COLLOQUIUM MATHEMATICUM

VOL. 87 2001 NO. 2

DERIVED TUBULAR ALGEBRAS AND APR-TILTS

BY

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Abstract. We show that each derived tubular algebra can be transformed by a finite sequence of Auslander–Platzeck–Reiten tilts and Auslander–Platzeck–Reiten cotilts to a canonical algebra.

1. Introduction. APR-tilting modules are probably the simplest ones in tilting theory. They were introduced by Auslander, Platzeck and Reiten in 1979 [2] and generalize the Coxeter functors for hereditary algebras.

In nice cases the iteration of APR-tilts (and APR-cotilts) leads to well known algebras. So it was shown by Happel that if $\vec{\Delta}$ is a Dynkin quiver (resp. an Euclidean quiver) and A an algebra derived equivalent to the path algebra $k\vec{\Delta}$, then A may be transformed by a sequence of APR-tilts (resp. a sequence of APR-tilts followed by a sequence of APR-cotilts) to $k\vec{\Delta}$ (see [8]). The main result of this article is the following:

THEOREM 1.1. *Let A be a finite-dimensional algebra over an algebraically closed field. Assume that A is derived equivalent to a tubular algebra. Then there is a sequence of algebras $A = A_0, A_1, \ldots, A_m$ and a sequence of modules $T^{(i)}_{A_i}$, where $0 \le i \le m$, such that each $T^{(i)}_{A_i}$ is an APR-tilting A_i-module, or an APR-cotilting A_i-module*, $\mathrm{End}(T^{(i)}_{A_i}) = A_{i+1}$ *and A_m is a canonical algebra* (*of tubular type*).

Observe that a similar result is not true, in general, for an algebra which is derived equivalent to a canonical algebra Λ of domestic type. Namely, in this case there is a tame hereditary algebra H which is derived equivalent to Λ. Obviously, from H we cannot reach Λ because any APR-tilt (resp. APR-cotilt) again yields a hereditary algebra.

We will also prove that a branch enlargement in the sense of Assem and Skowroński [1] of a concealed canonical algebra of arbitrary representation type can be transformed by APR-tilts and APR-cotilts to a concealed canonical algebra.

2000 *Mathematics Subject Classification*: Primary 16G70; Secondary 16G20, 14H60.

2. Preliminaries

2.1. Throughout this paper k will denote an algebraically closed field. By an *algebra* A we mean an associative, finite-dimensional k-algebra with identity. Let Q be the ordinary quiver of A and Q_0 the set of points of Q. By an A-*module* we mean a finite-dimensional right A-module. We will use freely facts about the category $\mathrm{mod}(A)$ of A-modules and the Auslander–Reiten translation τ_A. In particular, for any point $a \in Q_0$ we have a projective (resp. injective) A-module, which we will denote by $P(a)$ (resp. $I(a)$).

For an abelian category $\mathcal{A}$ we denote by $\mathcal{D}^{\mathrm{b}}(\mathcal{A})$ the derived category of bounded complexes over $\mathcal{A}$ and by $X \mapsto X[1]$ the translation functor of $\mathcal{D}^{\mathrm{b}}(\mathcal{A})$. Two algebras A and B are said to be *derived equivalent* if there is an equivalence of the triangulated categories $\mathcal{D}^{\mathrm{b}}(\mathrm{mod}(A)) \to \mathcal{D}^{\mathrm{b}}(\mathrm{mod}(B))$.

2.2. An A-module T is called a *tilting module* (resp. *cotilting module*) if the projective (resp. injective) dimension of T is ≤ 1, $\mathrm{Ext}^1_A(T,T) = 0$ and the number of isomorphism classes of its indecomposable direct summands equals the number of vertices of the quiver of A. It is well known that for a tilting module T in $\mathrm{mod}(A)$ the categories of modules over A and $B = \mathrm{End}(T)$ are closely related in the sense that some nice subcategories are equivalent [3], [9] and moreover, the algebras A and B are derived equivalent [7].

Let $P(i)$ be a simple projective A-module which is not injective. Then it is easy to see that the module $\tau_A^{-1}P(i) \oplus (\bigoplus_{j\neq i} P(j))$ is a tilting module, it is called an APR-*tilting module.* APR-cotilting modules are defined dually.

2.3. Tubular and canonical algebras, which are of wide interest in representation theory, were introduced by Ringel; for a definition we refer to [16]. We say that an algebra A is *derived tubular* (resp. *derived canonical*) if it is derived equivalent to a tubular (resp. canonical) algebra.

We will need the concept of weighted projective lines in the sense of Geigle and Lenzing. Roughly speaking a weighted projective line $\mathbb{X}$ is the projective line with a finite number $\lambda_1, \dots, \lambda_t$ of marked points with attached weights $p_1, \dots, p_t$ which are integers ≥ 2 (for details we refer to [5]). In particular we will use the fact that the category $\mathrm{coh}(\mathbb{X})$ of coherent sheaves on $\mathbb{X}$ is an abelian, hereditary and noetherian category with finite-dimensional morphism and extension spaces. Moreover $\mathrm{coh}(\mathbb{X})$ satisfies Serre duality in the form $\mathrm{D}\mathrm{Ext}^1_{\mathbb{X}}(X,Y) \cong \mathrm{Hom}_{\mathbb{X}}(Y, \tau_{\mathbb{X}}X)$, where $\tau_{\mathbb{X}}$ is the Auslander–Reiten translation in $\mathrm{coh}(\mathbb{X})$ and $\mathrm{D} = \mathrm{Hom}_k(-,k)$ is the standard duality. Moreover, the Auslander–Reiten translation is given by shift with the dualizing element $\vec{\omega}$. Each indecomposable coherent sheaf is either a vector bundle or a sheaf of finite length. We denote by $\mathrm{vect}(\mathbb{X})$ the category of vector bundles and by $\mathrm{coh}_0(\mathbb{X})$ the category of finite length sheaves on $\mathbb{X}$.

Observe that each indecomposable object of the derived category $\mathcal{D}^{\mathrm{b}}(\mathrm{coh}(\mathbb{X}))$ has the form $X[n]$ for an indecomposable object of $\mathrm{coh}(\mathbb{X})$, because $\mathrm{coh}(\mathbb{X})$ is hereditary. We will further use the notions of rank and slope for coherent sheaves. Note that if an indecomposable sheaf F is in $\mathrm{vect}(\mathbb{X})$ (resp. $\mathrm{coh}_0(\mathbb{X})$, $\mathrm{vect}(\mathbb{X})[1]$) then $\mathrm{rk}(F) > 0$ (resp. $\mathrm{rk}(F) = 0$, $\mathrm{rk}(F) < 0$).

One of the basic results of [5] is that the category $\mathrm{coh}(\mathbb{X})$ has a tilting sheaf such that its endomorphism algebra is the corresponding canonical algebra Λ and that this gives rise to an equivalence $\mathcal{D}^{\mathrm{b}}(\mathrm{coh}(\mathbb{X})) \to \mathcal{D}^{\mathrm{b}}(\mathrm{mod}(\Lambda))$.

For a weighted projective line $\mathbb{X}$ the *virtual genus* $g_{\mathbb{X}}$ of $\mathbb{X}$ is defined by

$$g_{\mathbb{X}} = 1 + \frac{1}{2}\Big((t-2)p - \sum_{i=1}^{t} p/p_i\Big), \quad p = \mathrm{l.c.m.}(p_1, \ldots, p_t).$$

Recall that $\mathbb{X}$ is called of *domestic* (resp. *tubular*, *wild*) *weight type* if $g_{\mathbb{X}} < 1$ (resp. $g_{\mathbb{X}} = 1$, $g_{\mathbb{X}} > 1$). It is well known that the tubular weight types are—up to permutation—the following: $(2,2,2,2)$, $(3,3,3)$, $(2,4,4)$, $(2,3,6)$. It was shown in [12, Prop. 3.6] that an algebra Σ is tubular if and only if it is an endomorphism algebra of a tilting sheaf (equivalently a tilting bundle) on a weighted projective line of tubular type.

We use the concept of perpendicular categories [6]. Recall that for a system of objects $\mathcal{S}$ in an abelian category $\mathcal{A}$ the *right perpendicular category* $\mathcal{S}^{\perp}$ is defined as the full subcategory of $\mathcal{A}$ consisting of all objects $A \in \mathcal{A}$ satisfying $\mathrm{Hom}(S, A) = 0$ and $\mathrm{Ext}^1(S, A) = 0$ for all $S \in \mathcal{S}$. Now, if S is a simple exceptional finite length sheaf on a weighted projective line $\mathbb{X}$, then the right perpendicular category $S^{\perp}$, formed in $\mathrm{coh}(\mathbb{X})$, is equivalent to a category of coherent sheaves $\mathrm{coh}(\mathbb{Y})$ on a weighted projective line $\mathbb{Y}$ of smaller weight type. Moreover, the embedding $S^{\perp} = \mathrm{coh}(\mathbb{Y}) \hookrightarrow \mathrm{coh}(\mathbb{X})$ is rank preserving.

2.4. We call an object T in $\mathcal{D}^{\mathrm{b}}(\mathrm{coh}(\mathbb{X}))$ a *tilting complex* if

$$\mathrm{Hom}_{\mathcal{D}^{\mathrm{b}}(\mathrm{coh}(\mathbb{X}))}(T, T[i]) = 0 \quad \text{ for } i \neq 0$$

and the indecomposable direct summands of T generate $\mathcal{D}^{\mathrm{b}}(\mathrm{coh}(\mathbb{X}))$ as a triangulated category. Let T be a tilting complex in $\mathcal{D}^{\mathrm{b}}(\mathrm{coh}(\mathbb{X}))$ with decomposition into indecomposable objects $T = \bigoplus T_i[n_i]$ where $T_i \in \mathrm{coh}(\mathbb{X})$ and denote $\Sigma = \mathrm{End}(T)$. As in the situation for algebras [15], a tilting complex yields an equivalence $\mathcal{D}^{\mathrm{b}}(\mathrm{coh}(\mathbb{X})) \overset{\cong}{\to} \mathcal{D}^{\mathrm{b}}(\mathrm{mod}(\Sigma))$. It is clear that under this equivalence the indecomposable projective Σ-modules correspond to the objects $T_i[n_i]$ of the complex and we will identify them further on. Further, it is easy to verify that the indecomposable injective Σ-modules correspond to the objects $\tau_{\mathbb{X}} T_i[n_i + 1]$ (see [12, Lemma 5.3], [13, 8.3.4]). An algebra which is given as the endomorphism algebra of a tilting bundle is called a *concealed canonical algebra* [12].

3. From derived tubular algebras to tubular algebras

3.1. In this section we will show that each derived tubular algebra can be transformed by APR-tilts and APR-cotilts to a tubular algebra.

The following fact is independent of the representation type of the category $\mathrm{coh}(\mathbb{X})$, in particular, it applies to the tubular case.

PROPOSITION 3.1. *Each derived canonical algebra can be transformed by a sequence of APR-tilts to a representation-infinite derived canonical algebra.*

This result was proved by Happel in [8, IV, 7.4] for algebras derived equivalent to hereditary algebras which are not of Dynkin type. One easily checks that the proof carries over to our situation, because the category $\mathrm{coh}(\mathbb{X})$ is hereditary, too.

3.2. The next result, which deals with branch enlargements (in the sense of [1]) of endomorphism algebras of tilting bundles is also valid for weighted projective lines of arbitrary type.

PROPOSITION 3.2. *Each branch enlargement of a concealed canonical algebra can be transformed by a sequence of APR-tilts followed by a sequence of APR-cotilts to a concealed canonical algebra.*

Proof. Let Σ a branch enlargement of a concealed canonical algebra. By [14], Σ can be regarded as the endomorphism ring of a tilting complex of the form

$$T = T_{-n}[-n] \oplus \ldots \oplus T_{-1}[-1] \oplus T_{+} \oplus T_0 \oplus T_1[1] \oplus \ldots \oplus T_m[m]$$

with $T_{+} \in \mathrm{vect}(\mathbb{X})$ and $T_i \in \mathrm{coh}_0(\mathbb{X})$ for all $-n \le i \le m$.

Applying APR-tilts and APR-cotilts we will reduce the number of indecomposable finite length sheaves belonging to the extension and coextension branches. Each extension branch is derived equivalent to a path algebra of a linear quiver of type $\mathbb{A}_n$. Therefore, by [8, IV, 6.2] we can obtain, after a finite sequence of APR-tilts, an algebra for which the extension root a of the extension branch considered is a source. Then the APR-tilt with respect to the simple projective module corresponding to a has the form

$$0 \to P(a) \to P' \oplus P'' \to R \to 0$$

where P' is in $\mathrm{vect}(\mathbb{X})$ and P'' is zero or an indecomposable projective corresponding to a point belonging to the branch. Now, $P(a) \in \mathrm{coh}_0(\mathbb{X})[-1]$, in particular, $\mathrm{rk}(P(a)) = 0$. Furthermore we have $\mathrm{rk}(P') > 0$ and $\mathrm{rk}(P'') = 0$, therefore $\mathrm{rk}(R) > 0$ because the rank function is additive on exact sequences. Since R is indecomposable it follows that it is an object of $\mathrm{vect}(\mathbb{X})$. Thus replacing $P(a)$ by R we get an algebra with a smaller number of points in the extension branch considered.

Applying this method to each extension branch and, dually, to each coextension branch we obtain an algebra which is an endomorphism algebra of a vector bundle, which finishes the proof. ∎

3.3. Now assume that $\mathbb{X}$ is of tubular weight type. It was shown by Assem and Skowroński [1, 2.5] that an algebra Σ is representation-infinite derived tubular if and only if Σ is obtained from a concealed-canonical algebra by branch enlargement. As a consequence of the Propositions 3.1 and 3.2 we obtain

COROLLARY 3.3. *Each derived tubular algebra can be transformed by a sequence of APR-tilts followed by a sequence of APR-cotilts to a tubular algebra.* ∎

4. From tubular to canonical algebras

4.1. The following lemma and proposition are due to Lenzing (unpublished). We will use a stronger version of Proposition 4.2(ii) in order to give a uniform proof of our main theorem (avoiding a case by case inspection).

LEMMA 4.1. *Let T be a tilting bundle on a weighted projective line $\mathbb{X}$ of arbitrary type. Then each line bundle L on $\mathbb{X}$ either satisfies* $\mathrm{Ext}^1_{\mathbb{X}}(T, L) = 0$ *or* $\mathrm{Hom}_{\mathbb{X}}(T, L) = 0$.

Proof. Assume L is a line bundle with $\mathrm{Ext}^1_{\mathbb{X}}(T, L) \neq 0$ and $\mathrm{Hom}_{\mathbb{X}}(T, L) \neq 0$. Because T is a vector bundle the first assumption yields a monomorphism $f : L \to T(\vec{\omega})$, the second one a non-zero morphism $g : T \to L$. Now the composition fg defines a non-zero morphism $T \to T(\vec{\omega})$, contradicting $\mathrm{Ext}^1_{\mathbb{X}}(T, T) = 0$. ∎

4.2. We have the following proposition:

PROPOSITION 4.2. (i) *Assume that $\mathbb{X}$ is a weighted projective line of domestic type. Then each tilting sheaf on $\mathbb{X}$ contains a direct summand that is a line bundle.*

(ii) *Each tilting sheaf T on a weighted projective line of tubular type $(p_1, \ldots, p_t)$ has an indecomposable direct summand whose τ-period equals $p = \mathrm{l.c.m.}(p_1, \ldots, p_t)$.*

Proof. (i) By perpendicular calculus we can first reduce to the situation that T is a bundle. Indeed, let T be a tilting sheaf and assume that $T = T_+ \oplus T_0$ with $T_+ \in \mathrm{vect}(\mathbb{X})$ and $T_0 \in \mathrm{coh}_0(\mathbb{X})$ with $T_0 \neq 0$. Then T_+ is contained in the right perpendicular category with respect to all simple composition factors of T_0, which is equivalent to a sheaf category $\mathrm{coh}(\mathbb{Y})$. Then $\mathbb{Y}$ is again of domestic type and, moreover, we can consider T_+ as a tilting bundle on $\mathbb{Y}$.

Thus assume that T is a tilting bundle on $\mathbb{X}$. We choose a line bundle L satisfying $\mathrm{Ext}^1_{\mathbb{X}}(T, L) = 0$ and $\mathrm{Ext}^1_{\mathbb{X}}(T, L(\vec{\omega})) \neq 0$. This is possible since the slopes of (indecomposable) bundles satisfying the first condition are bounded from below (by the minimal slope of an indecomposable direct factor from T) and, moreover, the shift by the dualizing element decreases the slope by the constant $|\delta(\vec{\omega})|$ (see [5]).

We show that in addition to $\mathrm{Ext}^1_{\mathbb{X}}(T, L) = 0$ also $\mathrm{Ext}^1_{\mathbb{X}}(L, T) = 0$ holds, which in turn implies that L is isomorphic to a direct summand of T. Indeed, we infer from Lemma 4.1 that $\mathrm{Hom}_{\mathbb{X}}(T, L(\vec{\omega})) = 0$, hence the claim follows. Thus L is a direct summand of T.

(ii) Applying, if necessary, a telescopic functor [12], [13], we can assume that $T = T_+ \oplus T_0$ with vector bundle part T_+ and non-zero finite length part T_0. Hence T_+ is realizable as a tilting bundle on a weighted projective line of domestic type. From (i) we conclude that T_+ contains a line bundle L. Then L, considered as a line bundle on the tubular curve, has the desired τ-period. ∎

4.3. Now we will change a tilting sheaf on a tubular weighted projective line in such a way that an object of maximal τ-period is at the "beginning" or "end" of the tilting complex. Of course this is relevant only in the cases $(2, 4, 4)$ and $(2, 3, 6)$.

PROPOSITION 4.3. *Each tubular algebra can be transformed by a sequence of APR-tilts and APR-cotilts to an algebra which is isomorphic to the endomorphism algebra of a tilting sheaf $T = T_+ \oplus S$ or of a tilting complex of the form $S[-1] \oplus T_+$, where T_+ is a vector bundle and S an indecomposable finite length sheaf of τ-period p.*

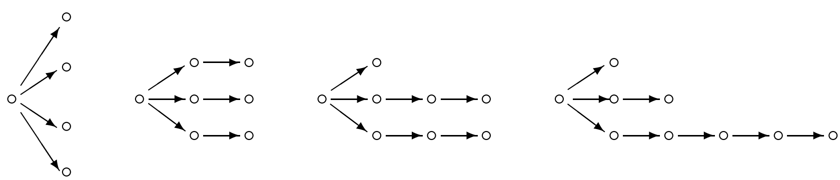
Proof. Let Σ be a tubular algebra, realized by a tilting sheaf T. Applying if necessary a telescopic functor, we can assume that $T = T_+ \oplus T_0$ with $T_+ \in \mathrm{vect}(\mathbb{X})$, $T_0 \in \mathrm{coh}_0(\mathbb{X})$ and $T_0 \neq 0$. We can further assume that T_0 is an indecomposable finite length sheaf S. Indeed, the objects from T_0 form truncated branches [16, 4.4], thus applying if necessary APR-cotilts (as in 3.2) we can replace all but one of the finite length sheaves by vector bundles.

We know from 4.2 that T contains an indecomposable direct summand L of τ-period p. Of course we are done if $L = S$. Furthermore, by dual arguments, the proof again is finished if the slope of L is minimal among the slopes of indecomposable direct summands of T.

Therefore we can assume that $T = T_+ \oplus S$, with $T_+ \in \mathrm{vect}(\mathbb{X})$, S indecomposable in $\mathrm{coh}_0(\mathbb{X})$, there is a direct summand $L \neq S$ of T_+ of τ-period p and there is an indecomposable direct summand of T_+ with slope smaller slope than the slope of L.

In this situation we consider the right perpendicular category $S^{\perp}$ which is equivalent to a sheaf category $\mathrm{coh}(\mathbb{Y})$ of domestic weight type. Since $T_+ \in S^{\perp}$ and the embedding $S^{\perp} = \mathrm{coh}(\mathbb{Y}) \hookrightarrow \mathrm{coh}(\mathbb{X})$ is rank preserving, the indecomposables of T_+ belong to the unique vector bundle component for $\mathbb{Y}$ which is of the form $\mathbb{Z}\Delta$ with Δ of Euclidean type. We denote by Σ_+ the endomorphism algebra of T_+. Using the same method as in the proof of 3.2 we can replace, by applying APR-tilts, bundles of minimal slope by bundles of bigger slope. Note that here APR-tilts with respect to Σ_+ are also APR-tilts with respect to Σ. Moreover, the bundles appearing in this way are again contained in $\mathrm{vect}(\mathbb{Y})$. Thus after finitely many steps we obtain the situation that L is of minimal slope. Reducing if necessary the number of objects of slope $\mu(L)$ and applying a telescoping functor we get a tilting complex of the form $S'[-1]\oplus T'_+$ with $S' \in \mathrm{coh}_0(\mathbb{X})$ and $T'_+ \in \mathrm{vect}(\mathbb{X})$, which finishes the proof. ∎

4.4. We now prove Theorem 1.1. Using the results of the previous section we can start with a tubular algebra Σ. Furthermore, by Proposition 4.3 we can, up to duality, assume that Σ is realized by a tilting sheaf $T = T_+ \oplus S$ where $T_+ \in \mathrm{vect}(\mathbb{X})$, $S \in \mathrm{coh}_0(\mathbb{X})$ and S is indecomposable of τ-period p. In this case the algebra $\Sigma_+ = \mathrm{End}(T_+)$ is a tame concealed algebra. Now, a result of Happel [8, IV, 7.7] states that each tame concealed algebra of type $\vec{\Delta}$ can be transformed by APR-tilts to a path algebra $H = k\vec{\Delta}$. Thus in our situation we can reach one of the path algebras H given by the following quivers:

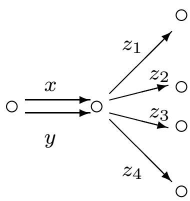


depending on the weight type $(2,2,2,2)$, $(3,3,3)$, $(2,4,4,)$ and $(2,3,6)$ of $\mathbb{X}$ respectively. Note again that APR-tilts with respect to Σ_+ are also APR-tilts with respect to Σ. Now, Σ is a one-point extension [16] by a module M which has to be simple regular [10, III, 3.9]. Further, denoting $S^\perp = \mathrm{coh}(\mathbb{Y})$ the weight type of $\mathbb{Y}$ is $(2,2,2)$, $(2,3,3)$, $(2,3,4)$, $(2,3,5)$ respectively. This means that M, considered as an H-module, belongs to a tube of rank 1, 2, 3, 5, respectively. By [4] these tubes contain the module N of dimension vector

$$\begin{bmatrix} & 1 \\ & 1 \\ 2 & \\ & 1 \\ & 1 \end{bmatrix}, \begin{bmatrix} & 1 & 1 \\ 2 & 1 & 1 \\ & 1 & 1 \end{bmatrix}, \begin{bmatrix} & 1 & & \\ 2 & 1 & 1 & 1 \\ & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} & 1 & & & & \\ 2 & 1 & 1 & & & \\ & 1 & 1 & 1 & 1 & 1 \end{bmatrix},$$

whose one-point extension $H[N]$ is the canonical algebra [10, III.4]. In case $M \cong N$ we are done. On the other hand, If $N \cong \tau_H^j M$ we can apply a sequence of APR-tilts in order to move from H to $\tau_H^{-j}H$ in the preprojective component of $\mathrm{mod}(H)$. Then $(\tau_H^{-j}H)[\tau_H^{j}M] \cong H[N]$ is again a canonical algebra. ■

4.5. The following example shows that it is not possible in general to transform a tubular algebra to a canonical algebra by using only APR-tilts. Let A be a squid algebra of type $(2,2,2,2)$, i.e. $A = k\vec{Q}$ where $\vec{Q}$ is the quiver



and I is the ideal generated by $z_i(x - \lambda_i y)$, for pairwise different parameters $\lambda_1, \ldots \lambda_4$. Then there is only one APR-tilting A-module T and $B = \mathrm{End}(T)$ is isomorphic to A.

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Received 3 March 2000;
revised 14 April 2000 (3895)