania* (Reports) in which many logical works were published (some of them are listed in the references at the end of this paper). Finally, let me mention that the first Polish philosophical journal in foreign languages was established in Lvov in 1936. Its first volume includes a German translation of Tarski's famous monograph about the concept of truth, published in Polish in 1933. Twardowski had also a definite idea of philosophical life in Poland. He was convinced that it would not be possible to build Polish philosophy solely on the base of a genuine national tradition. On the other hand, he warned against the uncritical adoption of foreign patterns and ideas. His prescription was as follows (Twardowski 1911, p. 114):

> What can the nation do which so strongly lacks established philosophical thought, which is only just laying the foundations for it? It will not want, even if it were possible, to enclose itself in a Chinese wall in order to prevent the influence of foreign thought, and thereby deprive itself of its beneficial effects. On the other hand, if the nation grants access to foreign philosophy, its own philosophical thought, still very weak, may be suppressed [...]. It seems that there is just one way out of this dilemma. Since it is impossible to escape the influence of foreign philosophy, and since our own philosophy is threatened by complete dominance by any more developed thought, the influence of foreign philosophy must be deprived of what is dangerous so that its benefits will remain untouched.

Guided by these rules, Twardowski stressed that Polish philosophy should be open to ideas coming from abroad and stay at an equal distance (or closeness, if one prefers this word) to the leading philosophical countries, that is, Germany (+ Austria), France and England. But this attitude assumes that foreign philosophy is well-known. Thus, he recommended studying philosophical novelties to his students. In particular, he lectured about new directions in logic in 1899/1900. Years later he recalled this course in the following words (Twardowski 1935, pp. 41–42):

> Dear Professor Scholz! It is my special pleasure that we can welcome you at the Polish Philosophical Society in Lvov. We are very grateful that you decided to accept our invitation [...] tell us about your results. In traveling from Warsaw to Lvov, you moved in the direction opposite to that in which interest in logistics and research in it moved in Poland. This does not mean that this interest and research left Lvov, because it is not so; but the point is that logistics in Poland had its beginning in Lvov. Here in Lvov the first Polish work devoted to logistics, that is, algebraic or mathematical logic, as it was called at that time, appeared in 1888. It was an essay "Algebra in Logic" published by [...] Stanisław Piątkiewicz. Eleven years later, in the academic year 1899–1900, I lectured in Lvov "On reforming tendencies in formal logic" and I informed the youth about those efforts, including that of George Boole, which prepared the present logistics. Jan Łukasiewicz participated in this course. Since that time, he has been faithful to mathematical logic and took it as one of the main fields of his research. When he later became a docent in Lvov, he was able to infuse his own interests into several of his own, as well as of my, students.

This passage brings us to Twardowski's pedagogical activities. He was a charismatic teacher and very soon attracted many students. Łukasiewicz belonged to the earliest of Twardowski's pupils. Łukasiewicz also studied mathematics with Puzyna. Unfortunately, it is not known whether and how Puzyna influenced Łukasiewicz. However, it is very likely that the latter learned the rudiments of set theory from the former. Puzyna 1900 appeared when Łukasiewicz was still a student. He obtained his PhD in 1902 and the habilitation in 1906. Łukasiewicz systematically lectured on logic since 1906. Other people strongly interested in logic and trained by both Twardowski and Łukasiewicz include (in alphabetic order; I mention major figures only) Kazimierz Ajdukiewicz, Tadeusz Czeżowski, Tadeusz Kotarbiński and Zygmunt Zawirski; all obtained their doctoral degrees between 1910–1913. In 1910, Leśniewski joined this group in order to work on his PhD under Twardowski's supervision. Hugo Steinhaus, one of the leaders (see below) of the Lvov mathematical school, also attended Twardowski's seminar. He remembers (see Steinhaus 2002, p. 107) that something of Russell was read at the meetings (about 1919).

4. Logic in Lvov before 1918. All logicians in Lvov in 1900–1918 were philosophers by training and although they knew mathematics quite well, it would be hard to call them mathematicians. Their interests were quite diversified and varied from mathematical logic to philosophy of science and semiotic. We can say that investigations belonged to logic *sensu largo*. Such questions as logical paradoxes (Ajdukiewicz, Czeżowski, Leśniewski, Łukasiewicz), existential sentences (Leśniewski), the logic of relations (Łukasiewicz), causality (Łukasiewicz), the concept of truth (Twardowski, Łukasiewicz, Leśniewski), induction and probability (Łukasiewicz), the principles of contradiction and excluded middle (Leśniewski, Łukasiewicz) the criticism of psychologism (Łukasiewicz), modalities (Zawirski), the problem of statements about the future (Kotarbiński, Leśniewski) represent typical directions of investigations in Lvov. No common program was realized and everybody chose their own as far as the matter concerns investigated topics, although methodologically the whole Lvov group shared Twardowski's principles. Important writings include (I mention only works belonging, at least partially, to the area of mathematical logic): Łukasiewicz 1910, Łukasiewicz 1913, Leśniewski 1914, Łukasiewicz 1915, Łukasiewicz 1916, Czeżowski 1918.

Five works deserve perhaps special attention. Łukasiewicz 1910 gives a detailed analysis of the principle of contradiction in Aristotle and anticipates later famous historical studies of this author; it also contains an addendum in which we find an outline of the algebra of logic (the level is probably related to Łukasiewicz's courses). Łukasiewicz 1913 is mostly devoted to the logical foundation of probability, but this study also outlines an algebra of logical values, anticipating some ideas of Łukasiewicz's many-valued logic. Łukasiewicz 1915 formulates the definition of logical entailment (A entails B, if B cannot be false, provided that A is true) and qualifies the Liar sentence as ill-constructed and thereby qualified as not suitable to be a value of propositional variable in logic. Łukasiewicz 1916 (the last piece written by Łukasiewicz in Lvov before his move to Warsaw) offers a detailed criticism of views of Stanisław Zaremba, a very important mathematician from Cracow. Historically speaking, this paper is extremely important, because it initiated a very hot polemic, which resulted in a great hostility between Zaremba and Warsaw mathematicians; the latter definitely defended Łukasiewicz. The issue much exceeds personal relations, because it concerned the style of doing mathematics, one based on logic and set theory, represented by Łukasiewicz and later developed by the Polish school and the other, more traditional, represented by Zaremba (see also below). Leśniewski 1914 gives a mereological analysis of the Liar paradox and thereby is a step toward mereology, completed in Leśniewski 1916. Leśniewski did not live in Lvov after 1912. However, his works were closely related to the Lvov philosophical circle.

5. Mathematics in Lvov in 1900–1918 (see Pawlikowska-Brożek 1995 for a general account of the development of mathematics in Lvov in 1851–1939). Sierpiński obtained his habilitation in Lvov in 1908. In 1910 he became professor of mathematics at Lvov University. Just one year earlier he began his lectures in set theory and very quickly a special textbook (Sierpiński 1912; it was preceded by lecture notes, published in 1910). Sierpiński's courses in set theory, as well as the mentioned textbook, belonged to the first in this field on an international scale. It is known that some philosophers, for example, Czeżowski and Zawirski attended his seminars and lectures. Sierpiński brought Janiszewski and Mazurkiewicz to Lvov. The former obtained his habilitation in 1913 (in topology on the basis of the work *On cutting the plane by continua*), the latter completed his PhD, also in 1913 (in topology on the basis of a dissertation concerning curves filling the square). According to rules, the last stage of the habilitation colloquium consisted in delivering a special lecture in order to check veniam legendi (the right for academic lectures). Sierpiński and Janiszewski chose topics from the foundations: Sierpiński on the concept of correspondence (Sierpiński 1909), Janiszewski on realism and idealism in mathematics (Janiszewski 1916). Although both lectures well document the foundational and philosophical interests and competence of their authors, this fact should not be exaggerated. Since the habilitation procedure of Sierpiński and Janiszewski was conducted before the Council of the Philosophical Faculty, having professors of various fields, including also the humanities, the lectures had to be general and relatively popular.

On the other hand, other thematic choices were also possible, for example concerning historical matters. However, we have other evidence that Sierpiński and Janiszewski had foundational interests at that time. I do not mention Mazurkiewicz, although he since published works in set theory. In fact, Mazurkiewicz's connection with Lvov was rather accidental.

Sierpiński's textbook contains the following material: denumerable sets, ordered sets, the continuum, inequalities for cardinals, point sets, well-ordered sets. The main ideology of the presentation is remarkable (p. 1; the same view is expressed in Sierpiński 1915, p. 222):

> We can courageously say that the whole contemporary Analysis is penetrated by set Theory, which contributed to enlightening and deepening many various problems and, today it is even indispensable for a presentation of the beginning of mathematics.

Thus, from the beginning Sierpiński looked at set theory from a broader perspective. In fact, Sierpiński 1912 contains only a part of the content of Sierpiński's courses in set theory. One of the lecture notes, published by the Mathematical-Physical Student Circle of the University of Lvov in 1911, was devoted to the applications of set theory in analysis. He also saw the methodological and philosophical problems of this field (see Introduction to Sierpiński 1912 and the end of Sierpiński 1915), although he was careful in expressing definite opinions in this respect. In 1912–1917 Sierpiński published several works on the axiom of choice and its applications; these investigations were summed up in Sierpiński 1918. It is perhaps worth adding that Sierpiński 1912 contains no mention of this axiom.

Except for his habilitation lecture, Janiszewski published two interesting papers about the foundations, namely Janiszewski 1915 and Janiszewski 1915a. The former stresses the role of logic in mathematics and philosophy. On the other hand, Janiszewski sees logic as something independent of its applications (p. 454):

> We note that logistics has no practical profits as its aim, at least directly. Logistic symbolism and analysis of concepts by their reduction to their primitive elements are not introduced in order to think, argue and write in this way. Similarly, physicists do not create the theory of sounds in order to help musicians in composing or write notes as mathematical equations (however, this does not exclude an indirect use of acoustic theories in the development

> of music). Logistics can contribute to the development of other sciences by discovering new forms of reasoning and training thinking, but it is only a possibility and would be its by-product. The direct aim of logistics in application to other sciences can only consist in explaining their logical structure.

This shows that Janiszewski, who studied in France, did not accept skepticism about logic, characteristic for French mathematicians, and explicitly rejected Poincaré's objections against logic as pointless. As far as the matter concerns the philosophy of mathematics Janiszewski limited its scope to (a) the nature of objects and theorems of mathematics (are they a priori or not?), and (b) the problem of the existence of mathematical objects and the correctness of some modes of reasoning in mathematics. There is an essential difference between (a) and (b) (Janiszewski 1915a, p. 470):

> The problems considered in previous sections [that is, concerning (a) – J. W.] are, so to speak, outside of the scope of a mathematician's activity. Independently of any view about these questions or its lack, this fact has no influence, at least no direct one, on the work inside mathematics and does not prevent communication between mathematicians. Disregarding what mathematicians think about the essence of natural numbers or mathematical induction, they will use them in the same way. On the other hand, there are controversial problems which have a direct influence on mathematical activity. They concern the validity of some mathematical arguments and the objective side of some mathematical concepts.

Janiszewski mentions the character of mathematical definitions (predicative or not) and the admissibility of the axiom of choice as examples of group (b). He rather reports controversies in the philosophy of mathematics without proposing solutions.

6. Janiszewski's program. In 1916 the Committee of the Mianowski Fund, a special institution supporting Polish science, invited scholars from various fields to formulate remarks concerning the most effective activities aiming at improving the organization of research. The organizers collected 44 papers. Mathematics was represented by the voices of Zaremba and Janiszewski. Although the former paper is almost forgotten, Janiszewski's contribution (Janiszewski 1918) gained more fame than any other from the rest of the submitted comments. The Janiszewski program is commonly regarded as the decisive factor of the subsequent development of mathematics in Poland, particularly in the years 1918-1939. The main idea of the program consisted in promoting various activities for achieving an autonomous position by Polish mathematics. Let me quote the end of Janiszewski 1918 (p. 18):

> If we do not like to always "to lag behind", we must apply radical means and go to the fundamentals of what is wrong. We must create a (mathematical) "workshop" at home! However, we may achieve this by concentrating the majority of our mathematicians in working in one selected branch of mathematics. In fact, this takes place automatically nowadays, but we have to help this process. Doubtless, establishing in Poland a special journal devoted to

> the only selected branch of mathematics, will attract many to research in this field.
> Yet there is also another advantage of such a journal in building the mentioned "workshop" in ourselves: we would became a technical center for publications in the related field. Others would send manuscripts of new works and have relations with us.
> If we want to capture the proper position in the world of science, let us come with own initiative.

Literally, Janiszewski's words are quite cryptic, because he did not point out which field should be chosen as the only branch of mathematics on which Poles could concentrate in the future, although he clearly alluded to something that "takes place automatically". Perhaps Janiszewski wanted to avoid a conflict with Zaremba, a great enemy of new trends in the foundations. Whatever caused Janiszewski's caution, his project became understood univocally: Polish mathematicians should concentrate on set theory and topology as well as their applications to other branches of mathematics. The new journal, materialized as *Fundamenta Mathematicae*, was devoted to this area of mathematical studies. Since it was published in foreign languages from the scratch, Janiszewski's expectation and hope was fulfilled almost immediately: *Fundamenta* became the main international journal of set-theoretical mathematics. According to the Janiszewski program logic and the foundations belonged to the heart of mathematics. This was documented by the fact that the Editorial Board of the journal consisted of two mathematicians (Sierpiński, Mazurkiewicz; Janiszewski died in 1920, before the first volume appeared) and two logicians (Leśniewski, Łukasiewicz). At the beginning, the editors wanted to publish two separate volumes, one devoted to logic and the foundations, and another to set theory and its applications, but finally this project was abandoned.

When Janiszewski formulated his program, Warsaw University was re-opened once again; it happened in 1915. Let me add that Lvov University had difficulties during Word War I and, due to Russian occupation of the city, was practically closed in 1914–1916. In the years 1915–1919, Janiszewski, Sierpiński, Mazurkiewicz, Leśniewski and Łukasiewicz became professors in Warsaw. Perhaps this fact gave rise to the opinion that the Janiszewski program was associated with Warsaw. A more proper view is that although its execution happened mainly in the capital of Poland, but its roots go to Lvov. Although we have no sources to reconstructing the discussions of Sierpiński and Janiszewski in 1913–1914 about mathematics and its general problems, it seems very likely that ideas expressed by Janiszewski in his program circulated in Lvov at that time. It would have been very strange, if Sierpiński had not communicated to Janiszewski his strong complaints (see Kuratowski 1980, p. 30) that he had no common scientific interests with any other influential Polish mathematicians about 1911; the Janiszewski program was obviously a response to this situation. As we remember, Sierpiński lectured on set theory as the fundation of mathematics and stressed its applications in analysis. Janiszewski considered logic as a theoretical science having its own problems, independently of its possible applications; this view concurred with Łukasiewicz's opinion about the place of logic in the system of sciences. He also identified the scope of philosophical and foundational problems which can be treated by mathematical methods. His view that philosophical problems of mathematics should not limit concrete works, was adopted by Polish mathematicians and became one of the hallmarks of the Polish school. In fact, Janiszewski's position was subsequently radicalized. He admitted that controversies over some modes of reasoning or some axioms could influence mathematical practice. Nobody denies that it happens, but most Polish mathematicians maintained that all mathematical constructions are allowed, provided that we have no reasons to expect that they lead to contradictions. It is also not excluded that Sierpiński and Janiszewski were influenced by Twardowski, in particular by his claim that although Polish philosophy should be open to various novelties coming from the rest of the world, we should nonetheless continuously act for its autonomous international position.

7. The Lvov mathematical school, the foundations of mathematics and logic. The first volume of *Fundamenta Mathematicae* (1920), included the papers by Stefan Banach (Lvov), Janiszewski (Warsaw), Kazimierz Kuratowski (Warsaw), Mazurkiewicz (Warsaw), Ruziewicz (Lvov), Steinhaus (Lvov), Sierpiński (Warsaw) and Witold Wilkosz (Kraków). Thus, Lvov and Warsaw, the two centers of the Polish mathematical school, were represented by 3 and 4 persons, that is, almost equally. However, an important difference between both places must be noted. Although mathematicians working in Lvov and Warsaw accepted the basic tenets of Janiszewski's program, the distribution of scientific interests was fairly different. Roughly speaking, the Warsaw circle specialized in set theory, topology and mathematical logic, whereas the Lvov School, led by Steinhaus and Banach, became much more involved in other parts of pure mathematics in which set theory and topology could be applied. Banach 1932, the basic treatise in functional analysis, full of set-theoretical and topological concepts, is a classical example of this feature of the Lvov school. The Lvov style of doing mathematics did not require supplementing it by special investigations in logic and the foundations. Another strong tendency in Lvov, represented by Steinhaus, favored applied mathematics. Perhaps Steinhaus 1923 gives a good picture of the understanding of mathematics, widespread among mathematicians working in Lvov (see also papers collected in Steinhaus 2000). This popular books touch on the following questions: the definition of mathematics, the development of mathematics, practical applications, mathematical method, differential and integral calculus, calculable mathematics, errors in mathematics, mathematics and life. Typical foundational problems are freely mixed with others, for example, historical, practical or with a presentation of calculus. Logic is treated with sympathy, but it enters in the book relatively late, more or less in its middle. More importantly, Steinhaus conceived logic mostly as a device of deduction, not a field with its own genuine theoretical problems.

The mentioned attitude of the Lvov mathematical school did not prevent its various representatives from working in logic and the foundations. However, these investigations, contrary to the situation in Warsaw, did not constitute a common enterprise, but depended on personal interests. Not very much is known about participating Lvov mathematicians in philosophical or logical meetings. Banach delivered a talk (on January 13, 1923) about apparent mathematical paradoxes at the Polish Philosophical Society in

Lvov (see Banach 1922–1923). Twardowski (see Twardowski 1992, v. I, pp. 201, 300, 323) reports three things about Banach: (a) Banach appeared at the inaugural meeting (on March 7, 1921) of the Section of Epistemology of the Polish Philosophical Society; (b) Banach appeared at Zawirski's talk (on March 26, 1927) on the relation between logic and mathematics at the Polish Philosophical Society; (c) Banach delivered a talk ("On the concept of limit") at the 1st Polish Mathematical congress in Lvov on September 7, 1927; this talk was included in the program of the Section of Mathematical Logic. The talk about paradoxes concerned puzzles of equicardinality of some sets (for example, the set of integers and the set of even numbers) and problems related to the Banach-Tarski paradox. Banach pointed out that infinite sets and the axiom of choice are responsible for such troubles which, however, are not formal contradictions. According to Banach, the solution of apparent paradoxes requires a construction of a logical system "without any objection". This is a good illustration of the mentioned attitude of Lvov mathematicians towards logic. In particular, Banach did not see any danger of the lack of a good logical system for mathematical practice. Let me mention some important contributions of Lvov mathematicians in logic and the foundations in 1918–1939:

- the Banach-Tarski paradox (Banach, Tarski 1924);
- the proof that there are only two Sheffer's functors, that is, two-termed sentential operations which define all connectives of propositional calculus (Żyliński 1925);
- the analysis of the Hilbert program (Żyliński 1935);
- the decision problem for first-order logic (Pepis 1937, Pepis 1938, Pepis 1938a);
- computable analysis (Banach, Mazur 1937).

As usual there is some arbitrariness in such reports. In particular, I will not mention Stanisław Ulam, who published some papers in general set theory (all are collected in Ulam 1974). The early Steinhaus' interests in game theory had their interesting outcome in the axiom of determinacy, discovered by him and Mycielski in the 1960s. In 1927–1933 Kuratowski was professor of mathematics at Lvov Technical University. Doubtless, his presence in Lvov was important for the intensification of foundational interests in this city. As a teacher he discovered Ulam. Although Kuratowski published important foundational papers at that time (also with Alfred Tarski), he was a typical representative of the Warsaw school. Contributions of philosophers as well as Chwistek and his collaborators will be reported in subsequent sections.

These works are of unequal rank. The second is interesting from the point of view of the early stage of the development of sentential calculus, but it is very elementary. Eustachy Żyliński's treatment of formalism was intended as a simplification of the Hilbert program, but, due to the war, it stopped at the level of propositional logic. The three remaining issues are much more important. Pepis' study provides new results about the decidability of first-order formulas; his results concerning the reducibility of some Skolem classes of formulas became classical. The Banach-Tarski paradox became one of the most famous results in set theory showing some very strange consequences of the axiom of choice. Moreover, it very well documents the earlier mentioned attitude of the Polish school allowing all fruitful mathematical methods, independently of their more or less controversial character, for example, from the point of view of constructivism. In

1936–1937, Banach and Stanisław Mazur achieved several results in computable analysis, that is, based on the concept of computable real numbers, but their joint contribution (Banach, Mazur 1937) is only an abstract (see Mazur 1963). Alan Turing achieved similar results at the same time as Banach and Mazur did. Today, computable analysis is a very developed branch of the theory of recursive functions.

8. Logic and the foundations in works of philosophers in Lvov. Twardowski became the main philosopher in Lvov, but he was faithful to his earlier views, somehow skeptical towards formalization (he termed formal tendencies in philosophy as symbolomonia and pragmatophobia). Kazimierz Ajdukiewicz played the main role in logic at the Lvov university (see Batóg 1995); he also lectured also for mathematicians before Chwistek was appointed as professor. Ajdukiewicz obtained his habilitation in 1921 on the basis of a dissertation about the methodology of the deductive sciences (Ajdukiewicz 1921), the first longer essay on the philosophical problems of mathematics, published in Poland, satisfying the standards introduced by mathematical logic. It consists of three parts: (I) The concept of proof in the logical sense; (II) On proofs of the consistency of axioms; (III) On the concept of existence in the deductive sciences. Ajdukiewicz's foundational considerations were very strongly influenced by Hilbert's formalism. In fact, Ajdukiewicz in attended Hilbert's classes in Göttingen in 1913. In particular he considered all discussed problems as pertaining to formal systems understood as well-defined complexes of formulas. Ajdukiewicz's dissertation, although it did not offer any new general views or proposals, systematized and make precise various issues of the philosophy of mathematics. Doubtless, the theory of syntactic categories is Ajdukiewicz's most famous discovery (see Ajdukiewicz 1936). He proposed a special formal quasi-arithmetical procedure for checking whether a formula is syntactically well-formed or not. This work was the first step toward contemporary categorical grammar.

Ajdukiewicz also had several interesting particular results and ideas. He worked on the concept of logical consequence (entailment). However, one point should be especially mentioned. Ajdukiewicz tried to define the concept of logical consequence. He says (see Ajdukiewicz 1921, p. 19) that '$A \Rightarrow B$' expresses the logical consequence if the implication $A \Rightarrow B$ is a logical theorem. This is a clear anticipation of the deduction theorem and the syntactic version of logical consequence. Ajdukiewicz also worked on the concept of logical consequence in his later publications. In Ajdukiewicz 1923 (p. 162), one can find the definition that $A(x)$ formally entails $B(x)$ when for any possible substitution of the variable x, either $A(x)$ is false or $B(x)$ is true (equivalently: for any possible substitution of x, if $A(x)$ is true, then $B(x)$ is true). Finally, he gave (see Ajdukiewicz 1934) the definition of logical entailment of a formula A from the set X in the case when X is finite. (These results influenced Tarski, who formulated the deduction theorem and the definition of entailment in an entirely general manner.) Ajdukiewicz also anticipated (Ajdukiewicz 1928, pp. 207–208) the rule of transfinite induction and showed (Ajdukiewicz 1926) that all the principles of traditional (Aristotelian) logic can be derived in predicate calculus under assumption of the existence of three objects, and offered (see Ajdukiewicz 1926a, Ajdukiewicz 1934) the first logical analysis of questions. Twardowski and

Ajdukiewicz trained a group of philosophers interested in logic in the 1920s and the 1930s: Izydora Dąmbska, Maria Kokoszyńska-Lutman, Seweryna Łuszczewska-Rohman, Henryk Mehlberg and Zygmunt Schmierer. Only the last worked (before War World II) in problems of mathematical logic. The rest rather represented what is called philosophical logic today. Of course, the full presentation of the history of logic in Lvov should also include works of this group. Let me add that Kokoszyńska-Lutnam and Łuszczewska-Rohman became professors of logic after 1945, the former in Wrocław and the latter in Poznań.

9. The Chwistek-Tarski competition. Lvov University decided to establish a professorship in mathematical logic in 1928; the position was located at the Faculty of Mathematics and Natural Sciences. As it was practiced at that time, other universities could point out candidates. Warsaw (Kotarbiński, Leśniewski and Łukasiewicz) nominated Tarski, but Cracow (mostly philosophers, but also Wilkosz) opted for Chwistek. Chwistek obtained his habilitation in 1928 in Cracow. It is perhaps interesting to know that he was allowed to do this, because Cracow mathematicians expected that he would go to Lvov. The situation that permission to start with the habilitation procedure was conditioned by such constraints was not unusual in Poland at that time. This shows that Chwistek's position in fact was not good. A typical explanation is that the conservative Cracow academic community was disappointed by some of his artistic activities (he was a remarkable painter and writer), in particular by acts (painted by him) of his wife. I think that an additional factor was even more decisive. In the Zaremba-Łukasiewicz controversy (see above), Chwistek took the side of the latter. The nomination of Tarski by his Warsaw teachers was uncontroversial. Since opinions were divided, the Council of the Faculty asked Brouwer, Hilbert and Russell to act as the foreign referees and Twardowski as the local one. The competition was very dramatic. In general, Twardowski and Ajdukiewicz supported (also at the request of Leśniewski and Łukasiewicz) Tarski, but Banach and Steinhaus opted for Chwistek. The situation of Banach and Steinhaus was not easy. The former collaborated with Tarski (see above), the latter's sister was Chwistek's wife and, doubtless, Steinhaus wanted avoid an impression that he supported Chwistek for family reason. Mrs. Alina Dawidowicz, Chwistek's daughter, told me once that Banach said to her: "Do not worry, we will make your father a professor". Russell send the following letter to the Dean of the Faculty (Żyliński at that time) (after Estreicher 1971, p. 212):

> 29th December, 1929
> Dear Sir,
>
> I much regret that owing to my absence in America, your letter on the 31-st October remained hitherto unanswered. I know the work of Dr. Chwistek and think very highly on it. The work of Mr. Tarski I do not at the moment remember, nor have I access to it at the present. In these circumstances, I can only say that in choosing Dr. Chwistek you will be choosing a man who will do you credit. But I am not in position to compare his merits with those of Mr. Tarski.
>
> Believe me with highest respect. Yours faithfully
>
> Bertrand Russell

It is not surprising that Russell appreciated Chwistek very much. Both worked on the theory of types, and the latter simplified the so-called ramified theory. In the introduction to the 2nd edition of the *Principia Mathematica* (Whitehead, Russell 1925, p. XIV) Russell says that Chwistek "took the heroic course of dispensing with the axiom [of reducibility] without adopting any substitute". On the other hand, Russell (p. XLVI) mentioned two papers by Tajtelbaum-Tarski. Unfortunately, the quoted letter to Żyliński is not sufficiently transparent in order to know whether Russell did not remember about Tarski's works at all or that he only forgot their content. Brouwer did not reply. Twardowski (see Twardowski 1997, v. II, pp. 112) informs that Hilbert also sent his report, but this document was never discovered. Twardowski, following his earlier preferences, wrote a report pointing out Tarski. Twardowski noted in his *Diary* (see Twardowski 1997, v. II, p. 111: "11. January 1930, Saturday. I discussed this matter (that is, Chwistek's – Tarski's competition – J. W.) with Kazik (that is, Ajdukiewicz – J. W.) and my conviction that Tarski is a much stronger candidate than Chwistek became strengthened." The Council of the Faculty finally chose Chwistek.

This competition and its result require some comments. I often heard that Chwistek won because Tarski was a Jew (see also A. B. Feferman, S. Feferman 2004, p. 68). Of course, Tarski's Jewish roots did not help him in his academic career in Poland. On the other hand, other aspects must also be taken into account. Anti-Semitism in Poland was not so strong in the late 1920s as it was in the 1930s. Thus, this factor should not be exaggerated. By the way, Chwistek's wife was also Jewish. Polish anti-Semites condemned family connections with Jews almost equally as being Jewish. Chwistek was older, his scientific reputation was certainly higher than Tarski's in 1928, the support of mathematicians was very important and, last but not least, Russell's letter, although careful and fairly conditional, certainly made a great impression in Lvov. Thus, nobody should be surprised by Chwistek's winning. The results of this affair were quite sad in Lvov. Twardowski became personally offended by the decision of the Faculty of Mathematics and Natural Sciences. On the other hand, the personal relations between Chwistek and Tarski were very good before the competition as well as after it.

10. Chwistek and his circle. As I have noted above, Chwistek worked on the Russell program. His simple theory of types became a remarkable achievement in modern logic. However, he changed views after coming to Lvov. Chwistek probably maintained that the simple theory is too weak, but the ramified theory is too complicated. Yet logicism was his stable view and he wanted to base the whole of mathematics on logic as the sole foundation. He began to develop a new foundational scheme. Although it was termed as semantics, one should distinguish it rather sharply from semantics in Tarski's meaning, that is, as the theory of the relations between languages and what they refer to. Chwistek's semantics was the theory of expressions and played the role of syntax; the idea was similar as Carnap's general syntax.

This theory was strictly nominalistic and finitary, based on one primitive sign and a sole operation, playing the role of concatenation. The construction was intended to begin with an elementary system and then to proceed to more advanced languages. The first version appeared in Chwistek 1935 (see also Chwistek 1946 and English translation of the book of 1935).

Chwistek brought his former Cracow students to Lvov. This small group included Władysław Hetper, Jan Herzberg and Jan Skarżeński. All collaborated with their teacher in developing the new logical system, and Hetper's contributions were particularly productive (see Chwistek, Hetper, Herzberg 1934, Chwistek, Hetper, Herzberg 1934a, Hetper 1937). Both Chwistek and Hetper were able to demonstrate that this formal system was sufficiently rich for arithmetic and to prove the Gödel incompleteness theorem (see Hetper 1934, Chwistek, Hetper 1938, Chwistek 1939). Unfortunately, this project was not completed and many open or unclear points remained for further investigation. Although Chwistek was very friendly with Lvov mathematicians and they liked him very much (see the previous section), his work, certainly not standard, did not gain major interest, except his mentioned Cracow students. In fact, although Chwistek lectured in Lvov on mathematical logic and tried to popularize his ideas, he did not attracted new students. Doubtless, the scientific collaboration of logicians and mathematicians was less intensive in Lvov than in Warsaw. And much less effective. In a sense, the position of logic in Lvov can be located between that in Warsaw and that in Cracow. It was certainly better and more appreciated that in the latter place, but worse than in Warsaw.

11. Final remarks. War World II ended the Lvov mathematical school. Until 1941 many professors kept their positions at Ivan Franko University (this name replaced 'Jan Kazimierz University'), but scientific activities were very limited. Germans closed the university just after coming to Lvov in September 1939. Many mathematicians and logicians from Lvov were killed or perished in 1939–1945. Of the persons mentioned, the Germans murdered Pepis and Schmierer (both were Jews), Hetper, Herzberg and Skarżeński perished in the Soviet Union (probably in lagers). Chwistek died in Moscow in 1944. Thus, his circle was entirely annihilated. Banach died in 1945. As we know, the Lvov School was restored in Wrocław to some extent. Logic and the foundations gained a high position in these new circumstances, perhaps even higher than in Lvov. However, that is another story.

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