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THE EULER AND HELMHOLTZ OPERATORS ON FIBERED MANIFOLDS WITH ORIENTED BASES

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Abstract. We study naturality of the Euler and Helmholtz operators arising in the variational calculus in fibered manifolds with oriented bases.

Given two fibered manifolds $Z_1 \to M$ and $Z_2 \to M$ over the same base M, we denote the space of all base preserving fibered manifold morphisms of Z_1 into Z_2 by $\mathcal{C}^\infty_M(Z_1, Z_2)$.

In [1], I. Kolář studied the Euler operator

$$E : \mathcal{C}^\infty_M(J^sY, \bigwedge^m T^*M) \to \mathcal{C}^\infty_Y(J^{2s}Y, V^*Y \otimes \bigwedge^m T^*M)$$

for fibered manifolds $p : Y \to M$. He deduced that all natural operators of this type are of the form cE, $c \in \mathbb{R}$, provided m is sufficiently large.

In [3], Kolář and Vitolo studied the Helmholtz operator

$$H : \mathcal{C}^\infty_Y(J^sY, V^*Y \otimes \bigwedge^m T^*M) \to \mathcal{C}^\infty_{J^sY}(J^{2s}Y, V^*J^sY \otimes V^*Y \otimes \bigwedge^m T^*M)$$

for fibered manifolds $p : Y \to M$. They deduced that all natural operators of this type are of the form cH, $c \in \mathbb{R}$, provided $s = 1, 2$. In [4], we extended this result to all s.

In the present paper, for a fibered manifold $p : Y \to M$ with oriented basis, we study the naturality of the Euler operator

$$\widetilde{E} : \mathcal{V}ol^+(M) \times \mathcal{C}^\infty(J^sY, \mathbb{R}) \to \mathcal{C}^\infty_Y(J^{2s}Y, V^*Y)$$

given by $\widetilde{E}(\eta, \lambda) \otimes \eta = E(\lambda \otimes \eta)$ for any $\eta \in \mathcal{V}ol^+(M)$ and $\lambda \in \mathcal{C}^\infty(J^sY, \mathbb{R})$, where $\mathcal{V}ol^+(M)$ is the set of all positive volume forms on M.

We also study, for fibered manifolds $p : Y \to M$ with oriented bases, the naturality of the Helmholtz operator

$$\widetilde{H} : \mathcal{V}ol^+(M) \times \mathcal{C}^\infty_Y(J^sY, V^*Y) \to \mathcal{C}^\infty_{J^sY}(J^{2s}Y, V^*J^sY \otimes V^*Y)$$

defined from H just as $\widetilde{E}$ from E.

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The first main result of the present paper is

THEOREM 1. *Let m, n, s be natural numbers. Any $\mathcal{FM}^+_{m,n}$-natural π^{2s}_s-local, regular operator*

$$D : \mathcal{V}ol^+(M) \times \mathcal{C}^\infty(J^sY, \mathbb{R}) \to \mathcal{C}^\infty_Y(J^{2s}Y, V^*Y),$$

$\mathbb{R}$-linear in the second factor and homogeneous of weight 0 in the first factor, is of the form $D = c\widetilde{E}$, $c \in \mathbb{R}$.

REMARK 1. $\mathcal{FM}^+_{m,n}$ denotes the category of all (m, n)-dimensional fibered manifolds with oriented bases and their fibered embeddings covering orientation preserving embeddings. The $\mathcal{FM}^+_{m,n}$-naturality of D means that for any $\mathcal{FM}^+_{m,n}$-map $f : Y_1 \to Y_2$, any Lagrangians $\lambda_1 \in \mathcal{C}^\infty(J^sY_1, \mathbb{R})$ and $\lambda_2 \in \mathcal{C}^\infty(J^sY_2, \mathbb{R})$ and any positive volume forms $\eta_1 \in \mathcal{V}ol^+(M_1)$ and $\eta_2 \in \mathcal{V}ol^+(M_2)$ if λ_1 and λ_2 are f-related and η_1 and η_2 are f-related, then $D(\eta_1, \lambda_1)$ and $D(\eta_2, \lambda_2)$ are f-related. The regularity means that D transforms smoothly parametrized families of Lagrangians and volume forms into smoothly parametrized families of respective morphisms. The locality means that $D(\eta, \lambda)_u$ depends on $\mathrm{germ}_{\pi^{2s}_s(u)}(\lambda)$ and $\mathrm{germ}_x(\eta)$ for any $u \in J^{2s}_xY$, $x \in M$, where $\pi^{2s}_s : J^{2s}Y \to J^sY$ is the jet projection. The linearity in the second factor means that $D(\eta, \lambda)$ depends $\mathbb{R}$-linearly on $\lambda \in \mathcal{C}^\infty(J^sY, \mathbb{R})$ for any fixed $\eta \in \mathcal{V}ol^+(M)$. The homogeneity of weight 0 in the first factor means that $D(t\eta, \lambda) = D(\eta, \lambda)$ for $t > 0$.

REMARK 2. Theorem 1 without the linearity assumption does not hold. For, let $h : \mathbb{R} \to \mathbb{R}$ be a non-constant function. Then the operator $\widetilde{E}^{[h]}(\eta, \lambda) = (h \circ \lambda \circ \pi^{2s}_s)\widetilde{E}(\eta, \lambda)$ is not linear in the second factor.

REMARK 3. If $C : \mathcal{C}^\infty_M(J^sY, \bigwedge^m T^*M) \to \mathcal{C}^\infty_Y(J^{2s}Y, V^*Y \otimes \bigwedge^m T^*M)$ is a natural $\mathbb{R}$-linear operator, then (similarly to $\widetilde{E}$) one can define the corresponding natural operator $\widetilde{C} : \mathcal{V}ol^+(M) \times \mathcal{C}^\infty(J^sY, \mathbb{R}) \to \mathcal{C}^\infty_Y(J^{2s}Y, V^*Y)$, $\mathbb{R}$-linear in the second factor and homogeneous of weight zero in the first factor. Using Theorem 1, we see that $\widetilde{C} = c\widetilde{E}$, and we recover the above mentioned result of [1] in the case of $\mathbb{R}$-linear operators. The inverse construction of C from $\widetilde{C}$ is impossible because we have no canonical surjection $\mathcal{C}^\infty_M(J^sY, \bigwedge^m T^*M) \to \mathcal{V}ol^+(M) \times \mathcal{C}^\infty(J^sY, \mathbb{R})$. So, Theorem 1 is not a consequence of the result of [1].

The second main result of the present paper is

THEOREM 2. *Let m, n, s be natural numbers. Any $\mathcal{FM}^+_{m,n}$-natural, π^{2s}_s-local, regular operator*

$$D : \mathcal{V}ol^+(M) \times \mathcal{C}^\infty_Y(J^sY, V^*Y) \to \mathcal{C}^\infty_{J^sY}(J^{2s}Y, V^*J^sY \otimes V^*Y),$$

$\mathbb{R}$-linear in the second factor and homogeneous with weight 0 in the first factor, is of the form $c\widetilde{H}$, $c \in \mathbb{R}$.

REMARK 4. Theorem 2 without the assumption of linearity does not hold. For, we have a natural operator $\widetilde{H}^0$ non-linear in the second factor given by $\langle \widetilde{H}^0(\eta, B)_{j^{2s}_x\sigma}, v \otimes w\rangle = \langle B_{j^s_x\sigma}, T\pi^s_0(v)\rangle\langle B_{j^s_x\sigma}, w\rangle$ for $j^{2s}_x\sigma \in J^{2s}Y$, $x \in M$, $v \in V_{j^s_x\sigma}J^sY$, $w \in V_{\sigma(x)}Y$.

Proof of Theorem 1. From now on $\mathbb{R}^{m,n}$ is the trivial bundle $\mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^m$ and $x^1, \ldots, x^m$, $y^1, \ldots, y^n$ are the usual coordinates on $\mathbb{R}^{m,n}$.

Let D be an operator in question.

Since an $\mathcal{FM}^+_{m,n}$-map $(x, y - \sigma(x))$ sends $j^{2s}_0(\sigma)$ to $\Theta = j^{2s}_0(0) \in J^{2s}_0(\mathbb{R}^m, \mathbb{R}^n) = J^{2s}_0(\mathbb{R}^{m,n})$, $J^{2s}(\mathbb{R}^{m,n})$ is the $\mathcal{FM}^+_{m,n}$-orbit of Θ. Therefore D is uniquely determined by the evaluations

$$\langle D(\eta, \lambda)_\Theta, v\rangle \in \mathbb{R}$$

for all $\lambda \in \mathcal{C}^\infty(J^s(\mathbb{R}^{m,n}), \mathbb{R})$, $\eta \in \mathcal{V}ol^+(\mathbb{R}^m)$ and $v \in T_0\mathbb{R}^n = V_{(0,0)}\mathbb{R}^{m,n}$.

Using the invariance of D with respect to $\mathcal{FM}^+_{m,n}$-morphisms of the form $\mathrm{id}_{\mathbb{R}^m} \times \psi$ for linear ψ we see that D is uniquely determined by the evaluations

$$\left\langle D(\eta, \lambda)_\Theta, \frac{\partial}{\partial y^1}_0 \right\rangle \in \mathbb{R}$$

for all $\lambda \in \mathcal{C}^\infty(J^s(\mathbb{R}^{m,n}), \mathbb{R})$ and $\eta \in \mathcal{V}ol^+(\mathbb{R}^m)$.

Consider an arbitrary positive volume form $\eta = f(x^1, \ldots, x^m)dx^1 \wedge \cdots \wedge dx^m$ on $\mathbb{R}^m$. There is a map $F : \mathbb{R}^m \to \mathbb{R}^m$ such that $\frac{\partial}{\partial x^1}F = f$ and $F(0) = 0$. Then the locally defined $\mathcal{FM}^+_{m,n}$-map $(F, x^2, \ldots, x^m, y^1, \ldots, y^n)^{-1}$ preserves Θ, $\frac{\partial}{\partial y^1}_0$ and sends $\mathrm{germ}_0(d^m x)$ into $\mathrm{germ}_0(\eta)$, where $d^m x = dx^1 \wedge \cdots \wedge dx^m$. Then by naturality D is uniquely determined by the evaluations

$$\left\langle D(d^m x, \lambda)_\Theta, \frac{\partial}{\partial y^1}_0 \right\rangle \in \mathbb{R}$$

for all $\lambda \in \mathcal{C}^\infty(J^s(\mathbb{R}^{m,n}), \mathbb{R})$.

By the $\mathbb{R}$-linearity in the second factor of D and by Corollary 19.8 in [1] we see that D is determined by the values

$$\left\langle D(d^m x, x^\beta M(y^j_\alpha))_\Theta, \frac{\partial}{\partial y^1}_0 \right\rangle, \tag{1}$$

where (x^i, y^j_α) is the induced coordinate system on $J^s(\mathbb{R}^{m,n})$ and M is an arbitrary monomial in the y^j_α's. (Here and below, α and β are arbitrary m-tuples with $|\alpha| \le s$ and $j = 1, \ldots, n$.)

Now, using the invariance of D with respect to the $\mathcal{FM}^+_{m,n}$-maps

$$(x^1, \ldots, x^m, \tau^1 y^1, \ldots, \tau^n y^n)$$

for $\tau^j > 0$, we get the homogeneity condition which gives that (1) is zero if

$M(y^j_\alpha)$ is not of the form y^1_α. So, D is determined by the values

$$\left\langle D(d^m x, x^\beta y^1_\alpha)_\Theta, \frac{\partial}{\partial y^1}_0 \right\rangle$$

for α and β as above.

Next, using the invariance of D with respect to the $\mathcal{FM}^+_{m,n}$-maps

$$(x^1, \ldots, \tau^i x^i, \ldots, x^m, y^1, \ldots, y^n)$$

for $\tau^i > 0$ and using the $\mathbb{R}$-linearity in the second factor and the homogeneity of weight 0 in the first factor of D we get

$$\left\langle D(d^m x, x^\beta y^1_\alpha)_\Theta, \frac{\partial}{\partial y^1}_0 \right\rangle = 0 \tag{2}$$

if only $\beta_i - \alpha_i \neq 0$ for some $i = 1, \ldots, m$ (i.e. if $\alpha \neq \beta$).

Let $\alpha = (\alpha_1, \ldots, \alpha_m)$ be an m-tuple with $|\alpha| \leq s$.

Suppose $\alpha_{i_1} > 0$ for some $i_1 = 1, \ldots, m$.

The locally defined $\mathcal{FM}^+_{m,n}$-map $\psi = (x^1, \ldots, x^m, y^1 + x^{i_1} y^1 \ldots, y^n)^{-1}$ preserves $x^1, \ldots, x^m$, Θ and $\frac{\partial}{\partial y^1}_0$ and sends y^1_α to $y^1_\alpha + x^{i_1} y^1_\alpha + y^1_{\alpha - 1_{i_1}}$ (as $y^1_\alpha \circ J^s\psi^{-1}(j^s_{x_o}\sigma) = \partial_\alpha(\sigma^1 + x^{i_1}\sigma^1)(x_o) = \partial_\alpha \sigma^1(x_o) + x^{i_1}_o \partial_\alpha \sigma^1(x_o) + \partial_{\alpha - 1_{i_1}} \sigma^1(x_o) = (y^1_\alpha + x^{i_1} y^1_\alpha + y^1_{\alpha - 1_{i_1}})(j^s_{x_o}\sigma)$ for $j^s_{x_o}\sigma \in J^s\mathbb{R}^{m,n}$, where ∂_α is the iterated partial derivative with respect to the index α multiplied by $1/\alpha!$). Then using the invariance of D with respect to ψ, from

$$\left\langle D(d^m x, x^{\alpha - 1_1} y^1_\alpha)_\Theta, \frac{\partial}{\partial y^1}_0 \right\rangle = 0$$

(see (2)) we see that

$$\left\langle D(d^m x, x^\alpha y^1_\alpha)_\Theta, \frac{\partial}{\partial y^1}_0 \right\rangle = -\left\langle D(d^m x, x^{\alpha - 1_{i_1}} y^1_{\alpha - 1_{i_1}})_\Theta, \frac{\partial}{\partial y^1}_0 \right\rangle.$$

Continuing this process we see that

$$\left\langle D(d^m x, x^\alpha y^1_\alpha)_\Theta, \frac{\partial}{\partial y^1}_0 \right\rangle = (-1)^{|\alpha|}\left\langle D(d^m x, y^1_{(0)})_\Theta, \frac{\partial}{\partial y^1}_0 \right\rangle.$$

Summing up, D is determined by the value

$$\left\langle D(d^m x, y^1_{(0)})_\Theta, \frac{\partial}{\partial y^1}_0 \right\rangle \in \mathbb{R}.$$

Thus the vector space of all D in question is of dimension less than or equal to 1. Hence $D = c\widetilde{E}$ for some $c \in \mathbb{R}$. ∎

Proof of Theorem 2. Let D be an operator in question. Let Θ be as in the proof of Theorem 1.

As in that proof, D is uniquely determined by

$$\left\langle D(\eta, B)_\Theta, \frac{d}{dt}_0 (tj_0^s(g(x), 0, \dots, 0)) \otimes \frac{\partial}{\partial y^2}_0 \right\rangle \in \mathbb{R}$$

for all $B \in \mathcal{C}^\infty_{\mathbb{R}^{m,n}}(J^s(\mathbb{R}^{m,n}), V^*\mathbb{R}^{m,n})$, $\eta \in \mathcal{V}ol^+(\mathbb{R}^m)$ and $g : \mathbb{R}^m \to \mathbb{R}$.

Using the invariance of D with respect to the $\mathcal{FM}^+_{m,n}$-maps $(x^1, \dots, x^m, y^1 + g(x)y^1, y^2, \dots, y^n)$ preserving Θ we find that D is uniquely determined by

$$\left\langle D(\eta, B)_\Theta, \frac{d}{dt}_0 (tj_0^s(1, 0, \dots, 0)) \otimes \frac{\partial}{\partial y^2}_0 \right\rangle \in \mathbb{R}$$

for all $B \in \mathcal{C}^\infty_{\mathbb{R}^{m,n}}(J^s(\mathbb{R}^{m,n}), V^*\mathbb{R}^{m,n})$.

Then similarly to the proof of Theorem 1 (using $\mathcal{FM}^+_{m,n}$-naturality), D is uniquely determined by

$$\left\langle D(d^m x, B)_\Theta, \frac{d}{dt}_0 (tj_0^s(1, 0, \dots, 0)) \otimes \frac{\partial}{\partial y^2}_0 \right\rangle \in \mathbb{R}$$

for all B as above.

Let $B \in \mathcal{C}^\infty_{\mathbb{R}^{m,n}}(J^s(\mathbb{R}^{m,n}), V^*\mathbb{R}^{m,n})$. Using the invariance of D with respect to the $\mathcal{FM}^+_{m,n}$-maps $\psi_\tau = (x^1, \dots, x^m, (1/\tau^1)y^1, \dots, (1/\tau^n)y^n)$ for $\tau^j \neq 0$ we get the homogeneity condition

$$\left\langle D(d^m x, (\psi_\tau)_* B)_\Theta, \frac{d}{dt}_0 (tj_0^s(1, 0, \dots, 0)) \otimes \frac{\partial}{\partial y^2}_0 \right\rangle \\ = \tau^1 \tau^2 \left\langle D(d^m x, B)_\Theta, \frac{d}{dt}_0 (tj_0^s(1, 0, \dots, 0)) \otimes \frac{\partial}{\partial y^2}_0 \right\rangle$$

for $\tau = (\tau^1, \dots, \tau^n)$. Then by the second factor linearity of D and by Corollary 19.8 in [2] of the Peetre theorem,

$$\left\langle D(d^m x, B)_\Theta, \frac{d}{dt}_0 (tj_0^s(1, 0, \dots, 0)) \otimes \frac{\partial}{\partial y^2}_0 \right\rangle$$

is determined by the values

$$\left\langle D(d^m x, x^\beta y^2_\alpha dy^1)_\Theta, \frac{d}{dt}_0 (tj_0^s(1, 0, \dots, 0)) \otimes \frac{\partial}{\partial y^2}_0 \right\rangle,$$
$$\left\langle D(d^m x, x^\beta y^1_\alpha dy^2)_\Theta, \frac{d}{dt}_0 (tj_0^s(1, 0, \dots, 0)) \otimes \frac{\partial}{\partial y^2}_0 \right\rangle$$

for all m-tuples α and β with $|\alpha| \leq s$.

Then by the invariance of D with respect to the $\mathcal{FM}^+_{m,n}$-maps

$$(\tau^1 x^1, \dots, \tau^m x^m, y^1, \dots, y^n)$$

for $\tau^i > 0$ and the first factor 0-weight homogeneity of D we get

$$(3)\quad \left\langle D(d^m x, x^\beta y^2_\alpha dy^1)_\Theta, \frac{d}{dt}{}_0(tj^s_0(1,0,\dots,0)) \otimes \frac{\partial}{\partial y^2}{}_0 \right\rangle = \left\langle D(d^m x, x^\beta y^1_\alpha dy^2)_\Theta, \frac{d}{dt}{}_0(tj^s_0(1,0,\dots,0)) \otimes \frac{\partial}{\partial y^2}{}_0 \right\rangle = 0$$

if only $\beta \neq \alpha$.

Suppose $\alpha = (\alpha_1, \dots, \alpha_m)$ is an m-tuple with $|\alpha| \leq s$ and $\alpha_i \neq 0$ for some i. Then using the invariance of D with respect to the locally defined $\mathcal{FM}^+_{m,n}$-map $\psi = (x^1, \dots, x^m, y^1, y^2 + x^i y^2, \dots, y^n)^{-1}$ preserving $x^1, \dots, x^m$, y^1, Θ, $j^s_0(1,0,\dots,0)$ and $\frac{\partial}{\partial y^2}{}_0$ and sending y^2_α to $y^2_\alpha + x^i y^2_\alpha + y^2_{\alpha-1_i}$, from

$$\left\langle D(d^m x, x^{\alpha-1_i} y^2_\alpha dy^1)_\Theta, \frac{d}{dt}{}_0(tj^s_0(1,0,\dots,0)) \otimes \frac{\partial}{\partial y^2}{}_0 \right\rangle = 0$$

(see (3)) we deduce that

$$\left\langle D(d^m x, x^\alpha y^2_\alpha dy^1)_\Theta, \frac{d}{dt}{}_0(tj^s_0(1,0,\dots,0)) \otimes \frac{\partial}{\partial y^2}{}_0 \right\rangle = -\left\langle D(d^m x, x^{\alpha-1_i} y^2_{\alpha-1_i} dy^1)_\Theta, \frac{d}{dt}{}_0(tj^s_0(1,0,\dots,0)) \otimes \frac{\partial}{\partial y^2}{}_0 \right\rangle.$$

Then for any m-tuple α with $|\alpha| \leq s$ we have

$$\left\langle D(d^m x, x^\alpha y^2_\alpha dy^1)_\Theta, \frac{d}{dt}{}_0(tj^s_0(1,0,\dots,0)) \otimes \frac{\partial}{\partial y^2}{}_0 \right\rangle = (-1)^{|\alpha|}\left\langle D(d^m x, y^2_{(0)} dy^1)_\Theta, \frac{d}{dt}{}_0(tj^s_0(1,0,\dots,0)) \otimes \frac{\partial}{\partial y^2}{}_0 \right\rangle.$$

By the same arguments (since ψ sends dy_2 to $dy^2 + x^i dy^2$), from

$$\left\langle D(d^m x, x^{\alpha-1_i} y^1_\alpha dy^2)_\Theta, \frac{d}{dt}{}_0(tj^s_0(1,0,\dots,0)) \otimes \frac{\partial}{\partial y^2}{}_0 \right\rangle = 0$$

we obtain

$$\left\langle D(d^m x, x^\alpha y^1_\alpha dy^2)_\Theta, \frac{d}{dt}{}_0(tj^s_0(1,0,\dots,0)) \otimes \frac{\partial}{\partial y^2}{}_0 \right\rangle = 0$$

if $\alpha \neq (0)$.

Using the invariance of D with respect to the local $\mathcal{FM}^+_{m,n}$-map

$$(x^1, \dots, x^m, y^1 + y^1 y^2, \dots, y^n)^{-1}$$

preserving Θ, $j^s_0(1,0,\dots,0)$ and $\frac{\partial}{\partial y^2}{}_0$, from

$$\left\langle D(d^m x, dy^1)_\Theta, \frac{d}{dt}{}_0(tj^s_0(1,0,\dots,0)) \otimes \frac{\partial}{\partial y^2}{}_0 \right\rangle = 0$$

we deduce that

$$\left\langle D(d^m x, y^2_{(0)} dy^1)_\Theta, \frac{d}{dt}_0 (tj^s_0(1,0,\dots,0)) \otimes \frac{\partial}{\partial y^2}_0 \right\rangle = -\left\langle D(d^m x, y^1_{(0)} dy^2)_\Theta, \frac{d}{dt}_0 (tj^s_0(1,0,\dots,0)) \otimes \frac{\partial}{\partial y^2}_0 \right\rangle.$$

Thus D is uniquely determined by

$$\left\langle D(d^m x, y^2_{(0)} dy^1)_\Theta, \frac{d}{dt}_0 (tj^s_0(1,0,\dots,0)) \otimes \frac{\partial}{\partial y^2}_0 \right\rangle \in \mathbb{R}.$$

Therefore the vector space of all D in question is of dimension less than or equal to 1. Hence $D = c\widetilde{H}$ for some $c \in \mathbb{R}$. ∎

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