CAUSTICS IN GREEK ANTIQUITY

ALAIN JOETS

Laboratoire de Physique des Solides, bât. 510
Université de Paris-Sud, CNRS, UMR 8502, F-91405 Orsay Cedex, France
E-mail: joets@lps.u-psud.fr

Abstract. The word *caustic* was introduced by Tschirnhausen in 1686, in the Latin expression *caustica curva*. We show that the study of the optical caustics goes back well before, at least to the hellenistic period. We present a small Greek text, whose author is perhaps Geminus (1st cent. B.C.), describing an optical phenomenon called *achilles*. We show that the term *achilles*, which has appeared only once, to our knowledge, in the literature, means caustics by reflection. We complete the description of the *achilles* thanks to another text, a passage of the poem *Argonautika* of Apollonius Rhodius. Finally, we attempt to explain the association between the mythical hero *Achilles* and the optical phenomenon called *achilles*.

1. Introduction. The term *caustic* was introduced by W. Tschirnhausen, who studied the reflection of sun rays in a circular mirror. He observes that the concentration of light occurs not only at the focus, but also along an "entire curved line, which is produced by the intersections of reflected rays" [Tschirn82]. In fact, the name itself appears later in the Tschirnhausen's papers [Tschirn86], in the Latin expression *caustica curva* (caustic curve), quickly abbreviated to *caustica*. "Caustica" recalls the framework of Tschirnhausen's study: the problem of the burning mirrors, i.e. *causticum speculum* in Latin.

That the name *caustica* appears only in 1686 does not mean that caustics were unknown before. One finds examples of caustics in Huygens works (circa 1659), or in those of Maurolico, da Vinci, etc. We want here to go back in time as far as possible and to examine whether optical caustics were recognized by the Greeks in Antiquity. To our knowledge this question has not been considered yet.

2. The achilles. In his *Géométrie Grecque*, the French historian P. Tannery says that the Greeks studied "bright points" (points brillants) [Tannery]. Tannery does not use the

---

ὁποία γὰρ	ἡ	τῶν	ὄψεων	πρόπτωσις,				
equal	the	of-the	sights	motion,				

τοιοῦτος καὶ	ὁ	καταφωτισμὸς	ὑπὸ	τοῦ	ἡλίου	γίνεται,		
just-as	the	lighting	by	the	sun	happens,		

καὶ τότε μὲν	κατ᾽	εὐθείας	ἀκλάστους,					
either	according-to	straight [lines]	not-broken,					

τότε δὲ	κατὰ	δυομένας,	ὥσπερ	ἐπὶ	τῶν	ὑέλων;		
either	according-to	entering [lines],	such as	in-the-case	of-the	glasses:		

κατακλώμεναι γὰρ	καὶ	εἰς	ἕν	συννεύουσαι	ἐξάπτουσι			
refracted	and	towards	one [point]	converging	[they] set fire			

παρὰ	τὰ ποιὰ	σχήματα;						
according-to	such	configurations;						

τότε δὲ	κατὰ	ἀνάκλασιν,						
either	according-to	reflected [lines],						

ὥσπερ	οἱ	**ἀχιλλεῖς**	φαίνονται	ἐπὶ	τῶν	ὀροφῶν		
such as	the	**achilles**	appearing	on	the	ceilings		

Table 1. Greek text and our interlinear translation of the passage containing the *achilles*. Some particles, like δὲ, are not translated, because they yield slight nuances, not important for the meaning of the text but difficult to translate. Conversely one Greek word may be translated by several words, linked here by the symbol "-". The implied words are written in brackets [...] in the translation.

name caustic and gives us to understand that he is talking about focuses. However he gives an interesting information: the Greek name for bright points is, rather strangely, "achilles" (ἀχιλλεῖς). For more information, Tannery invites the reader to consult two references, where the text of the *achilles* is given and translated: the *Variæ Collectiones* edited by F. Hultsch [Hultsch] (1864) and a work of H. Martin on Heron of Alexandria [Martin] (1854). For the modern reader, a more accessible reference is the *Heronis Alexandrini Opera*, edited by J. L. Heiberg (1912) and re-edited more recently by Teubner [Heiberg] (1976). A digital copy of the last reference is now downloadable through the digital library Gallica on the site www.bnf.fr of the Bibliothèque Nationale de France.

The most ancient version of the text of the *achilles* dates from the Middle Ages. It is found in three manuscripts of the Bibliothèque Nationale de France: ms 2385, ms. 2475 and ms. 387, and also in other manuscripts of the libraries of Leiden, Oxford, El Escorial. Table 1 shows the text, accompanied with our interlinear word for word translation. A detailed philological analysis of the text will be published elsewhere [FederspielJoets].

The text belongs to the second part of a work entitled *Origin of the geometric terms of Heron*. Only the first part may be attributed to Heron (1st cent. A.D.). The second part is a compilation – "une compilation indigeste", says Martin – of several authors not cited (this is usual at that time). According to Martin, our text belongs to the Optika of

Damianus (4th cent. A.D.). The modern philology attributes the text rather to Geminus (1st cent. B.C.) but the attribution is not certain. Some philologists prefer to present the author of our text as a "pseudo-Geminus". It remains that the text itself dates back at least to the 1st cent. B.C.

With the meaning of an optical phenomenon, the word *achilles* appears only once in the Greek literature, precisely in this text. In this sense, *achilles* is a hapax, a word having a unique occurrence in the ancient literature and thus generally difficult to translate. Heiberg translates *achilles* by "Sonnenreflexe" and Martin by "lumières mobiles auxquelles on donne le nom d'*achilles*" (moving lights called *achilles*).

In order to understand correctly the meaning of the passage, we have to recall the usual conception of the Greeks concerning the sight [Ronchi]. Rays are emitted by the eyes. These visual rays fall on objects and "sense" them, procuring the sensation of sight. Of course, the Greeks were aware that there exist also natural rays, like the sun rays. However optics, ὀπτική τέχνη or τὰ ὀπτικά, means above all the science of the sight and not the science of the natural rays. The aim of the text is precisely to show that what is known about the visual rays is also valid for the sun rays, despite their different nature.

In the first sentence the author says that the propagation (the transport, the progress, the motion) of the sun rays is totally similar to that of the visual rays. The remaining text specifies this general statement. Three situations implying sun rays are considered. The first one is the free, direct propagation. The author simply says that the rays are not-broken (ἀ-κλάστους). For the second and third situation, respectively the refraction (κατά-κλασις) and the reflection (ἀνά-κλασις), the author gives explicit examples. The example of the refraction of sun rays is provided by sun rays falling on glasses (or lenses): they converge towards a focus and set fire, when the glasses are correctly positioned. We find here the burning lens, a classical theme in the ancient scientific literature. Finally the example chosen for the reflection of the sun rays is an optical phenomenon called *achilles* by the author and appearing on the ceilings.

We claim that *achilles* correspond to caustics by reflection. We know that a generic congruence of rays, for example reflected rays, produces a caustic surface. The caustic is not directly observable. One usually uses a screen to observe the sections of the caustic with a plane. The whole caustic is then reconstructed by varying the position of the screen. It is clear that the ceiling plays precisely the role of a fixed screen. Moreover the ceiling suggests that the caustic is observed inside a house, the rays coming from the outside. In this situation, the optical contrast is better and makes the observation of the caustic easier.

The author says that the *achilles* are "appearing", but nothing is said about the cause of their appearance. We want now to answer this question with the help of another text, more ancient, belonging to the poetic literature: *Argonautika* of Apollonius Rhodius.

3. An optical metaphor in *Argonautika* of Apollonius of Rhodes. The poem *Argonautika* of Apollonius Rhodius (3rd cent. B.C.) is the story of Jason and the Argonauts and their quest for the Golden Fleece. In book III, the magician Medea falls in love with Jason. The poet lengthily describes her emotional turmoil. At some time, he

| ἠελίου | ὥς | τίς τε | δόμοις | ἐνὶ | πάλλεται | αἴγλη | 756 |
| of-the-sun | like | some | residence | in | dances | [a] brightness | |

| ὕδατος | ἐξανιοῦσα, | τὸ | δὴ | νέον | ἠὲ | λέβητι | 757 |
| of-the-water | going out, | which | indeed | new | or | [a] cauldron | |

| ἠέ | που | ἐν | γαυλῷ | κέχυται· | ἡ δ′ | ἔνθα καὶ ἔνθα | 758 |
| or | perhaps | in | [a] vase | is poured : | it | hither and thither | |

| ὠκείῃ | στροφάλιγγι | τινάσσεται | ἀίσσουσα; | | | | 759 |
| [by a] swift | eddy | dances | being shaken | | | | |

Table 2. *Argonautika*, book III, 756-759 of Apollonius of Rhodes. The apparent disorder in the position of the words comes from the fact that, in Greek, the ending is sufficient to express the relation between them, and from the necessity to follow the rules of the metrics. τὸ (verse 757) represents the water (τὸ ὕδωρ) and ἡ (verse 758) represents the brightness (ἡ αἴγλη).

uses the rhetorical process of metaphor, making an analogy between the interior world of the emotions and the exterior world of the phenomena. The interest of this passage, for us, is that the metaphor is optical (see Table 2).

The emotion felt by Medea is compared with a sun brightness dancing in the house, when flung up from water. The scene then corresponds to the text of the *achilles*: the sun light enters the interior of a house and produces there rapidly changing luminous forms. The term *achilles* is not used. The poet talks only about sun brightness. The important point is that the reflector is described: water in a container. To express the reflection, Apollonius does not use the scientific term (ἀνάκλασις), but a poetic term (ἐξανιοῦσα), which has nearly the same meaning. The water is said to be just poured in a cauldron or a vase. In other words, the surface of the water is not at rest; it is continuously varying with the time. Finally, the poet describes roughly the phenomenon: the brightness, shaken by the eddy (of the water), appears here and there.

The metaphor of Apollonius usefully completes the scientific text of the *achilles*. It answers about the nature of the reflector: *achilles* may be produced by the reflection of the sun rays on a wavy reflecting surface (Fig. 1).
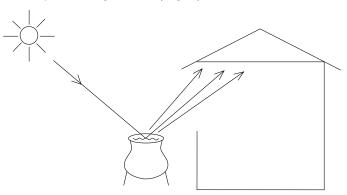

Fig. 1. The reflection of the sun rays on the water contained in a cauldron produces *achilles* on the ceiling.

At this point we may try to explain the name *achilles*. The eponymous hero *Achilles* (Ἀχιλλεύς), the king of the Myrmidons, killed at the end of the Trojan war, is usually called *swift-footed Achilles*: πόδας ὠκύς Ἀχιλλεύς. We encounter once more the adjective *swift* (ὠκύς) used in *Argonautika* to characterize the eddy (ὠκείη). Thus the natural assumption is that the characteristic feature of the optical phenomenon called *achilles* is its variability in time, its fleetingness. The surface of the moving water varies continuously and a small variation in the orientation of a given surface element generates an appreciable variation in the section of the caustic by the plane of the ceiling. The optical form changes continuously and rapidly, as the warrior *Achilles* when he is running.

We note that, inversely, the word *achilles* is less appropriate for caustics by refraction. In effect, the bending of the rays towards the normal generates a spreading of the rays lesser than that in the case of a reflection. For this reason, one may believe that the *achilles* do not designate the caustics in general, but rather only the caustics by reflection. To our knowledge, there is no special name and even no description for the caustics by refraction in the Greek literature.

It is a pleasure for me to thank Prof. M. Federspiel, for his help in the translations and very interesting discussions about the Greek literature.

References

[FederspielJoets]   M. Federspiel and A. Joets, *Sur le phénomène optique des «achilles» dans la littérature grecque*, in preparation.

[Heiberg]   *Heronis Alexandrini Opera quae supersunt omnia*, vol. 4, J. L. Heiberg (ed.), B. G. Teubner, Stuttgart, 1976.

[Hultsch]   *Heronis Alexandrini geometricorum et stereometricorum reliquiae*, F. Hultsch (ed.), Berlin, 1864.

[Martin]   Th. Henri Martin, *Recherches sur la vie et les ouvrages d'Héron d'Alexandrie, etc*, Mémoires présentés par divers savants à l'Académie des Inscriptions et Belles-Lettres, Première Série, t. IV, 1854.

[Ronchi]   V. Ronchi, *Histoire de la lumière*, Ed. Jacques Gabay, 1996.

[Tannery]   P. Tannery, *La Géométrie Grecque*, Ed. Jacques Gabay, Sceaux, 1988.

[Tschirn82]   W. Tschirnhausen [his name does not appear in the article], *Nouvelles découvertes proposées à Messieurs de l'Académie Royale des Sciences, par ***, Journal des Scavans, 176, 1682; Acta Eruditorum, I, 364, 1682.

[Tschirn86]   W. Tschirnhausen, *Curva Geometrica quæ se ipsam sui evolutione describit*, Acta Eruditorum, IX, 169, 1686.