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WEAK COMPACTNESS AND ORLICZ SPACES

BY

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Abstract. We give new proofs that some Banach spaces have Pełczyński's property (V).

1. Introduction. Recall that a Banach space X is said to have *Pełczyński's property* (V) if one has a good weak-compactness criterion in the dual space X^* of X, namely: every subset A of X^* is relatively weakly compact whenever it has the following property (easily seen to be necessary):

$$\lim_{n\to\infty}\sup_{x^*\in A}|x^*(x_n)|=0$$

for every weakly unconditionally Cauchy sequence $(x_n)_n$ in X (i.e. such that $\sum_{n\geq 1}|x^*(x_n)|<\infty$ for any $x^*\in X^*$). Equivalently, X has Pełczyński's property (V) if and only if for every Banach space Z and every non-weakly compact operator $T\colon X\to Z$, there exists a subspace X_0, isomorphic to c_0, such that T is an isomorphism between X_0 and $T(X_0)$. Besides the reflexive spaces (and in particular the L^p spaces for $1<p<\infty$), the spaces $\mathcal{C}(S)$ of continuous functions on compact spaces S have property (V); in particular L^∞ has (V). Another general class of Banach spaces having property (V) is that of Banach spaces which are M-ideals in their bidual, i.e. those for which the canonical decomposition of their third dual is an ℓ_1 decomposition:

$$X^{***}=X^*\oplus_1 X^\perp$$

(see [8, 9]). Note that every subspace of a Banach space M-ideal of its bidual is itself an M-ideal of its bidual; hence every such subspace has property (V).

On the contrary, a non-reflexive Banach space that does not contain c_0 cannot have property (V). In particular, L^1 does not have this property. Thus, the L^p spaces have (V) for $1<p\leq\infty$, whereas L^1 does not.

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For the Orlicz spaces, which are, in a natural sense, intermediate between L^1 and L^∞, D. Leung [12] proved that, when the dual space is weakly sequentially complete, not only does the Orlicz space have property (V), but it actually has the local property (V), i.e. all its ultrapowers have property (V).

D. Leung's proof uses non-trivial properties of Banach lattices. In this paper, we shall give an elementary proof of the (weaker) result that the Orlicz space L^Ψ has property (V) when the complementary function of Ψ satifies the Δ_2 condition.

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We are grateful to the referee for having simplified the proof of Theorem 2, making it shorter and much more elegant and conceptual, by giving us the statement and proof of Proposition 5.

2. The Morse–Transue space. In this paper, we shall consider Orlicz spaces defined on a probability space $(\Omega, \mathbb{P})$, which we shall assume to be non-purely atomic.

By an *Orlicz function*, we shall mean a non-decreasing convex function $\Psi\colon [0,\infty] \to [0,\infty]$ such that $\Psi(0) = 0$ and $\Psi(\infty) = \infty$. To avoid pathologies, we shall assume that we work with an Orlicz function Ψ having the following additional properties: Ψ is continuous at 0, strictly convex (hence *strictly* increasing), and such that

$$\Psi(x)/x \underset{x\to\infty}{\longrightarrow} \infty.$$

This is essentially to exclude the case of $\Psi(x) = ax$. The *Orlicz space* $L^\Psi(\Omega)$ is the space of all (equivalence classes of) measurable functions $f\colon \Omega \to \mathbb{C}$ for which there is a constant $C > 0$ such that

$$\int_\Omega \Psi(|f(t)|/C)\, d\mathbb{P}(t) < \infty,$$

and then $\|f\|_\Psi$ (the *Luxemburg norm*) is the infimum of all possible constants C such that this integral is ≤ 1.

To every Orlicz function is associated the *complementary Orlicz function* $\Phi = \Psi^*\colon [0,\infty] \to [0,\infty]$ defined by

$$\Phi(x) = \sup_{y\ge 0}(xy - \Psi(y)).$$

The extra assumptions on Ψ ensure that Φ is itself strictly convex.

Throughout this paper, we shall assume that the *complementary* Orlicz function satisfies the Δ_2 *condition* ($\varPhi \in \Delta_2$), i.e., for some constant $K > 0$ and some $x_0 > 0$ we have

$$\varPhi(2x) \le K\,\varPhi(x), \qquad \forall x \ge x_0.$$

This is usually expressed by saying that $\varPsi$ satisfies the ∇_2 *condition* ($\varPsi \in \nabla_2$). This is equivalent to the fact that for some $\beta > 1$ and $x_0 > 0$, one has $\varPsi(x) \le \varPsi(\beta x)/(2\beta)$ for $x \ge x_0$, and that implies that $\varPsi(x)/x \to \infty$ as $x \to \infty$. In particular, this excludes the case $L^\varPsi = L^1$.

When $\varPhi$ satisfies the Δ_2 condition, $L^\varPsi$ is the dual space of $L^\varPhi$.

We shall denote by $M^\varPsi$ the closure of L^∞ in $L^\varPsi$. Equivalently (see [15, p. 75]), $M^\varPsi$ is the space of (classes of) functions such that

$$\int_\varOmega \varPsi(|f(t)|/C)\, d\mathbb{P}(t) < \infty, \qquad \forall C > 0.$$

This space is the *Morse–Transue space* associated to $\varPsi$, and $(M^\varPsi)^* = L^\varPhi$, isometrically if $L^\varPhi$ is provided with the Orlicz norm, and isomorphically if it is equipped with the Luxemburg norm (see [15, Chapter IV, Theorem 1.7, p. 110]).

We have $M^\varPsi = L^\varPsi$ if and only if $\varPsi$ satisfies the Δ_2 condition, and $L^\varPsi$ is reflexive if and only if both $\varPsi$ and $\varPhi$ satisfy the Δ_2 condition. When the complementary function $\varPhi = \varPsi^*$ satisfies it (but $\varPsi$ does not, to exclude the reflexive case), we have (see [15, Chapter IV, Proposition 2.8, p. 122, and Theorem 2.11, p. 123])

$$(*) \qquad (L^\varPsi)^* = (M^\varPsi)^* \oplus_1 (M^\varPsi)^\perp,$$

or, equivalently, $(L^\varPsi)^* = L^\varPhi \oplus_1 (M^\varPsi)^\perp$, isometrically, with the Orlicz norm on $L^\varPhi$.

For more information about Orlicz functions and Orlicz spaces, we refer to [15] or [11].

It follows from $(*)$ that $M^\varPsi$ is an M-ideal in its bidual. Hence $M^\varPsi$ and all its subspaces have Pełczyński's property (V) ([8, 9]; see also [10, Chapter III, Theorem 3.4], and the end of this paper). This result was shown by D. Werner ([19]; see also [10, Chapter III, Example 1.4(d), p. 105]), in different way, using the ball intersection property (in these references, it is assumed moreover that $\varPsi$ does not satisfy the Δ_2 condition, but if it does, the space $L^\varPsi$ is reflexive, and so the result is obvious).

The proof given in [8, 9] of the fact that Banach spaces which are M-ideals in their bidual have property (V) uses local reflexivity and the notion of *pseudo-ball*. Below we give a slightly different proof, which does not use this notion, and seems more transparent. Note, however, that a stronger property, namely Pełczyński's property (u), has since been shown to be satisfied by the spaces that are M-ideals in their bidual (see [7] and, in a more general setting, [6]; that also follows from [17]).

THEOREM 1 (Godefroy–Saab, [8, 9]). *Every Banach space which is an M-ideal in its bidual has property (V).*

Proof. Assume that $X^{***} = X^* \oplus_1 X^\perp$ and let $T\colon X \to Y$ be a non-weakly compact map. By Gantmacher's theorem, $T^{**}\colon X^{**} \to Y^{**}$ is not weakly compact either. This means that $T^{(4)}(X^{(4)}) \not\subseteq Y^{**}$. Since $X^{(4)} = X^{**} \oplus (X^*)^\perp$ (canonical decomposition of the third dual of X^*), there exists some $u \in (X^*)^\perp$ with $\|u\| = 1$ such that $T^{(4)}(u) \neq 0$. Now the M-ideal property of X gives $X^{(4)} = (X^*)^\perp \oplus_\infty X^{\perp\perp}$. It follows that

$$\|x + au\| = \max\{\|x\|, |a|\}, \qquad \forall x \in X,\ \forall a \in \mathbb{C}.$$

By local reflexivity, we can construct a sequence $(x_n)_{n\ge1}$ in X equivalent to the canonical basis of c_0 and such that $\|Tx_n\| \ge \delta > 0$ for every $n \ge 1$.

For that, let $0 < \delta < \|T^{(4)}u\|$, $\varepsilon_n > 0$ be such that $(1-\varepsilon_n)\|T^{(4)}u\| > \delta$ and $\prod_{n\ge1}(1+\varepsilon_n) \le 2$, $\prod_{n\ge1}(1-\varepsilon_n) \ge 1/2$.

Assume that $x_1, \dots, x_n$ have been constructed in such a way that $\|Tx_k\| > \delta$ and

$$\prod_{k=1}^{n}(1-\varepsilon_k)\max\{|a_1|,\dots,|a_n|\} \le \|a_1x_1 + \dots + a_nx_n\| \le \prod_{k=1}^{n}(1+\varepsilon_k)\max\{|a_1|,\dots,|a_n|\}$$

for any scalars $a_1, \dots, a_n$.

Let V_n be the linear subspace of $X^{(4)}$ generated by $\{u, x_1, \dots, x_n\}$. By Bellenot's version of the principle of local reflexivity ([1, Corollary 7]), there exists an operator $A_n\colon V_n \to X$ such that $\|A_n\|, \|A_n^{-1}\| \le 1 + \varepsilon_{n+1}$, A_n is the identity on the linear span of $\{x_1, \dots, x_n\}$ and

$$|\,\|T^{(4)}u\| - \|TA_nu\|\,| \le \varepsilon_{n+1}\|T^{(4)}u\|.$$

If $x_{n+1} = A_nu$, it is now clear that

$$\prod_{k=1}^{n+1}(1-\varepsilon_k)\max\{|a_1|,\dots,|a_{n+1}|\} \le \|a_1x_1 + \dots + a_{n+1}x_{n+1}\| \le \prod_{k=1}^{n+1}(1+\varepsilon_k)\max\{|a_1|,\dots,|a_{n+1}|\}$$

for any scalars $a_1, \dots, a_{n+1}$ and $\|Tx_{n+1}\| > \delta$. Hence

$$\frac{1}{2}\max\{|a_1|,\dots,|a_n|\} \le \|a_1x_1 + \dots + a_nx_n\| \le 2\max\{|a_1|,\dots,|a_n|\}$$

for any scalars $a_1, \dots, a_n$. Since $\|Tx_n\| > \delta$, this ends the proof. ∎

3. Pełczyński's property (V) for L^Ψ. As we said, the following result is a particular case of that of D. Leung ([12]), but we shall give an elementary proof.

THEOREM 2 ([12]). *Suppose that the conjugate function Φ of Ψ satisfies the Δ_2 condition. Then the space L^Ψ has Pełczyński's property (V).*

As is well-known (and easy to prove), every dual space with Pełczyński's property (V) is a Grothendieck space: every weak-star convergent sequence in its dual is weakly convergent. Hence, we have:

COROLLARY 3. *Suppose that the conjugate function Φ of Ψ satisfies the Δ_2 condition. Then the space L^Ψ is a Grothendieck space.*

Proof of Theorem 2. We may assume that L^Ψ is a *real* Banach space.

The proof comes directly from the following two results, since $E = M^\Psi$ is a Banach lattice having property (V) and $L^\Psi = (M^\Psi)^{**}$.

LEMMA 4. *Suppose that the Orlicz function Ψ does not satisfy the Δ_2 condition. Then for every sequence $(g_n)_n$ in the unit ball of L^Ψ, there exist a sequence $(f_n)_n$ in M^Ψ and a positive function $g \in L^\Psi$ such that $|g_n - f_n| \le g$.*

PROPOSITION 5. *Let E be a Banach lattice that has property (V). Suppose that for every sequence $(x_n^{**})_n$ in $B_{E^{**}}$, there are a sequence $(x_n)_n$ in E and a positive $x^{**} \in E^{**}$ such that $|x_n^{**} - x_n| \le x^{**}$. Then E^{**} has property (V).*

Proof of Lemma 4. Since, by dominated convergence,

$$\lim_{t\to\infty} \int_\Omega \Psi(|g_n|\,\mathbb{1}_{\{|g_n|>t\}})\,d\mathbb{P} = 0,$$

we can choose, for every $n \ge 1$, a positive number t_n so large that

$$\int_\Omega \Psi(|g_n|\,\mathbb{1}_{\{|g_n|>t_n\}})\,d\mathbb{P} \le \frac{1}{2^n},$$

and

$$\sum_{n=1}^{\infty} \mathbb{P}(|g_n| > t_n) < \infty.$$

This last condition implies, by Borel–Cantelli's lemma, that, almost surely, $|g_n| \le t_n$ for n large enough. Equivalently, by setting

$$\widetilde{g}_n = g_n\,\mathbb{1}_{\{|g_n|>t_n\}},$$

we have, almost surely, $\widetilde{g}_n = 0$ for n large enough. It follows that almost surely $\sup_n |\widetilde{g}_n|$ is attained. Set now

$$A_n = \{\omega \in \Omega;\ |\widetilde{g}_1(\omega)|, \dots, |\widetilde{g}_{n-1}(\omega)| < |\widetilde{g}_n(\omega)| \text{ and } |\widetilde{g}_k(\omega)| \le |\widetilde{g}_n(\omega)|,\ \forall k \ge n\}$$

($\omega \in A_n$ if and only if n is the first time for which $\sup_k |\widetilde{g}_k(\omega)|$ is attained).

The sets A_n are disjoint and

$$\sup_{n\ge 1} |\widetilde{g}_n| = \sum_{n=1}^{\infty} |\widetilde{g}_n| \mathbb{1}_{A_n}.$$

Hence, if we set

$$g = \sup_{n\ge 1} |\widetilde{g}_n|,$$

we have $g \in L^\Psi$, since, by the disjointness of the A_n's,

$$\int_\Omega \Psi(g)\, d\mathbb{P} = \sum_{n=1}^{\infty} \int_{A_n} \Psi(|\widetilde{g}_n|)\, d\mathbb{P} \le \sum_{n=1}^{\infty} \int_\Omega \Psi(|\widetilde{g}_n|)\, d\mathbb{P} \le \sum_{n=1}^{\infty} \frac{1}{2^n} = 1.$$

That proves the lemma, by taking $f_n = g_n - \widetilde{g}_n$, which is in $L^\infty \subseteq M^\Psi$. ■

Proof of Proposition 5. Suppose that $T\colon E^{**} \to Y$ is not weakly compact. Then there exists a sequence $(x_n^{**})_n$ in $B_{E^{**}}$ such that $(Tx_n^{**})_n$ is not relatively weakly compact. Choose $(x_n)_n$ and x^{**} as in the statement of the proposition, and set $y_n^{**} = x_n^{**} - x_n$ for all n. Then either:

(a) $(Tx_n)_n$ is not weakly compact, or
(b) $(Ty_n^{**})_n$ is not weakly compact.

If (a) holds, $T_{|E}\colon E \to Y$ is not weakly compact; hence $T_{|E}$ fixes a copy of c_0.

If (b) holds, let I be the closed lattice ideal generated by x^{**} in E^{**}, normed so that $[-x^{**}, x^{**}]$ is the unit ball, and let $i\colon I \to E^{**}$ be the inclusion map. Since $(y_n^{**})_n$ lies in $[-x^{**}, x^{**}]$, $T \circ i$ is not weakly compact. But I is a lattice isomorphic to a $C(K)$ space, and hence has property (V). Thus $T \circ i$ fixes a copy of c_0. So T fixes a copy of c_0. ■

REMARK. We cannot expect that, for t_n large enough, the functions $\widetilde{g}_n$ could have a small norm. For example, let G be a standard Gaussian random variable $\mathcal{N}(0,1)$. For $\Psi = \Psi_2$ ($\Psi_2(x) = e^{x^2} - 1$), we have, for every $t > 0$,

$$\int_\Omega \Psi_2(|G|\mathbb{1}_{\{|G|>t\}}/\varepsilon)\, d\mathbb{P} = \frac{1}{\sqrt{2\pi}} \int_{|x|>t} (e^{x^2/\varepsilon^2} - 1)e^{-x^2/2}\, dx = \infty$$

for every $\varepsilon < \sqrt{2}$; that means that $\|G\mathbb{1}_{\{|G|>t\}}\|_{\Psi_2} \ge \sqrt{2}$ for every $t > 0$ (recall that $\|G\|_{\Psi_2} = \sqrt{8/3}$; see [13, p. 31]).

4. Concluding remarks and questions

1. The full result of D. Leung that L^{Ψ} has the *local property* (V), i.e. every ultrapower of L^{Ψ} has property (V) (see [3]), cannot be obtained straightforwardly from our proof. Indeed, since $L^{\Psi}=(M^{\Psi})^{**}$ is 1-complemented in every ultrapower of M^{Ψ}, it would suffice to prove that every such ultrapower has property (V); but if $[(M^{\Psi})_{\mathcal{U}}]^*$ contains $(L^{\Phi})_{\mathcal{U}}$ as a w^*-dense subspace, it is bigger. The ultraprower $(L^{\Phi})_{\mathcal{U}}$ is not exactly known in general. In the particular case of $\Psi=\Psi_2$ ($\Psi_2(x)=e^{x^2}-1$), we have ([4, Propositions 4.1 and 4.2]):

$$(L^{\Phi_2})_{\mathcal{U}}\cong L^{\Phi_2}(\mathbb{P}_{\mathcal{U}})\oplus L^1(\mu_{\mathcal{U}}).$$

However, since $(L^{\Psi})^*=(L^{\Phi})^{**}\cong L^{\Phi}\oplus_1 L^1(\mu)$, all the odd duals of L^{Ψ} can be written

$$(L^{\Psi})^{(2n+1)}\cong(L^{\Psi})^*\oplus_1 L^1(\mu_n).$$

Hence all the even duals of L^{Ψ} have property (V).

2. We can define the Hardy–Orlicz space H^{Ψ} in a natural way: it is the subspace of L^{Ψ} consisting of the functions on the unit circle $\mathbb{T}=\partial\mathbb{D}$ which have an analytic extension to $\mathbb{D}$; equivalently, it is the subspace of L^{Ψ} whose negative Fourier coefficients vanish. In [2], J. Bourgain proved that H^{∞} has property (V). Does H^{Ψ} have property (V)?

Note that the answer cannot follow trivially from our Theorem 2 since H^{Ψ} is complemented in L^{Ψ} if and only if L^{Ψ} is reflexive: indeed, the Riesz projection from L^{Ψ} onto H^{Ψ} is bounded if and only if L^{Ψ} is reflexive ([18]; see [16, Chapter VI, Theorem 2.8, p. 196]), and we have:

PROPOSITION 6. *Assume that $\Psi\in\nabla_2$. Then the Hardy–Orlicz space H^{Ψ} is complemented in L^{Ψ} if and only if the Riesz projection is bounded on L^{Ψ}. Hence H^{Ψ} is complemented in L^{Ψ} if and only if L^{Ψ} is reflexive.*

Proof. Only the necessity needs a proof. Assume that there is a bounded projection P from L^{Ψ} onto H^{Ψ}. For all $f\in M^{\Psi}$ and $g\in L^{\Phi}$, the translations $t\mapsto f_t$ and $t\mapsto g_t$ are continuous. Hence we can define $\widetilde{P}$ by setting

$$\langle\widetilde{P}f,g\rangle=\int_{\mathbb{T}}\langle P(f_t),g_t\rangle\,dt.$$

One has $\|\widetilde{P}f\|_{\Psi}\le\|P\|\,\|f\|_{\Psi}$, so that $\widetilde{P}$ is bounded from M^{Ψ} into L^{Ψ}. On the other hand, it is immediate that for every trigonometric polynomial f and $e_n(x)=e^{inx}$,

$$\widetilde{P}(f)=\sum_{n\in\mathbb{Z}}\widehat{f}(n)\widehat{P(e_n)}(n)e_n.$$

Since P is a projection, we have $P(e_n)=e_n$ for $n\ge 0$; and since P takes its values in H^{Ψ}, we have $\widehat{P(e_n)}(k)=0$ for $k<0$; in particular, $\widehat{P(e_n)}(n)=0$ for $n<0$.

Therefore we get

$$\widetilde{P}(f) = \sum_{n\geq 0} \widehat{f}(n) e_n,$$

that is, $\widetilde{P}$ is the restriction to M^Ψ of the Riesz projection. Hence the Riesz projection is bounded on M^Ψ. By taking its bi-adjoint, we see that it is bounded on L^Ψ. ∎

In Ryan's paper ([18]), it is assumed that Ψ is an N-function, that is, $\lim_{x\to 0} \Psi(x)/x = 0$. But we may modify Ψ on $[0,1]$ to get an N-function Ψ_1. Since we work on a probability space $(\Omega, \mathbb{P})$, the new space L^{Ψ_1} is equal, as a vector space, to L^Ψ, but with an equivalent norm. Hence Ryan's result remains true without this assumption.

Note that, when the probability space $(\Omega, \mathbb{P})$ is separable, since we have assumed that $\Psi \in \nabla_2$, the reflexivity of L^Ψ is equivalent to its separability (see [15, Chapter III, Theorem 5.1, pp. 87–88]).

3. Property (V) allows us to say that L^Ψ looks like L^p, $1 < p \leq \infty$. In some sense, it may be seen as being close to L^∞ when $\Psi \notin \Delta_2$, since it is not reflexive. However, from other points of view, it is closer to L^p with $p < \infty$; on the one hand, it is a bidual space; on the other hand, one has:

PROPOSITION 7. *If $\Psi \in \nabla_2$, then L^Ψ never has the Dunford–Pettis property.*

Proof. We are actually going to show that M^Ψ does not have the Dunford–Pettis property. That will prove the proposition, since $L^\Psi = (M^\Psi)^{**}$.

Since $\Psi \in \nabla_2$, there are $\alpha > 1$ and $c > 0$ such that $\Psi(x) \geq cx^\alpha$. It follows that $L^\Psi \subseteq L^\alpha$ and the natural injection $i\colon L^\Psi \to L^\alpha$ is bounded, and hence weakly compact, since L^α is reflexive.

Take now an orthonormal sequence $(r_n)_{n\geq 1}$ in L^2 with constant modulus equal to 1 (for example, an independent sequence of random variables taking the values ± 1 each with probability 1/2). One has $\int_\Omega r_n f \, d\mathbb{P} \to 0$ as $n \to \infty$ for every $f \in L^2$. By density, this remains true for every $f \in L^1$, and in particular for every $f \in L^\Phi$, since $L^\Phi \subseteq L^1$. Therefore $(r_n)_{n\geq 1}$ weakly converges to 0 in M^Ψ. Since $\|r_n\|_\alpha = 1$, $(i(r_n))_n$ does not norm-converge to 0, and hence the weakly compact map $i\colon M^\Psi \to L^\alpha$ is not a Dunford–Pettis operator. Therefore M^Ψ does not have the Dunford–Pettis property. ∎

A slightly different way to prove this is to use the fact that for every Banach space X which has the Dunford–Pettis property and which does not contain ℓ_1, its dual X^* has the Schur property ([5, 14]; see also [13, Chapitre 7, Exercice 7.2]). But M^Ψ does not contain ℓ_1 (because all its subspaces have property (V); or because its dual L^Φ is separable). Hence L^Φ

would have the Schur property. The same argument as above shows that is not the case.

4. We have required in this paper that the complementary function Φ satisfies the Δ_2 condition. Hence, in some sense, the space L^Ψ is far from L^1. We may ask what happens when we are at the other end of the scale, namely when L^Ψ is close to L^1. But if Ψ satisfies the Δ_2 condition, then $L^\Psi = (M^\Phi)^*$ and M^Φ, being an M-ideal in its bidual, has property (V), as said in the introduction. It follows that L^Ψ is weakly sequentially complete (and in fact has property (V^*)), and if we assume that $\Phi \notin \Delta_2$ (so that L^Ψ is not reflexive), then L^Ψ does not have property (V).

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