COLLOQUIUM MATHEMATICUM

VOL. 105 2006 NO. 2

STRONG MIXING MARKOV SEMIGROUPS ON $\mathcal{C}_1$ ARE MEAGER

BY

WOJCIECH BARTOSZEK and BEATA KUNA (Gdańsk)

Abstract. We show that the set of those Markov semigroups on the Schatten class $\mathcal{C}_1$ such that in the strong operator topology $\lim_{t\to\infty} T(t) = Q$, where Q is a one-dimensional projection, form a meager subset of all Markov semigroups.

An important problem in the theory of dynamical (open quantum) systems is the description of their asymptotic behaviour. Given a class of possible evolutions (scenario), what is the nature of a generic element of the class? Of course, in order to describe genericity, first we have to define the notion itself. In our case it is based on the category theorem of Baire. A set which is a countable intersection of dense and open sets (called *residual*) is commonly recognized as a large object. Generic evolutions are those which belong to a residual subset. We will use this concept to describe the size of specific classes of semigroups of positive operators on the Schatten class 1. This space plays a central role in the von Neumann model of quantum mechanics. It has recently been proved (see [5] and [15]) that for the uniform norm topology asymptotically stable semigroups are generic. In this article we consider related questions from the point of view of the strong operator topology. It turns out that here the situation is different. Generic are those semigroups which do not possess an absorbing state (hence are not stable).

Let $(\mathcal{H}, \langle\cdot,\cdot\rangle)$ be a separable (infinite-dimensional) complex Hilbert space. As usual the norm is denoted by $\|\cdot\|$ and the Banach algebra of all bounded linear operators on $(\mathcal{H}, \|\cdot\|)$ is denoted by $\mathcal{L}(\mathcal{H})$. Without confusion the operator norm in $\mathcal{L}(\mathcal{H})$ will also be denoted by $\|\cdot\|$. The paper is devoted to positive contraction semigroups of linear operators acting on the ordered Banach space of trace-class operators on $\mathcal{H}$. For all the basic facts the reader is referred to any standard book on operators on Hilbert spaces (for instance

2000 *Mathematics Subject Classification*: Primary 46L55, 47A35; Secondary 37A30, 37A55.

Key words and phrases: Schatten classes, Markov semigroup, mixing, first category.

The first author would like to express his gratitude for hospitality and support to Prof. J. Myjak, from the Department of Mathematics of L'Aquila University, where part of this work was done.

[6], [17]–[19] and [23]). Our notation and definitions come from [5]. For the convenience of the reader we recall some of them.

The compact operators on $\mathcal{H}$ are denoted by $\mathcal{C}_0$. They form a (closed) ideal in $\mathcal{L}(\mathcal{H})$. A compact operator $X \in \mathcal{L}(\mathcal{H})$ is *trace-class* if for each (or some; see [19] for the details) orthonormal basis $e_1, e_2, \ldots \in \mathcal{H}$ we have $\sum_{j=1}^{\infty} \langle |X| e_j, e_j \rangle < \infty$. The trace is defined as $\sum_{j=1}^{\infty} \langle X e_j, e_j \rangle$ and it is denoted by $\mathrm{tr}(X)$. Then the functional

$$X \mapsto \mathrm{tr}(|X|) = \|X\|_1$$

defines a norm (stronger than the operator norm; see [18], [19]). The trace-class operators form a two-sided ideal in $\mathcal{L}(\mathcal{H})$, called the *Schatten class* 1 (see [17]–[19] and [23]) and denoted by $\mathcal{C}_1$. The trace norm is complete on $\mathcal{C}_1$. It may be easily verified that because $\mathcal{H}$ is not finite-dimensional, $\mathcal{C}_1$ is not closed in the operator norm in $\mathcal{L}(\mathcal{H})$. It is well known (see [19]) that by means of the dual operation $\langle A, X \rangle = \mathrm{tr}(XA)$, where $A \in \mathcal{C}_0$ and $X \in \mathcal{C}_1$, the adjoint space to $(\mathcal{C}_0, \|\cdot\|)$ may be identified with $(\mathcal{C}_1, \|\cdot\|_1)$. Further, the dual space to $(\mathcal{C}_1, \|\cdot\|_1)$ is $(\mathcal{L}(\mathcal{H}), \|\cdot\|)$ (denoted in this context by $\mathcal{C}_\infty$) with the dual operation $\langle X, B \rangle = \mathrm{tr}(BX)$, where $B \in \mathcal{C}_\infty$ and $X \in \mathcal{C}_1$. In particular, $\mathcal{C}_1$ is not reflexive. The space $\mathcal{C}_1$ is commonly recognized as the noncommutative counterpart of the ℓ^1 space (however, it should be pointed out that $\mathcal{C}_1$ is neither a Riesz space nor a space with the Schur property). Since the operators of finite rank are norm dense in $\mathcal{C}_1$ and the Hilbert space $\mathcal{H}$ is separable (by our assumption), $\mathcal{C}_1$ is also separable. The norm $\|\cdot\|_1$ has the following additivity property (sometimes called (AL) condition when we deal with Banach lattices):

$$\forall_{X_1, X_2 \in \mathcal{C}_1} \quad (X_1, X_2 \geq 0 \Rightarrow \|X_1 + X_2\|_1 = \|X_1\|_1 + \|X_2\|_1).$$

A noncommutative analog of the ℓ^p space, called the Schatten class $\mathcal{C}_p$, also exists but it is not used in this paper.

DEFINITION 1. A positive operator X from $\mathcal{C}_1$ is called a *state* if $\mathrm{tr}(X) = 1$. The set of all states is denoted by S.

It is easy to verify that S is a convex and closed subset of $\mathcal{C}_1$ for the weak topology (hence for both operator and trace norms). By direct inspection it can be shown that it is not closed for the weak* topology ($\dim \mathcal{H} = \infty$).

DEFINITION 2. A bounded linear operator $P : \mathcal{C}_1 \to \mathcal{C}_1$ is said to be *positive* if $P(\mathcal{C}_{1+}) \subseteq \mathcal{C}_{1+}$. A positive operator P is called *Markov* (*markovian*) if for every $X \in \mathcal{C}_{1+}$ we have $\|P(X)\|_1 = \|X\|_1$ (equivalently $P(\mathrm{S}) \subseteq \mathrm{S}$). The set of all markovian operators on $\mathcal{C}_1$ is denoted by $\mathcal{S}$.

It may be calculated directly from the above definition that the hermitian part $\mathcal{C}_{1,H}$ of $\mathcal{C}_1$ is invariant for Markov operators T and that $T\restriction_{\mathcal{C}_{1,H}}$ is a contraction. The asymptotic properties of positive contractions (Markov operators) and one-parameter semigroups on C*-algebras or von Neumann algebras have recently been intensively studied (see [2], [11], [12] and [22]). The present paper is devoted to the Baire properties of one-parameter markovian semigroups acting on $\mathcal{C}_1$. We are motivated by the papers [3], [4], [7], [8], [13], [14] and [16], where the authors discuss similar questions for markovian semigroups acting on $L^1(\mu)$. Let us emphasize that in [20] R. Rudnicki showed more generally that convergence of powers a^n is a generic property (for the norm topology) in a wide class of (natural) closed and convex subsets $\mathcal{N}$ of Banach algebras. Roughly speaking, the set $\mathcal{N}_0$ of those elements $a \in \mathcal{N}$ whose iterates a^n converge in the norm contains a norm dense G_δ. The residuality of asymptotically stable multiplicative functions $T : [0, \infty) \to \mathcal{N}$ was obtained in [20] as well. Here we study the Baire properties from the point of view of the strong operator topology. Let us begin with:

Definition 3. We say that a family of Markov operators $T_t : \mathcal{C}_1 \to \mathcal{C}_1$, indexed by $t \in [0, \infty)$, forms a *one-parameter continuous semigroup* $\mathfrak{T}$ if:

(1) $T_0 = \mathrm{Id}$, the identity operator,
(2) for all $t, s \geq 0$ we have $T_{t+s} = T_t \circ T_s$ $(= T_s \circ T_t)$,
(3) for each $X \in \mathcal{C}_1$ we have $\lim_{t\to 0^+} \|T_t(X) - X\|_1 = 0$.

The family of all Markov semigroups $\mathfrak{T}$ on $\mathcal{C}_1$ is denoted by $\mathfrak{M}$.

There are several natural topologies used in studying the geometry of the set $\mathfrak{M}$ (and its subsets). First of all we have the uniform topology inherited from the operator norm topology on the Banach space $\mathcal{L}(\mathcal{C}_1, \mathcal{C}_1)$ of all bounded linear operators on $\mathcal{C}_1$. Namely the metric

$$\widetilde{\varrho}(\mathfrak{T}, \mathfrak{U}) = \sum_{m=1}^{\infty} \frac{1}{2^m} \frac{\sup_{0\le t\le m} \|T_t\restriction_{\mathcal{C}_{1,H}} - U_t\restriction_{\mathcal{C}_{1,H}}\|}{1 + \sup_{0\le t\le m} \|T_t\restriction_{\mathcal{C}_{1,H}} - U_t\restriction_{\mathcal{C}_{1,H}}\|}$$

(used in [15]) and an equivalent one,

$$\varrho(\mathfrak{T}, \mathfrak{U}) = \sum_{m=1}^{\infty} \frac{1}{2^{m+1}} \sup_{0\le t\le m} \|T_t\restriction_{\mathcal{C}_{1,H}} - U_t\restriction_{\mathcal{C}_{1,H}}\| \qquad \text{(if we are confined to } \mathfrak{M})$$

introduce on $\mathfrak{M}$ a complete metric structure (again the norm in $\mathcal{L}(\mathcal{C}_1, \mathcal{C}_1)$ is denoted simply by $\|\cdot\|$). The Baire properties of $\mathfrak{M}$ for this metric have been studied in [15]. We recall that a semigroup $\mathfrak{T} \in \mathfrak{M}$ is said to be (*uniform*) *mixing* if there exists a state $X_* \in \mathrm{S}$ such that $\lim_{t\to\infty} \|T_t - Q_{X_*}\| = 0$, where $Q_{X_*}(X) = \mathrm{tr}(X)X_*$. It has been proved in [15] (see also [5] and [20]) that the set $\mathfrak{M}_1$ of all uniform mixing semigroups is a ϱ-open and dense subset of $\mathfrak{M}$. If we additionally require that the limit projection is on a strictly positive state (i.e. $\langle X_* x, x\rangle > 0$ for all $x \neq 0$) or equivalently that X_* is one-to-one then we get a set $\mathfrak{M}_{1,+}$ which is a dense G_δ for ϱ. In this paper we discuss the mixing property for the so-called strong operator topology.

DEFINITION 4. Given a $\|\cdot\|_1$-dense sequence $X_1, X_2, \ldots$ in S we define on $\mathfrak{M}$ the metric

$$\varrho_{\mathrm{s}}(\mathfrak{T},\mathfrak{U}) = \sum_{\substack{m=1\\ l=1}}^{\infty} \frac{1}{2^{m+l+2}} \sup_{0\le t\le m} \|T_t(X_l) - U_t(X_l)\|_1.$$

Clearly $(\mathfrak{M}, \varrho_{\mathrm{s}})$ is a Polish metric space (i.e. complete, separable) and ϱ is stronger than ϱ_{s} (simply $\varrho_{\mathrm{s}}(\mathfrak{T},\mathfrak{U}) \le \varrho(\mathfrak{T},\mathfrak{U})$). Of course the topology defined by ϱ_{s} on $\mathfrak{M}$ does not depend on the specific family of states $X_1, X_2, \ldots$ and it is the strong operator topology.

DEFINITION 5. We say that a Markov semigroup $\mathfrak{T}$ is *almost mixing for the strong operator topology* if for each pair of states $X, Y \in$ S we have

$$\lim_{t\to\infty} \|T_t(X) - T_t(Y)\|_1 = 0.$$

The set of all such semigroups is denoted by $\mathfrak{M}_{\mathrm{ams}}$.

It well known (see [1, p. 59]) that semigroups of endomorphisms $T_t = \alpha_t$ acting on a von Neumann algebra $\mathcal{M}$ are almost mixing if and only if $\bigcap_{t\ge 0} \alpha_t(\mathcal{M}) = \mathbb{C}\cdot\mathbf{1}$ and in this case they are called *pure*. For classical contraction semigroups pure Markov operators are sometimes called *exact* (corresponding results for contraction semigroups on general Banach spaces were obtained much earlier by Y. Derriennic in [9]). We continue by showing

THEOREM 1. *$\mathfrak{M}_{\mathrm{ams}}$ is a dense G_δ subset of $\mathfrak{M}$ for the metric ϱ_{s}.*

Proof. It follows from [15, Theorem 1] that $\mathfrak{M}_{\mathrm{ams}}$ is ϱ_{s}-dense in $\mathfrak{M}$. Since all states belong to $\mathcal{C}_{1,H}$, it follows that $\mathfrak{T} \in \mathfrak{M}_{\mathrm{ams}}$ if and only if

$$\forall_{j\in\mathbb{N}}\ \forall_{k,m\in\mathbb{N}}\ \forall_{N\in\mathbb{N}}\ \exists_{t\ge N} \quad \|T_t(X_k) - T_t(X_m)\|_1 < 1/j$$

(we recall that T_t are contractions on $\mathcal{C}_{1,H}$). It remains to observe that the sets

$$\{\mathfrak{T} \in \mathfrak{M} : \|T_t(X_k) - T_t(X_m)\|_1 < 1/j\}$$

are ϱ_{s}-open. ∎

As in [5], we introduce

DEFINITION 6. A semigroup $\mathfrak{T} \in \mathfrak{M}$ is called *strong mixing* if there exists a state $X_* \in$ S such that for every $X \in$ S,

$$\lim_{t\to\infty} \|T_t(X) - X_*\|_1 = 0.$$

The set of all strong mixing Markov semigroups is denoted by $\mathfrak{M}_{\mathrm{ms}}$.

We notice that X_* is invariant for all T_t (in [1] the invariant X_* are called *absorbing*). Clearly a pure semigroup of endomorphisms $\alpha_t = T_t$ possessing an absorbing state is exactly strong mixing (see [1, p. 61]).

In what follows we will show that the strong mixing semigroups are meager in $\mathfrak{M}$. For this we consider the set

$$\mathfrak{M}_{0,\mathrm{w}^*} = \{\mathfrak{T} \in \mathfrak{M} : \forall_{Z\in\mathcal{C}_0} \lim_{t\to\infty} \operatorname{tr}(T_t(X)Z) = 0\}.$$

LEMMA 1. *$\mathfrak{M}_{0,\mathrm{w}^*}$ is ϱ_{s}-dense in $\mathfrak{M}$.*

Proof. It follows from the classical theory of one-parameter semigroups that for each $X \in \mathcal{C}_1$ we have

$$T_t(X) = \lim_{h\to 0^+} T_t^{(h)}(X),$$

where

$$T_t^{(h)}(X) = e^{tA_h}(X) = e^{-t/h}\sum_{k=0}^{\infty}\frac{(t/h)^k}{k!}\,T_h^k(X) \quad \text{and} \quad A_h = \frac{T_h - \mathrm{Id}}{h}.$$

The above convergence is uniform on compact subsets of $\mathbb{R}_+$ (see [10] for all details). We denote by $\mathfrak{T}^h = \{T_t^{(h)} : t \ge 0\}$ the (Markov) semigroup with generator A_h. Given $\varepsilon > 0$ and $M > \log_2(16/\varepsilon)$ we fix $h > 0$ such that $\sup_{0\le t\le M}\|T_t - T_t^{(h)}\| \le \varepsilon/8$. In particular,

$$\varrho_{\mathrm{s}}(\mathfrak{T},\mathfrak{T}^h) \le \varrho(\mathfrak{T},\mathfrak{T}^h) < \frac{\varepsilon}{8} + \frac{1}{2^M} < \frac{\varepsilon}{4}.$$

We find $K \ge 1$ such that $\sum_{k=K+1}^{\infty}(t/h)^k/k! < \varepsilon/16$ for all $t \in [0,M]$. It follows from Lemma 3.3 in [5] that for a finite family of states $X_1,\dots,X_L$, where $L > \log_2(8/\varepsilon)$ is fixed, there exists a Markov operator $\widetilde{T}$ such that

$$\sup\{\|\widetilde{T}^k(X_l) - T_h^k(X_l)\|_1 : 1 \le l \le L,\ 1 \le k \le K\} < \varepsilon/8$$

and $\lim_{k\to\infty}\operatorname{tr}(Z\widetilde{T}^k(X)) = 0$ for any state $X \in \mathrm{S}$ and any compact operator $Z \in \mathcal{C}_0$. Let $\widetilde{\mathfrak{T}}$ be the Markov semigroup defined by the generator $(\widetilde{T} - \mathrm{Id})/h$. For all $0 \le t \le M$ and $1 \le l \le L$ we obtain

$$\begin{aligned}
\|\widetilde{T}_t(X_l) - T_t^{(h)}(X_l)\|_1 &= e^{-t/h}\left\|\sum_{k=0}^{\infty}\frac{(t/h)^k}{k!}(\widetilde{T}^k(X_l) - T_h^k(X_l))\right\|_1 \\
&\le e^{-t/h}\sum_{k=1}^{K}\frac{(t/h)^k}{k!}\,\frac{\varepsilon}{8} + e^{-t/h}\sum_{k=K+1}^{\infty}\frac{(t/h)^k\cdot 2}{k!} \\
&\le \frac{\varepsilon}{8} + 2\sum_{k=K+1}^{\infty}\frac{(t/h)^k}{k!} \le \frac{\varepsilon}{8} + \frac{\varepsilon}{8} = \frac{\varepsilon}{4}.
\end{aligned}$$

It follows that

$$\varrho_{\mathrm{s}}(\mathfrak{T}^h,\widetilde{\mathfrak{T}}) \le \frac{\varepsilon}{4} + \sum_{m=M+1}^{\infty}\frac{1}{2^m} + \sum_{l=L+1}^{\infty}\frac{1}{2^l} = \frac{\varepsilon}{4} + \frac{1}{2^M} + \frac{1}{2^L}.$$

Now,

$$\varrho_{\mathrm{s}}(\mathfrak{T},\widetilde{\mathfrak{T}}) \le \varrho_{\mathrm{s}}(\mathfrak{T},\mathfrak{T}^h) + \varrho_{\mathrm{s}}(\mathfrak{T}^h,\widetilde{\mathfrak{T}}) < \frac{\varepsilon}{4} + \frac{\varepsilon}{4} + \frac{1}{2^M} + \frac{1}{2^L} < \varepsilon.$$

It remains to show that $\widetilde{\mathfrak{T}} \in \mathfrak{M}_{0,\mathrm{w}^*}$. For this let $Z \in \mathcal{C}_0$, $X \in \mathcal{C}_1$ be arbitrary. Since the Poisson convolution semigroup of measures

$$\mu_t(k) = e^{-t/h}\,\frac{(t/h)^k}{k!}$$

tends weakly to δ_∞ as $t \to \infty$, we obtain

$$\begin{aligned}\lim_{t\to\infty} \mathrm{tr}(Z\widetilde{T}_t(X)) &= \lim_{t\to\infty} \mathrm{tr}\Big(Ze^{-t/h}\sum_{k=0}^{\infty}\frac{(t/h)^k}{k!}\,\widetilde{T}^k(X)\Big)\\ &= \lim_{t\to\infty} e^{-t/h}\sum_{k=0}^{\infty}\frac{(t/h)^k}{k!}\,\mathrm{tr}(Z\widetilde{T}^k(X)) = 0. \ \blacksquare\end{aligned}$$

Define

$$\mathfrak{M}_{\mathrm{inv}} = \{\mathfrak{T} \in \mathfrak{M} : \exists_{X\in \mathrm{S}}\,\forall_{t\in\mathbb{R}_+}\ T_t(X) = X\},$$
$$\widetilde{\mathfrak{M}}_{0,\mathrm{w}^*} = \{\mathfrak{T} \in \mathfrak{M} : \forall_{l\in\mathbb{N}}\ \forall_{Z\in\mathcal{C}_0}\ \forall_{X\in\mathrm{S}}\ \forall_{n\in\mathbb{N}}\ \exists_{t\ge n}\ \ \mathrm{tr}(ZT_t(X)) < 1/l\}.$$

Finite intersections of the sets $\mathcal{U}_{\varepsilon,Z,X,t} = \{\mathfrak{T} : \mathrm{tr}(ZT_t(X)) < \varepsilon\} \subset \mathfrak{M}$, where $\varepsilon > 0$, $Z \in \mathcal{C}_0$, $X \in \mathcal{C}_1$, and $t \ge 0$, form a base of the so-called weak* operator topology (w*.o.t.) on $\mathfrak{M}$. Of course w*.o.t. is weaker than the metric topology generated by ϱ_{s}. Since the Banach spaces $\mathcal{C}_0$, $\mathcal{C}_1$ are separable, we may choose countable dense sequences $Z_1, Z_2, \ldots \in \mathcal{C}_0$ and $X_1, X_2, \ldots \in \mathrm{S}$. It follows that

$$\widetilde{\mathfrak{M}}_{0,\mathrm{w}^*} = \bigcap_{l=1}^{\infty}\bigcap_{i=1}^{\infty}\bigcap_{j=1}^{\infty}\bigcap_{n=1}^{\infty}\bigcup_{t>n}\{\mathfrak{T} : \mathrm{tr}(Z_iT_t(X_j)) < 1/l\}$$

is a w*.o.t. G_δ subset of $\mathfrak{M}$. Applying Lemma 1 we obtain

THEOREM 2. *$\widetilde{\mathfrak{M}}_{0,\mathrm{w}^*}$ is a ϱ_{s}-dense and w*.o.t. G_δ subset of $\mathfrak{M}$.*

It is not hard to notice that $\mathfrak{M}_{\mathrm{ms}} \subseteq \mathfrak{M}_{\mathrm{inv}} = (\widetilde{\mathfrak{M}}_{0,\mathrm{w}^*})^{\mathrm{c}}$. Hence Theorem 2 implies

COROLLARY 1. *In the strong operator topology* (*i.e. for the metric ϱ_{s}*) *the set $\mathfrak{M}_{\mathrm{ms}}$ is a meager subset of $\mathfrak{M}$.*

REFERENCES

[1] W. Arveson, *Noncommutative Dynamics and E-Semigroups*, Springer, 2003.

[2] Sh. A. Ayupov and T. A. Sarymsakov, *Markov operators on quantum probability spaces*, in: Probability Theory and Applications, Proc. World Congr. Bernoulli Soc. (Tashkent, 1986), Vol. 1, 1987, 445–454.

[3] W. Bartoszek, *Norm residuality of ergodic operators,* Bull. Polish Acad. Sci. Math. 29 (1981), 165–167.

[4] —, *One parameter positive contraction semigroups are convergent*, Univ. Iagel. Acta Math. 33 (1996), 49–57.

[5] W. Bartoszek and B. Kuna, *On residualities in the set of Markov operators on* $\mathcal{C}_1$, Proc. Amer. Math. Soc. 133 (2005), 2119–2129.

[6] O. Bratteli and D. W. Robinson, *Operator Algebras and Quantum Statistical Mechanics 1*, Springer, 1979.

[7] J. R. Choksi, S. J. Eigen and V. S. Prasad, *In general a stochastic operator does not preserve a measure*, Indiana Univ. Math. J. 36 (1987), 617–630.

[8] J. R. Choksi and V. S. Prasad, *Approximation and Baire category theorems in ergodic theory*, in: Measure Theory and Its Applications (Sherbrooke, Que., 1982), Lecture Notes in Math. 1033, Springer, 1993, 94–113.

[9] Y. Derriennic, *Lois «zéro ou deux» pour les processus de Markov – applications aux marches aléatoires*, Ann. Inst. H. Poincaré Sect. B 12 (1976), 111–129.

[10] N. Dunford and J. T. Schwartz, *Linear Operators*, *Part I*, Interscience, New York, 1958.

[11] E. Yu. Emel'yanov and M. P. H. Wolff, *Positive operators on Banach spaces ordered by strongly normal cones*, Positivity 7 (2003), 3–22.

[12] —, —, *Asymptotic properties of Markov semigroups on preduals of von Neumann algebras*, J. Math. Anal. Appl. 314 (2006), 749–763.

[13] A. Iwanik, *Approximation theorems for stochastic operators*, Indiana Univ. Math. J. 29 (1980), 415–425.

[14] —, *Baire category of mixing for stochastic operators*, Rend. Circ. Mat. Palermo (2) 28 (1992), 201–217.

[15] B. Kuna, *On residualities in the set of Markov continuous semigroups on* $\mathcal{C}_1$, Demonstratio Math. 39 (2006), 439–453.

[16] A. Lasota and J. Myjak, *Generic properties of stochastic semigroups*, Bull. Polish Acad. Sci. Math. 40 (1992), 283–292.

[17] M. Reed and B. Simon, *Methods of Modern Mathematical Physics*, *Vol. 1*, *Functional Analysis*, Academic Press, 1972.

[18] J. R. Retherford, *Hilbert Space*: *Compact Operators and the Trace Theorem*, London Math. Soc. Student Texts 27, Cambridge Univ. Press, 1993.

[19] J. R. Ringrose, *Compact Non-Selfadjoint Operators*, Van Nostrand, 1971.

[20] R. Rudnicki, *Generic properties of multiplicative functions and stochastic semigroups*, Bull. Polish Acad. Sci. Math. 45 (1997), 7–16.

[21] U. Sachdeva, *On category of mixing in infinite measure spaces*, Math. Systems Theory 5 (1971), 319–330.

[22] T. A. Sarymsakov and N. P. Zimakov, *Ergodic principle for the Markov semigroup in ordered normed spacs with basis*, Dokl. Akad. Nauk SSSR 289 (1986), 554–558 (in Russian).

[23] B. Simon, *Trace Ideals and their Applications*, Cambridge Univ. Press, 1979.

Department of Mathematics
Gdańsk University of Technology
Narutowicza 11/12
80-952 Gdańsk, Poland
E-mail: bartowk@mifgate.mif.pg.gda.pl

Received 19 August 2005;
revised 9 December 2005 (4654)