COLLOQUIUM MATHEMATICUM

VOL. 123 2011 NO. 2

ON SELFINJECTIVE ALGEBRAS WITHOUT SHORT CYCLES IN THE COMPONENT QUIVER

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Abstract. We give a complete description of all finite-dimensional selfinjective algebras over an algebraically closed field whose component quiver has no short cycles.

Introduction and the main result. Throughout the paper, by an *algebra* we mean a basic, connected, finite-dimensional algebra over an algebraically closed field k. For an algebra A, we denote by $\operatorname{mod} A$ the category of finitely generated right A-modules, and by $\operatorname{ind} A$ the full subcategory of $\operatorname{mod} A$ given by the indecomposable modules. An algebra A is called *selfinjective* if A_A is an injective module, or equivalently, the projective and injective modules in $\operatorname{mod} A$ coincide.

An important combinatorial and homological invariant of the module category $\operatorname{mod} A$ of an algebra A is its Auslander–Reiten quiver $\Gamma_A = \Gamma(\operatorname{mod} A)$. It describes the structure of the quotient category $\operatorname{mod} A/\operatorname{rad}^\infty(\operatorname{mod} A)$, where $\operatorname{rad}^\infty(\operatorname{mod} A)$ is the infinite Jacobson radical of $\operatorname{mod} A$. In particular, by a result of Auslander [4], A is of finite representation type if and only if $\operatorname{rad}^\infty(\operatorname{mod} A) = 0$. In general, it is important to study the behavior of the components of Γ_A in the category $\operatorname{mod} A$. Following [25], a component $\mathcal{C}$ of Γ_A is called *generalized standard* if $\operatorname{rad}^\infty(X, Y) = 0$ for all modules X and Y in $\mathcal{C}$. It has been proved in [25] that every generalized standard component $\mathcal{C}$ of Γ_A is *almost periodic*, that is, all but finitely many DTr-orbits in $\mathcal{C}$ are periodic. Moreover, by a result of [32], the additive closure $\operatorname{add}(\mathcal{C})$ of a generalized standard component $\mathcal{C}$ of Γ_A is closed under extensions in $\operatorname{mod} A$. A component of Γ_A of the form $\mathbb{Z}\mathbb{A}_\infty/((\mathrm{DTr})^r)$, where r is a positive integer, is called a *stable tube* of rank r. We note that, for A selfinjective, every infinite, generalized standard component $\mathcal{C}$ of Γ_A is either acyclic with finitely many DTr-orbits or is a *quasi-tube* (the stable part $\mathcal{C}^s$ of $\mathcal{C}$ is a stable tube). Following [24], a *component quiver* Σ_A of an algebra A has the components of Γ_A as the vertices, and two components $\mathcal{C}$ and $\mathcal{D}$ of Γ_A are linked in Σ_A

2010 *Mathematics Subject Classification*: Primary 16D50, 16G60, 16G70.

Key words and phrases: selfinjective algebra, orbit algebra, quasi-tilted algebra, quasi-tube, short cycle, component quiver.

DOI: 10.4064/cm123-2-6

by an arrow $\mathcal{C} \to \mathcal{D}$ if $\mathrm{rad}_A^\infty(X,Y) \neq 0$ for some modules X in $\mathcal{C}$ and Y in $\mathcal{D}$. In particular, a component $\mathcal{C}$ of Γ_A is generalized standard if and only if Σ_A has no loop at $\mathcal{C}$. By a *short cycle* in Σ_A we mean a cycle $\mathcal{C} \to \mathcal{D} \to \mathcal{C}$, where possibly $\mathcal{C} = \mathcal{D}$. We also mention that by a result in [14] the component quiver Σ_A of a selfinjective algebra A of infinite representation type is fully cyclic, that is, any finite number of components of Γ_A lies on a common cycle in Σ_A. We also mention that the structure of all selfinjective algebras of finite representation type is completely understood (see [28, Section 3]).

The aim of this paper is to prove the following theorem characterizing the class of representation-infinite selfinjective algebras whose component quiver Σ_A contains no short cycles.

THEOREM. *Let A be a representation-infinite selfinjective algebra such that Σ_A has no short cycles. Then A is isomorphic to an orbit algebra $\widehat{B}/(\varphi\nu_{\widehat{B}}^2)$, where $\widehat{B}$ is the repetitive algebra of an algebra B which is a tilted algebra of Euclidean type or a tubular algebra, φ is a strictly positive automorphism of $\widehat{B}$, and $\nu_{\widehat{B}}$ is the Nakayama automorphism of $\widehat{B}$.*

The assumptions of the theorem imply that the module category $\mathrm{mod}\, A$ contains no infinite short cycles, and no component $\mathcal{C}$ in Γ_A has external short paths. By a *short cycle* in $\mathrm{mod}\, A$ we mean a sequence $M \xrightarrow{f} N \xrightarrow{g} M$ of non-zero non-isomorphisms between indecomposable modules in $\mathrm{mod}\, A$ [19], and such a cycle is said to be *infinite* if at least one of the homomorphisms f or g belongs to $\mathrm{rad}^\infty(\mathrm{mod}\, A)$. Moreover, following [18], by an *external short path* of a component $\mathcal{C}$ of Γ_A we mean a sequence $X \to Y \to Z$ of non-zero homomorphisms between indecomposable modules in $\mathrm{mod}\, A$ with X and Z in $\mathcal{C}$ but Y not in $\mathcal{C}$.

We also note that, if A is a representation-infinite selfinjective algebra such that Σ_A contains no short cycles, then every short cycle in $\mathrm{mod}\, A$ is finite and hence, by [27], A is a tame algebra of polynomial growth.

The paper is organized as follows. In Section 1 we recall the related background on the orbit algebras of the repetitive algebras of selfinjective algebras, and the almost concealed canonical algebras. Section 2 is devoted to the proof of Theorem.

For basic background on the representation theory of algebras applied in the paper we refer to the books [2], [5], [20], [21], [22] and to the survey articles [28], [31], [33].

1. Preliminaries. Let A be an algebra and $\mathcal{C}$ be a family of components of Γ_A. Then $\mathcal{C}$ is said to be *sincere* if any simple A-module occurs as a composition factor of a module in $\mathcal{C}$, and *faithful* if its annihilator $\mathrm{ann}_A(\mathcal{C})$ in A (the intersection of the annihilators of all modules in $\mathcal{C}$) is zero. Observe that if $\mathcal{C}$ is faithful then it is sincere. Moreover, the family $\mathcal{C}$ is said to be *separating* in mod A if the indecomposable modules in mod A split into three disjoint classes $\mathcal{P}^A$, $\mathcal{C}^A = \mathcal{C}$ and $\mathcal{Q}^A$ such that:

(S1) $\mathcal{C}^A$ is a sincere generalized standard family of components;
(S2) $\mathrm{Hom}_A(\mathcal{Q}^A, \mathcal{P}^A) = 0$, $\mathrm{Hom}_A(\mathcal{Q}^A, \mathcal{C}^A) = 0$, $\mathrm{Hom}_A(\mathcal{C}^A, \mathcal{P}^A) = 0$;
(S3) any homomorphism from $\mathcal{P}^A$ to $\mathcal{Q}^A$ factors through the additive category $\mathrm{add}\,\mathcal{C}^A$ of $\mathcal{C}^A$.

Let Λ be a canonical algebra in the sense of Ringel [20]. Then the quiver Q_Λ of Λ has a unique sink and a unique source. Denote by Q_Λ^* the quiver obtained from Q_Λ by removing the unique source of Q_Λ and the arrows attached to it. Then Λ is said to be a *canonical algebra of Euclidean type* (respectively, *of tubular type*) if Q_Λ^* is a Dynkin quiver (respectively, a Euclidean quiver). The general shape of the Auslander–Reiten quiver Γ_Λ, described in [20, Sections 3 and 4], is as follows:

$$\Gamma_\Lambda = \mathcal{P}^\Lambda \vee \mathcal{T}^\Lambda \vee \mathcal{Q}^\Lambda,$$

where $\mathcal{P}^\Lambda$ is a family of components containing a unique preprojective component $\mathcal{P}(\Lambda)$ and all indecomposable projective Λ-modules, $\mathcal{Q}^\Lambda$ is a family of components containing a unique preinjective component $\mathcal{Q}(\Lambda)$ and all indecomposable injective Λ-modules, and $\mathcal{T}^\Lambda$ is an infinite family of pairwise orthogonal, generalized standard, faithful stable tubes, separating $\mathcal{P}^\Lambda$ from $\mathcal{Q}^\Lambda$, and with all but finitely many stable tubes of rank one. An algebra C of the form $\mathrm{End}_\Lambda(T)$, where T is a multiplicity-free tilting module from the additive category $\mathrm{add}(\mathcal{P}^\Lambda)$ of $\mathcal{P}^\Lambda$, is said to be a *concealed canonical algebra* of type Λ. More generally, an algebra B of the form $\mathrm{End}_\Lambda(T)$, where T is a multiplicity-free tilting module from $\mathrm{add}(\mathcal{P}^\Lambda \cup \mathcal{T}^\Lambda)$, is said to be an *almost concealed canonical algebra* of type Λ. An almost concealed canonical algebra B of tubular type is called a *tubular algebra*. Moreover, an almost concealed canonical algebra of Euclidean type is a representation-infinite, tilted algebra of Euclidean type whose preinjective component contains all indecomposable injective modules (see [1]).

For an algebra A, we denote by D the standard duality $\mathrm{Hom}_k(-, k)$ on mod A. Then an algebra A is *selfinjective* if and only if $A \cong \mathrm{D}(A)$ in mod A.

Let A be a selfinjective algebra and $\{e_i \mid 1 \le i \le s\}$ a complete set of orthogonal primitive idempotents of A. We denote by $\nu = \nu_A$ the *Nakayama automorphism* of A inducing an A-bimodule isomorphism $A \cong \mathrm{D}(A)_\nu$, where $\mathrm{D}(A)_\nu$ denotes the right A-module obtained from $\mathrm{D}(A)$ by changing the right operation of A as follows: $f \cdot a = f\nu(a)$ for each $a \in A$ and $f \in \mathrm{D}(A)$. Hence we have $\mathrm{soc}(\nu(e_i)A) \cong \mathrm{top}(e_iA)$ $(= e_iA/\mathrm{rad}(e_iA))$ as right A-modules for all $i \in \{1, \ldots, s\}$. Since $\{\nu(e_i)A \mid 1 \le i \le s\}$ is a complete set of representatives of indecomposable projective right A-modules, there is a (*Nakayama*) *permutation* of $\{1, \ldots, s\}$, denoted again by ν, such that

$\nu(e_i)A \cong e_{\nu(i)}A$ for all $i \in \{1, \ldots, s\}$. Invoking the Krull–Schmidt theorem, we may assume that $\nu(e_iA) = \nu(e_i)A = e_{\nu(i)}A$ for all $i \in \{1, \ldots, s\}$.

Let B be an algebra. The *repetitive algebra* $\widehat{B}$ of B (see [13]) is an algebra (without identity) whose k-vector space structure is that of

$$\bigoplus_{m\in\mathbb{Z}} (B_m \oplus \mathrm{D}(B)_m),$$

where $B_m = B$ and $\mathrm{D}(B)_m = \mathrm{D}(B)$ for all $m \in \mathbb{Z}$, and the multiplication is defined by

$$(a_m, f_m)_m \cdot (b_m, g_m)_m = (a_m b_m, a_m g_m + f_m b_{m-1})_m$$

for $a_m, b_m \in B_m$, $f_m, g_m \in \mathrm{D}(B)_m$. For a fixed set $\mathcal{E} = \{e_i \mid 1 \le i \le n\}$ of orthogonal primitive idempotents of B with $1_B = e_1 + \cdots + e_n$, consider the canonical set $\widehat{\mathcal{E}} = \{e_{m,i} \mid m \in \mathbb{Z},\ 1 \le i \le n\}$ of orthogonal primitive idempotents of $\widehat{B}$ such that $e_{m,i}\widehat{B} = (e_iB)_m \oplus (e_i\,\mathrm{D}(B))_m$ for $m \in \mathbb{Z}$ and $1 \le i \le n$. By an automorphism of $\widehat{B}$ we mean a k-algebra automorphism of $\widehat{B}$ which fixes the set $\widehat{\mathcal{E}}$. A group G of automorphisms of $\widehat{B}$ is said to be *admissible* if the induced action of G on $\widehat{\mathcal{E}}$ is free and has finitely many orbits. Then the *orbit algebra* $\widehat{B}/G$ is a finite-dimensional selfinjective algebra (see [11], [13]) and the G-orbits in $\widehat{\mathcal{E}}$ form a canonical set of orthogonal primitive idempotents of $\widehat{B}/G$ whose sum is the identity of $\widehat{B}/G$. We also denote by $\nu_{\widehat{B}}$ the *Nakayama automorphism* of $\widehat{B}$ defined by $\nu_{\widehat{B}}(e_{m,i}) = e_{m+1,i}$ for all $m \in \mathbb{Z}$, $1 \le i \le n$. Then the infinite cyclic group $(\nu_{\widehat{B}})$ generated by $\nu_{\widehat{B}}$ is admissible and $\widehat{B}/(\nu_{\widehat{B}})$ is the trivial extension $B \ltimes \mathrm{D}(B)$ of B by $\mathrm{D}(B)$. An automorphism φ of $\widehat{B}$ is said to be *positive* (respectively, *rigid*) if $\varphi(B_m) \subseteq \sum_{j\ge m} B_j$ (respectively, $\varphi(B_m) = B_m$) for any $m \in \mathbb{Z}$. Finally, φ is said to be *strictly positive* if φ is positive but not rigid.

We refer to [29] and [30] for criteria for a selfinjective algebra to be an orbit algebra $\widehat{B}/G$ with G an infinite cyclic group generated by a strictly positive automorphism of $\widehat{B}$.

2. Proof of the Theorem. Let A be a representation-infinite selfinjective algebra such that the component quiver Σ_A of A contains no short cycles. Then the Auslander–Reiten quiver Γ_A of A consists of modules which do not lie on infinite short cycles, and all components in Γ_A are generalized standard.

Given a module M in $\operatorname{mod} A$, we denote by $[M]$ the image of M in the Grothendieck group $K_0(A)$ of A. Thus $[M] = [N]$ if and only if the modules M and N have the same composition factors including multiplicities. We also mention that, by a result proved in [19], every indecomposable module M in

$\operatorname{mod} A$ which does not lie on a short cycle is uniquely determined by $[M]$ (up to isomorphism). In addition, recall that, following [26], a family $\mathcal{C} = (\mathcal{C}_i)_{i \in I}$ of components of Γ_A is said to have *common composition factors* if, for each pair i, j in I, there are modules $X_i \in \mathcal{C}_i$ and $X_j \in \mathcal{C}_j$ with $[X_i] = [X_j]$. Moreover, $\mathcal{C}$ is *closed under composition factors* if, for any indecomposable modules M and N in $\operatorname{mod} A$ with $[M] = [N]$, $M \in \mathcal{C}$ forces $N \in \mathcal{C}$.

PROPOSITION 2.1. *Let A be a representation-infinite selfinjective algebra such that the component quiver Σ_A of A contains no short cycles. Then the Auslander–Reiten quiver Γ_A of A admits a family $\mathcal{C} = (\mathcal{C})_{\lambda \in \mathbb{P}_1(k)}$ of quasi-tubes having common composition factors, closed under composition factors and consisting of modules which do not lie on infinite short cycles in* $\operatorname{mod} A$.

Proof. From the validity of the second Brauer–Thrall conjecture (see [6], [8]), for an algebraically closed field k, we know that there exists an infinite sequence of numbers $d_i \in \mathbb{N}$ such that, for each i, there exists an infinite number of non-isomorphic indecomposable modules with k-dimension d_i. Moreover, it was proved in [9, Corollary E] that if an algebra A is tame, then, for any dimension d, all but a finite number of isomorphism classes of indecomposable A-modules of dimension d lie in stable tubes of rank 1. Therefore, there is at least one stable tube $\mathcal{T}$ in Γ_A. Then, by [27, Corollary 1.3], there is an idempotent e of A such that $B = A/AeA$ is tame concealed or tubular and $\mathcal{T}$ is a faithful stable tube of Γ_B. In addition, from [26], there exists a family $\mathcal{T}^B = (\mathcal{T}^B_\lambda)_{\lambda \in \mathbb{P}_1(k)}$ of stable tubes in Γ_B with common composition factors. Hence, there is a family $\mathcal{C}^A = (\mathcal{C}^A_\lambda)_{\lambda \in \mathbb{P}_1(k)}$ of quasi-tubes in Γ_A such that $\mathcal{T}^B \subseteq \mathcal{C}^A$ and $\mathcal{T}^B_\lambda = \mathcal{C}^A_\lambda$ for almost all $\lambda \in \mathbb{P}_1(k)$ (see [15, Section 2]). Obviously, because $\mathcal{T}^B$ is a family of stable tubes in Γ_B with common composition factors, $\mathcal{C}^A$ is a family of quasi-tubes with common composition factors. We claim that $\mathcal{C}^A$ is closed under composition factors.

Let N be a module in Γ_A and M a module in $\mathcal{C}^A = (\mathcal{C}^A_\lambda)_{\lambda \in \mathbb{P}_1(k)}$. Assume that $[M] = [N]$. We will show that N belongs to $\mathcal{C}^A$. Let $\mathcal{C}^A_\lambda$, for some $\lambda \in \mathbb{P}_1(k)$, be a quasi-tube, in the family $\mathcal{C}^A$, containing M. There are pairwise orthogonal idempotents e and f of A such that $A = eA \oplus fA$ and the simple summands of $eA/e(\operatorname{rad} A)$ are exactly the simple composition factors of modules in $\mathcal{C}^A_\lambda$. Consider the quotient algebra $A' = A/AfA$. Then $\mathcal{C}^A_\lambda$ is a component in $\Gamma_{A'}$. Moreover, the A-module N is also a module over A'. Further, because A is tame, so is A'.

Finally, since $\mathcal{C}^A_\lambda$ is a generalized standard quasi-tube without external short paths, applying [16, Theorem A] we conclude that A' is a quasi-tube enlargement of a concealed canonical algebra C and there is a separating family $\mathcal{C}^{A'}$ of quasi-tubes containing the quasi-tube $\mathcal{C}^A_\lambda$. In particular, we have a decomposition $\Gamma_{A'} = \mathcal{P}^{A'} \vee \mathcal{C}^{A'} \vee \mathcal{Q}^{A'}$. Therefore, by dual arguments, we may assume that N belongs to $\mathcal{P}^{A'} \vee \mathcal{C}^{A'}$. By [16, Theorem C], there is a unique factor algebra A'_l of A' which is a tilted algebra of Euclidean type or a tubular algebra having a separating family $\mathcal{T}^{A'_l}$ of coray tubes (see [22, XV.2]) such that $\Gamma_{A'_l} = \mathcal{P}^{A'_l} \vee \mathcal{T}^{A'_l} \vee \mathcal{Q}^{A'_l}$ and $\mathcal{P}^{A'} = \mathcal{P}^{A'_l}$ consists of all proper predecessors of $\mathcal{C}^{A'}$ in $\operatorname{ind} A'$, that is, of those indecomposable modules X such that $\operatorname{Hom}_A(X, \mathcal{C}^{A'}) \neq 0$ and X does not belong to $\mathcal{C}^{A'}$. We have two cases to consider.

Assume that A'_l is a tilted algebra of Euclidean type. Then $\mathcal{P}^{A'_l}$ is a postprojective component and hence all modules from $\mathcal{P}^{A'} = \mathcal{P}^{A'_l}$ are uniquely determined by their composition factors, because they do not lie on a short cycle. Therefore, N belongs to $\mathcal{C}^{A'}$ in $\Gamma_{A'}$. Thus N is a module from the family $\mathcal{C}^A$ in Γ_A.

Assume that A'_l is a tubular algebra. Then $\mathcal{P}^{A'_l}$ consists of all indecomposable modules which precede the family $\mathcal{T}_\infty^{A'_l}$ of coray tubes of $\Gamma_{A'_l}$ and $\mathcal{T}_\infty^{A'_l} \subseteq \mathcal{C}^{A'}$. Moreover, because $[N] = [M]$ and N is an A'_l-module, M belongs to the family $\mathcal{T}_\infty^{A'_l}$. Therefore, by [20, (5.2)], we conclude that N belongs to $\mathcal{T}_\infty^{A'_l}$. Thus N is a module from the family $\mathcal{C}^A$ in Γ_A.

Summing up, the family $\mathcal{C}^A$ consists of quasi-tubes having common composition factors, is closed under composition factors and, from our assumptions on Σ_A, consists of modules which do not lie on infinite short cycles. ■

It now follows from Lemma 2.1 and [15, Theorem 1.1] that the algebra A is of the form $\widehat{B}/(\varphi\nu_{\widehat{B}}^2)$, where B is an almost concealed canonical algebra. Moreover, since B is a tame algebra, it is either a tilted algebra of Euclidean type or a tubular algebra. Thus, in order to prove the Theorem, it remains to show that φ is a strictly positive automorphism of $\widehat{B}$. This will be a consequence of Propositions 2.3 and 2.4.

We need the following general result which is a consequence of results proved in [1], [10], [11], [17] and [23].

THEOREM 2.2. *Let B be a quasi-tilted algebra of canonical type, G an admissible torsion-free group of automorphisms of $\widehat{B}$, and $A = \widehat{B}/G$ the associated orbit algebra. Then:*

(i) *G is an infinite cyclic group generated by a strictly positive automorphism ψ of $\widehat{B}$.*
(ii) *The push-down functor $F_\lambda : \operatorname{mod} \widehat{B} \to \operatorname{mod} A$ associated to the Galois covering $F : \widehat{B} \to \widehat{B}/G = A$ with Galois group G is dense.*
(iii) *The Auslander–Reiten quiver Γ_A is isomorphic to the orbit quiver $\Gamma_{\widehat{B}}/G$ with respect to the induced action of G on $\Gamma_{\widehat{B}}$.*

PROPOSITION 2.3. *Let B be a tubular algebra, G an infinite cyclic admissible group of automorphisms of $\widehat{B}$, and $A = \widehat{B}/G$. Then the following statements are equivalent:*

(i) *The component quiver Σ_A has no short cycles.*
(ii) *$G = (\varphi\nu_{\widehat{B}}^2)$ for a strictly positive automorphism φ of $\widehat{B}$.*

Proof. It follows from the results established in [12], [17], [23] (see also [7]) that the Auslander–Reiten quiver $\Gamma_{\widehat{B}}$ has a decomposition

$$\Gamma_{\widehat{B}} = \bigvee_{q\in\mathbb{Q}} \mathcal{C}_q^{\widehat{B}} = \bigvee_{q\in\mathbb{Q}} \bigvee_{\lambda\in\mathbb{P}_1(k)} \mathcal{C}_{q,\lambda}^{\widehat{B}}$$

such that:

(1) For each $q \in \mathbb{Z}$, $\mathcal{C}_q^{\widehat{B}}$ is an infinite family $\mathcal{C}_{q,\lambda}^{\widehat{B}}$, $\lambda \in \mathbb{P}_1(k)$, of quasi-tubes containing at least one projective module.
(2) For each $q \in \mathbb{Q}\setminus\mathbb{Z}$, $\mathcal{C}_q^{\widehat{B}}$ is an infinite family $\mathcal{C}_{q,\lambda}^{\widehat{B}}$, $\lambda \in \mathbb{P}_1(k)$, of stable tubes.
(3) For each $q \in \mathbb{Q}$, $\mathcal{C}_q^{\widehat{B}}$ is a family of pairwise orthogonal generalized standard quasi-tubes with common composition factors, closed under composition factors, and consisting of modules which do not lie on infinite short cycles in $\operatorname{mod}\widehat{B}$.
(4) There is a positive integer m such that $3 \le m \le \operatorname{rk} K_0(B)$ and $\nu_{\widehat{B}}(\mathcal{C}_q^{\widehat{B}}) = \mathcal{C}_{q+m}^{\widehat{B}}$ for any $q \in \mathbb{Q}$.
(5) $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_q^{\widehat{B}}, \mathcal{C}_r^{\widehat{B}}) = 0$ for all $q > r$ in $\mathbb{Q}$.
(6) $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_q^{\widehat{B}}, \mathcal{C}_r^{\widehat{B}}) = 0$ for all $r > q + m$ in $\mathbb{Q}$.
(7) For $q \in \mathbb{Q}$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_q^{\widehat{B}}, \mathcal{C}_{q+m}^{\widehat{B}}) \neq 0$ if and only if $q \in \mathbb{Z}$.
(8) For $p < q$ in $\mathbb{Q}$ with $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_p^{\widehat{B}}, \mathcal{C}_q^{\widehat{B}}) \neq 0$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_p^{\widehat{B}}, \mathcal{C}_r^{\widehat{B}}) \neq 0$ and $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_r^{\widehat{B}}, \mathcal{C}_q^{\widehat{B}}) \neq 0$ for any $r \in \mathbb{Q}$ with $p \le r \le q$.
(9) For all $p \in \mathbb{Q}\setminus\mathbb{Z}$ and all $q \in \mathbb{Q}$ with $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_p^{\widehat{B}}, \mathcal{C}_q^{\widehat{B}}) \neq 0$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_{p,\lambda}^{\widehat{B}}, \mathcal{C}_{q,\mu}^{\widehat{B}}) \neq 0$ for all $\lambda, \mu \in \mathbb{P}_1(k)$.
(10) For all $p \in \mathbb{Q}$ and all $q \in \mathbb{Q}\setminus\mathbb{Z}$ with $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_p^{\widehat{B}}, \mathcal{C}_q^{\widehat{B}}) \neq 0$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_{p,\lambda}^{\widehat{B}}, \mathcal{C}_{q,\mu}^{\widehat{B}}) \neq 0$ for all $\lambda, \mu \in \mathbb{P}_1(k)$.

We also know from [23] that G is generated by a strictly positive automorphism g of $\widehat{B}$. Consider the canonical Galois covering $F : \widehat{B} \to \widehat{B}/G = A$ and the associated push-down functor $F_\lambda : \operatorname{mod}\widehat{B} \to \operatorname{mod} A$. Since F_λ is dense, we obtain natural isomorphisms of k-modules

$$\bigoplus_{i\in\mathbb{Z}} \operatorname{Hom}_{\widehat{B}}(X, {}^{g^i}Y) \xrightarrow{\sim} \operatorname{Hom}_A(F_\lambda(X), F_\lambda(Y)),$$

$$\bigoplus_{i\in\mathbb{Z}} \operatorname{Hom}_{\widehat{B}}({}^{g^i}X, Y) \xrightarrow{\sim} \operatorname{Hom}_A(F_\lambda(X), F_\lambda(Y)),$$

for all indecomposable modules X and Y in $\operatorname{mod} \widehat{B}$.

We first show that (ii) implies (i). Assume that $g = \varphi\nu_{\widehat{B}}^2$ for some strictly positive automorphism φ of $\widehat{B}$. Then it follows from (4) that there is a positive integer $l > 2m$ such that $g(\mathcal{C}_q^{\widehat{B}}) = \mathcal{C}_{q+l}^{\widehat{B}}$ for any $q \in \mathbb{Q}$. Since $g = \varphi\nu_{\widehat{B}}^2 = (\varphi\nu_{\widehat{B}})\nu_{\widehat{B}}$ with $\varphi\nu_{\widehat{B}}$ a strictly positive automorphism of $\widehat{B}$, invoking the knowledge of the supports of indecomposable modules in $\operatorname{mod} \widehat{B}$ (see [17, Section 3]), we conclude that the images $F_\lambda(S)$ and $F_\lambda(T)$ of any non-isomorphic simple $\widehat{B}$-modules S and T which occur as composition factors of modules in a fixed family $\mathcal{C}_q^{\widehat{B}}$ are non-isomorphic simple A-modules. Therefore, it follows from Theorem 2.2 and properties (1)–(4) that, for each $q \in \mathbb{Q}$, $\mathcal{C}_q^A = F_\lambda(\mathcal{C}_q^{\widehat{B}})$ is an infinite family $\mathcal{C}_{q,\lambda}^A = F_\lambda(\mathcal{C}_{q,\lambda}^{\widehat{B}})$, $\lambda \in \mathbb{P}_1(k)$, of quasi-tubes of Γ_A with common composition factors and closed under composition factors. Take now $p \in \mathbb{Q}$. We claim that $\mathcal{C}_{p,\lambda}^A$, for any $\lambda \in \mathbb{P}_1(k)$, is a quasi-tube without external short paths in $\operatorname{mod} A$. Observe first that, for two indecomposable modules M and L in $\mathcal{C}_p^A$, we have $M = F_\lambda(X)$ and $L = F_\lambda(Z)$ for some indecomposable modules X and Z in $\mathcal{C}_p^{\widehat{B}}$, and F_λ induces an isomorphism of k-vector spaces $\operatorname{Hom}_A(M, L) \xrightarrow{\sim} \operatorname{Hom}_{\widehat{B}}(X, Z)$, by (5), (6) and the inequalities $q + l > q + 2m > q + m$.

Suppose now that there is an external short path $M \to L \to N$ in $\operatorname{mod} A$ with M and N in $\mathcal{C}_{p,\lambda}^A$ for some $\lambda \in \mathbb{P}_1(k)$ and L not in $\mathcal{C}_{p,\lambda}^A$. If L lies in $\mathcal{C}_p^A$, then $0 \neq \operatorname{rad}_A^\infty(M, L) \cong \operatorname{Hom}_A(M, L) \xrightarrow{\sim} \operatorname{Hom}_{\widehat{B}}(X, Z)$ contradicts (3). Therefore, $M = F_\lambda(X)$, $N = F_\lambda(Y)$ for some X and Y in $\mathcal{C}_{p,\lambda}^{\widehat{B}}$ and $L = F_\lambda(Z)$ for some Z in $\mathcal{C}_r^{\widehat{B}}$ with $r > p$. We have an isomorphism of k-vector spaces, induced by F_λ,

$$\operatorname{Hom}_A(M, L) \xrightarrow{\sim} \bigoplus_{i\in\mathbb{Z}} \operatorname{Hom}_{\widehat{B}}(X, {}^{g^i}Z).$$

Since $\operatorname{Hom}_A(M, L) \neq 0$, we may choose, invoking (5), a minimal $r > p$ and $Z \in \mathcal{C}_r^{\widehat{B}}$ such that $L = F_\lambda(Z)$ and $\operatorname{Hom}_{\widehat{B}}(X, Z) \neq 0$. Since $p \in \mathbb{Z}$ and X lies in $\mathcal{C}_p^{\widehat{B}}$, applying (6) and (7) we infer that $p < r \leq p + m$. Further, we also have an isomorphism of k-vector spaces, induced by F_λ,

$$\operatorname{Hom}_A(L, N) \xrightarrow{\sim} \bigoplus_{i\in\mathbb{Z}} \operatorname{Hom}_{\widehat{B}}(Z, {}^{g^i}Y).$$

Observe that, for each $i \in \mathbb{Z}$, ${}^{g^i}Y$ is an indecomposable module from $\mathcal{C}_{p+li}^{\widehat{B}}$, and clearly $F_\lambda({}^{g^i}Y) = F_\lambda(Y) = N$. Since $\operatorname{Hom}_A(L, N) \neq 0$, $L = F_\lambda(Z)$

for $Z \in \mathcal{C}_r^{\widehat{B}}$ with $r > p$, and $Y \in \mathcal{C}_p^{\widehat{B}}$, applying (5) we conclude that $\operatorname{Hom}_{\widehat{B}}(Z, {}^{g^i}Y) \neq 0$ for some $i \geq 1$. But then $p + li \geq p + l > p + 2m > p + m \geq r > p$, because $r \leq p + m$, and we obtain a contradiction with (6).

Summing up, we have proved that all quasi-tubes in Γ_A are generalized standard and consist of modules which do not lie on external short paths in $\operatorname{mod} A$. Thus, the component quiver Σ_A of A has no short cycles.

Therefore, (ii) implies (i).

We will now show that (i) implies (ii). Assume that the component quiver Σ_A has no short cycles. Then, by Proposition 2.1, Γ_A admits a family $\mathcal{C} = (\mathcal{C}_\lambda)_{\lambda \in \mathbb{P}_1(k)}$ of quasi-tubes with common composition factors, closed under composition factors and consisting of modules which do not lie on infinite short cycles in $\operatorname{mod} A$. We know from property (3) that, for each $q \in \mathbb{Q}$, $\mathcal{C}_q^A = F_\lambda(\mathcal{C}_q^{\widehat{B}})$ is a family $\mathcal{C}_{q,\lambda}^A = F_\lambda(\mathcal{C}_{q,\lambda}^{\widehat{B}})$, $\lambda \in \mathbb{P}_1(k)$, of quasi-tubes with common composition factors. Moreover, the push-down functor F_λ induces an isomorphism of translation quivers $\Gamma_{\widehat{B}}/G \xrightarrow{\sim} \Gamma_A$ (see Theorem 2.2), and hence every component of Γ_A is a quasi-tube of the form $\mathcal{C}_{q,\lambda}^A = F_\lambda(\mathcal{C}_{q,\lambda}^{\widehat{B}})$ for some $q \in \mathbb{Q}$ and $\lambda \in \mathbb{P}_1(k)$. Then, since $\mathcal{C}$ is closed under composition factors, we conclude that there is $r \in \mathbb{Q}$ such that $\mathcal{C}$ contains all quasi-tubes $\mathcal{C}_{r,\lambda}^A$, $\lambda \in \mathbb{P}_1(k)$, of $\mathcal{C}_r^A$. This forces, by [15, Proposition 6.4], g to be of the form $g = \varphi\nu_{\widehat{B}}^2$ for some positive automorphism φ of $\widehat{B}$.

Suppose that φ is a rigid automorphism of $\widehat{B}$. Then, from [23, Lemma 3.5], we know that the restriction of φ to B fixes a projective module, say P. Let P be a corresponding projective-injective $\widehat{B}$-module and $\mathcal{C}_{p,\lambda}$, for some $p \in \mathbb{Z}$ and $\lambda \in \mathbb{P}_1(k)$, the quasi-tube containing P. Without loss of generality, we may assume that $p = 0$. We have a short cycle of modules in $\operatorname{mod} \widehat{B}$ of the form $P \xrightarrow{f} \nu_{\widehat{B}}(P) \xrightarrow{g} \nu_{\widehat{B}}^2(P)$, where f and g are the compositions

$$P \to \operatorname{top}(P) \cong \operatorname{soc}(\nu_{\widehat{B}}(P)) \to \nu_{\widehat{B}}(P)$$

and

$$\nu_{\widehat{B}}(P) \to \operatorname{top}(\nu_{\widehat{B}}(P)) \cong \operatorname{soc}(\nu_{\widehat{B}}^2(P)) \to \nu_{\widehat{B}}^2(P),$$

respectively. Consequently, we obtain a short cycle in Σ_A, because

$$\operatorname{rad}_A^\infty(F_\lambda(\mathcal{C}_{0,\lambda}), F_\lambda(\mathcal{C}_{m,\mu})) \neq 0$$

and

$$\operatorname{rad}_A^\infty(F_\lambda(\mathcal{C}_{m,\mu}), F_\lambda(\mathcal{C}_{2m,\lambda})) = \operatorname{rad}_A^\infty(F_\lambda(\mathcal{C}_{m,\mu}), F_\lambda(\mathcal{C}_{0,\lambda})) \neq 0,$$

where $\nu_{\widehat{B}}(P) \in \mathcal{C}_{m,\mu}$ for some $\mu \in \mathbb{P}_1(k)$, which contradicts (i). ■

PROPOSITION 2.4. *Let B be a tilted algebra of Euclidean type, G an infinite cyclic admissible group of automorphisms of $\widehat{B}$, and $A = \widehat{B}/G$. Then the following statements are equivalent:*

(i) *The component quiver Σ_A has no short cycle.*
(ii) $G = (\varphi\nu_{\widehat{B}}^2)$ *for a strictly positive automorphism φ of $\widehat{B}$.*

Proof. It follows from [1], [3] and [23] that the Auslander–Reiten quiver $\Gamma_{\widehat{B}}$ has a decomposition

$$\Gamma_{\widehat{B}} = \bigvee_{q\in\mathbb{Z}} (\mathcal{C}_q^{\widehat{B}} \vee \mathcal{X}_q^{\widehat{B}})$$

such that:

(1) For each $q \in \mathbb{Z}$, $\mathcal{C}_q^{\widehat{B}}$ is an infinite family $\mathcal{C}_{q,\lambda}^{\widehat{B}}$, $\lambda \in \mathbb{P}_1(k)$, of quasi-tubes.
(2) For each $q \in \mathbb{Z}$, $\mathcal{X}_q^{\widehat{B}}$ is an acyclic component of Euclidean type.
(3) For each $q \in \mathbb{Z}$, $\mathcal{C}_q^{\widehat{B}}$ is a family $\mathcal{C}_{q,\lambda}^{\widehat{B}}$, $\lambda \in \mathbb{P}_1(k)$, of pairwise orthogonal generalized standard quasi-tubes with common composition factors, closed under composition factors, and consisting of modules which do not lie on infinite short cycles in $\operatorname{mod}\widehat{B}$.
(4) For each $q \in \mathbb{Z}$, we have $\nu_{\widehat{B}}(\mathcal{C}_q^{\widehat{B}}) = \mathcal{C}_{q+2}^{\widehat{B}}$ and $\nu_{\widehat{B}}(\mathcal{X}_q^{\widehat{B}}) = \mathcal{X}_{q+2}^{\widehat{B}}$.
(5) For each $q \in \mathbb{Z}$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{X}_q^{\widehat{B}}, \mathcal{C}_q^{\widehat{B}} \vee \bigvee_{r<q}(\mathcal{C}_r^{\widehat{B}} \vee \mathcal{X}_r^{\widehat{B}})) = 0$ and $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_q^{\widehat{B}}, \bigvee_{r<q}(\mathcal{C}_r^{\widehat{B}} \vee \mathcal{X}_r^{\widehat{B}})) = 0$.
(6) For each $q \in \mathbb{Z}$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_q^{\widehat{B}}, \mathcal{X}_{q+2}^{\widehat{B}} \vee \bigvee_{r>q+2}(\mathcal{C}_r^{\widehat{B}} \vee \mathcal{X}_r^{\widehat{B}})) = 0$ and $\operatorname{Hom}_{\widehat{B}}(\mathcal{X}_q^{\widehat{B}}, \bigvee_{r>q+2}(\mathcal{C}_r^{\widehat{B}} \vee \mathcal{X}_r^{\widehat{B}})) = 0$.
(7) For $q \in \mathbb{Z}$ and $\lambda, \mu \in \mathbb{P}_1(k)$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_{q,\lambda}^{\widehat{B}}, \mathcal{C}_{q+2,\mu}^{\widehat{B}}) \neq 0$ if and only if the quasi-tube $\mathcal{C}_{q,\lambda}^{\widehat{B}}$ is non-stable and $\nu_{\widehat{B}}(\mathcal{C}_{q,\mu}^{\widehat{B}}) = \mathcal{C}_{q+2,\lambda}^{\widehat{B}}$.
(8) For all $q \in \mathbb{Z}$ and $\lambda, \mu \in \mathbb{P}_1(k)$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_{q,\lambda}^{\widehat{B}}, \mathcal{C}_{q+1,\mu}^{\widehat{B}}) \neq 0$.
(9) For each $r \in \mathbb{Z}$, $\mathcal{X}_r$ contains at least one projective module.

We also know from [1], [3] and [23] that G is generated by a strictly positive automorphism g of $\widehat{B}$. Hence there exists a positive integer l such that $g(\mathcal{C}_q^{\widehat{B}}) = \mathcal{C}_{q+l}^{\widehat{B}}$ and $g(\mathcal{X}_q^{\widehat{B}}) = \mathcal{X}_{q+l}^{\widehat{B}}$ for any $q \in \mathbb{Z}$. Consider the canonical Galois covering $F : \widehat{B} \to \widehat{B}/G = A$ and the associated push-down functor $F_\lambda : \operatorname{mod}\widehat{B} \to \operatorname{mod} A$. Since F_λ is dense, we obtain natural isomorphisms of k-vector spaces

$$\bigoplus_{i\in\mathbb{Z}} \operatorname{Hom}_{\widehat{B}}(X, {}^{g^i}Y) \xrightarrow{\sim} \operatorname{Hom}_A(F_\lambda(X), F_\lambda(Y)),$$

$$\bigoplus_{i\in\mathbb{Z}} \operatorname{Hom}_{\widehat{B}}({}^{g^i}X, Y) \xrightarrow{\sim} \operatorname{Hom}_A(F_\lambda(X), F_\lambda(Y)),$$

for all indecomposable modules X and Y in $\operatorname{mod}\widehat{B}$.

We show first that (i) implies (ii). Assume that the component quiver Σ_A has no short cycles. Then, by Proposition 2.1, Γ_A admits a family $\mathcal{C} =$

$(\mathcal{C}_\lambda)_{\lambda\in\mathbb{P}_1(k)}$ of quasi-tubes with common composition factors, closed under composition factors and consisting of modules which do not lie on infinite short cycles in $\operatorname{mod} A$. It follows from [15, Proposition 6.5] that $g = \varphi\nu_{\widehat{B}}^2$ for some positive automorphism φ of $\widehat{B}$. We claim that φ is a strictly positive automorphism.

Assume that φ is a rigid automorphism of $\widehat{B}$. Take $q = 0$ and, invoking (9), some projective-injective module $P = e\widehat{B}$ in $\mathcal{X}_0$. Let f, g be the compositions

$$P \to \operatorname{top}(P) \cong \operatorname{soc}(\nu_{\widehat{B}}(P)) \to \nu_{\widehat{B}}(P)$$

and

$$\nu_{\widehat{B}}(P) \to \operatorname{top}(P) \cong \operatorname{soc}(\nu_{\widehat{B}}^2(P)) \to \nu_{\widehat{B}}^2(P),$$

respectively. Then we have a short path of indecomposable modules

$$P \xrightarrow{f} \nu_{\widehat{B}}(P) \xrightarrow{g} \nu_{\widehat{B}}^2(P)$$

in $\widehat{B}$, where, by (4), $P \in \mathcal{X}_0$, $\nu_{\widehat{B}}(P) \in \mathcal{X}_2$ and $\nu_{\widehat{B}}^2(P) \in \mathcal{X}_4$. Thus, it follows from Theorem 2.2 that we have a short path of indecomposable modules $F_\lambda(P) \to F_\lambda(\nu_{\widehat{B}}(P)) \to F_\lambda(\nu_{\widehat{B}}^2(P))$ in $\operatorname{mod} A$. Because φ is a rigid automorphism of $\widehat{B}$ we conclude that $F_\lambda(P)$ and $F_\lambda(\nu_{\widehat{B}}^2(P))$ belong to the same component $F_\lambda(\mathcal{X}_0)$. Obviously, $\operatorname{rad}_A^\infty(F_\lambda(P), F_\lambda(\nu_{\widehat{B}}(P))) \neq 0$ and $\operatorname{rad}_A^\infty(F_\lambda(\nu_{\widehat{B}}(P)), F_\lambda(\nu_{\widehat{B}}^2(P))) \neq 0$. Therefore, the component quiver Σ_A contains a short cycle $F_\lambda(\mathcal{X}_0) \to F_\lambda(\mathcal{X}_2) \to F_\lambda(\mathcal{X}_0)$, and we get a contradiction.

This finishes the proof that (i) implies (ii).

Assume now that (ii) holds. In particular, $g = \varphi\nu_{\widehat{B}}^2$ for a strictly positive automorphism φ of $\widehat{B}$. Then it follows from (4) that there is a positive integer $l > 4$ such that $g(\mathcal{C}_q^{\widehat{B}}) = \mathcal{C}_{q+l}^{\widehat{B}}$ and $g(\mathcal{X}_q^{\widehat{B}}) = \mathcal{X}_{q+l}^{\widehat{B}}$ for any $q \in \mathbb{Z}$. By (3) and Theorem 2.2, to show that Σ_A has no short cycles, we must show that no component in Γ_A has external short paths. Assume that there is a component $\mathcal{C}$ in Γ_A and an external short path $M \to N \to L$ with M and L in $\mathcal{C}$ but N not in $\mathcal{C}$. By Theorem 2.2, there are indecomposable modules X, Y and Z in $\operatorname{mod}\widehat{B}$ such that $M = F_\lambda(X)$, $N = F_\lambda(Y)$ and $L = F_\lambda(Z)$. Moreover, X belongs to $\mathcal{C}_{p,\lambda}$ for some $p \in \mathbb{Z}$ and $\lambda \in \mathbb{P}_1(k)$, or to $\mathcal{X}_p$ for some $p \in \mathbb{Z}$. Then $\mathcal{C} = F_\lambda(\mathcal{C}_{p,\lambda})$ or $\mathcal{C} = F_\lambda(\mathcal{X}_p)$. Without loss of generality, we may assume that $p = 0$. Therefore, we have two cases to consider.

Assume that $X \in \mathcal{C}_{0,\lambda}$. We have an isomorphism of k-vector spaces, induced by F_λ,

$$\operatorname{Hom}_A(M, N) \xrightarrow{\sim} \bigoplus_{i\in\mathbb{Z}} \operatorname{Hom}_{\widehat{B}}(X, {}^{g^i}Y).$$

Since $\mathrm{Hom}_A(M,N) \neq 0$, invoking (6), we conclude that Y belongs to

$$\mathcal{X}_0 \vee \mathcal{C}_1 \vee \mathcal{X}_1 \vee \mathcal{C}_2.$$

We also have an isomorphism of k-vector spaces

$$\mathrm{Hom}_A(N,L) \xrightarrow{\sim} \bigoplus_{i\in\mathbb{Z}} \mathrm{Hom}_{\widehat{B}}(Y, {}^{g^i}Z).$$

Again, since $\mathrm{Hom}_A(N,L) \neq 0$, we conclude from (6) that Z belongs to

$$\mathcal{C}_2 \vee \mathcal{C}_3 \vee \mathcal{C}_4.$$

On the other hand, by (5) and our assumption on φ, we find that $Z \in \mathcal{C}_{l,\lambda}$ for some $l > 4$, a contradiction.

Assume that $X \in \mathcal{X}_0$. We have an isomorphism of k-vector spaces, induced by F_λ,

$$\mathrm{Hom}_A(M,N) \xrightarrow{\sim} \bigoplus_{i\in\mathbb{Z}} \mathrm{Hom}_{\widehat{B}}(X, {}^{g^i}Y).$$

Since $\mathrm{Hom}_A(M,N) \neq 0$, invoking (6), we infer that Y belongs to

$$\mathcal{C}_1 \vee \mathcal{X}_1 \vee \mathcal{C}_2 \vee \mathcal{X}_2.$$

We also have an isomorphism of k-vector spaces

$$\mathrm{Hom}_A(N,L) \xrightarrow{\sim} \bigoplus_{i\in\mathbb{Z}} \mathrm{Hom}_{\widehat{B}}(Y, {}^{g^i}Z).$$

Again, since $\mathrm{Hom}_A(N,L) \neq 0$, we conclude from (6) that Z belongs to

$$\mathcal{X}_1 \vee \mathcal{X}_2 \vee \mathcal{X}_3 \vee \mathcal{X}_4.$$

On the other hand, by (5) and our assumption on φ, we infer that $Z \in \mathcal{X}_l$ for some $l > 4$, a contradiction. ∎

Acknowledgements. The author gratefully acknowledges support from Research Grant No. N N201 263 135 of Polish Ministry of Science and Higher Education.

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Received 27 January 2011;
revised 8 April 2011 (5464)