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HYPERSPACES OF UNIVERSAL CURVES AND 2-CELLS ARE TRUE $F_{\sigma\delta}$-SETS

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Abstract. It is shown that the following hyperspaces, endowed with the Hausdorff metric, are true absolute $F_{\sigma\delta}$-sets:

(1) $\mathcal{M}_1^2(X)$ of Sierpiński universal curves in a locally compact metric space X, provided $\mathcal{M}_1^2(X) \neq \emptyset$;

(2) $\mathcal{M}_1^3(X)$ of Menger universal curves in a locally compact metric space X, provided $\mathcal{M}_1^3(X) \neq \emptyset$;

(3) 2-cells in the plane.

Introduction. All spaces are assumed to be metric separable. There are results obtained over the last decade which fully characterize certain subspaces of the hyperspaces 2^X or $\mathcal{C}(X)$ of all non-empty compact or compact connected subsets, respectively, equipped with the Hausdorff metric, of spaces X such as, e.g., $\mathbb{R}^k$, I^k and the Hilbert cube $Q = I^\infty$, where $I = [0,1]$ (see [5, 7, 8, 10]).

Recall [10] that the subspace $\mathrm{LC}(X)$ of $\mathcal{C}(X)$ of all locally connected continua in X, where X is either of the above-mentioned spaces (for $k \geq 3$), is an $F_{\sigma\delta}$-absorber, so it is homeomorphic to $\widehat{c}_0 = \{(x_i) \in Q : \lim x_i = 0\}$. If X is a compact space containing a harmonic fan or comb, then $\mathrm{LC}(X)$ is a true $F_{\sigma\delta}$-set (see [12, 13]), i.e., it is $F_{\sigma\delta}$ but not $G_{\delta\sigma}$. It is known from [5] that the subspace $\mathrm{AR}(\mathbb{R}^2)$ of $\mathcal{C}(\mathbb{R}^2)$ of all absolute retracts in $\mathbb{R}^2$ is an $F_{\sigma\delta}$-absorber.

The first essential step in establishing results of that kind is to determine the exact Borel class of a given subspace.

Locally connected continua with no local cut points. A point $x \in X$ is a *local cut point* of a locally connected space X if there is an open connected subset $U \subset X$ such that $U \setminus \{x\}$ is not connected. Let X be a compact space. Denote by $\mathcal{A}(X)$ the subspace of $\mathcal{C}(X)$ consisting of all

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locally connected continua in X with no local cut points. Fix a countable open base $U_1, U_2, \ldots$ in X and let

$$T = \{(k,l,m) : \mathrm{cl}\, U_k \cup \mathrm{cl}\, U_l \subset U_m \text{ and } \mathrm{cl}\, U_k \cap \mathrm{cl}\, U_l = \emptyset\}.$$

We say that, for $(k,l,m) \in T$ and $C \in \mathcal{C}(X)$, the set $C \cap U_m$ is *connected between* U_k *and* U_l if there exists a continuum $D \subset C \cap U_m$ intersecting both U_k and U_l; we say that the set $C \cap U_m$ is *cyclicly connected between* U_k *and* U_l if it contains two continua D_1, D_2 each of which intersects both U_k and U_l and $D_1 \cap D_2 \subset U_k \cup U_l$.

For each $(k,l,m) \in T$ put

$$Z(k,l,m) = \{C \in \mathcal{C}(X) : C \cap U_m \text{ is not connected between } U_k \text{ and } U_l\}$$

and

$$W(k,l,m) = \{C \in \mathcal{C}(X) : C \cap U_m \text{ is cyclicly connected between } U_k \text{ and } U_l\}.$$

LEMMA 1. *The set $Z(k,l,m)$ is a G_δ-set and $W(k,l,m)$ is an F_σ-set in $\mathcal{C}(X)$.*

Proof. The set $Z_1 = \{(C,D) \in \mathcal{C}(X) \times \mathcal{C}(X) : D \subset C\}$ is closed in $\mathcal{C}(X) \times \mathcal{C}(X)$. The set $Z_2 = \mathcal{C}(X) \times \{D \in \mathcal{C}(X) : D \subset U_m, D \cap U_k \neq \emptyset \neq D \cap U_l\}$ is open in $\mathcal{C}(X) \times \mathcal{C}(X)$. The set $\mathcal{C}(X) \setminus Z(k,l,m)$ is the projection of $Z_1 \cap Z_2$ on the first coordinate space, so it is F_σ.

Similarly, $W(k,l,m)$ is F_σ.

LEMMA 2. *We have*

$$\mathcal{A}(X) = \mathrm{LC}(X) \cap \bigcap_{(k,l,m) \in T} (Z(k,l,m) \cup W(k,l,m)).$$

Proof. Suppose $C \in \mathcal{A}(X)$ and $C \cap U_m$ is connected between U_k and U_l for $(k,l,m) \in T$. This means there are a continuum $D \subset C \cap U_m$ and two points $v_0 \in D \cap U_k$ and $v_1 \in D \cap U_l$. It follows by the local connectivity of C that the component E of $C \cap U_m$ containing D is an open arcwise connected subset of C. The Arc Doubling Lemma of [15, p. 21] says that v_0, v_1 lie on a simple closed curve in E. Hence, $C \cap U_m$ is cyclicly connected between U_k and U_l.

Now, suppose $C \subset X$ is a locally connected continuum which belongs to the right hand side set in Lemma 2. Let c be an arbitrary point of C and G be an open subset of X such that $c \in G$ and $C \cap G$ is connected. Choose a basic set $U_m \subset G$ containing c. The component E of $C \cap U_m$ that contains c is an arcwise connected open subset of C.

We claim that $E \setminus \{c\}$ is connected. Indeed, let $a, b \in E \setminus \{c\}$ be two distinct points and let $V_a, V_b \subset E \setminus \{c\}$ be their respective connected open neighborhoods in C. Choose basic sets $U_k \subset \mathrm{cl}\, U_k \subset U_m$ and $U_l \subset \mathrm{cl}\, U_l \subset U_m$

containing a and b, respectively, such that $C \cap U_k \subset V_a$ and $C \cap U_l \subset V_b$. Since the set $C \cap U_m$ is connected between U_k and U_l, it is cyclicly connected between them. Hence, there exist two continua $D_1, D_2 \subset C \cap U_m$ each of which intersects both U_k and U_l and $D_1 \cap D_2 \subset U_k \cup U_l$. For $i = 1, 2$, we have $D_i \cap U_k \subset C \cap U_k \subset E$, thus $D_i \subset E$. At least one of the continua D_1, D_2, say D_1, omits c. Then $V_a \cup V_b \cup D_1$ is a connected subset of $E \setminus \{c\}$ joining a and b. Since c does not cut $E \subset C \cap G$, it does not cut $C \cap G$ either.

If X is a locally compact, non-compact space, then taking a one-point compactification $X' = X \cup \{p\}$ we get $\mathcal{A}(X) = \mathcal{A}(X') \setminus \mathcal{C}(X', p)$, where $\mathcal{C}(X', p) = \{C \in \mathcal{C}(X') : p \in C\}$ is compact. Thus we obtain

Proposition 1. *$\mathcal{A}(X)$ is an absolute $F_{\sigma\delta}$-set for any locally compact space X.*

Suppose now that X is an arbitrary Polish space containing a continuum M, where

(1) M is a copy of the $(k-1)$-dimensional Sierpiński continuum $M^k_{k-1} \subset \mathbb{R}^k$ universal for all $(k-1)$-dimensional compacta in $\mathbb{R}^k$ $(k > 1)$, or

(2) M is a copy of the Menger k-dimensional continuum $M^{2k+1}_k \subset \mathbb{R}^{2k+1}$ universal for k-dimensional compacta.

(See [9, p. 122] for their description.) Denote by $\mathcal{M}(X)$ the subspace of $\mathcal{C}(X)$ of all topological copies of M in X. We are going to show that $\mathcal{M}(X)$, as well as $\mathcal{A}(X)$, are not $G_{\delta\sigma}$. Denote by $\mathbb{N}$ the set of all positive integers. We will exploit the set

$$P = \{f \in \{0,1\}^{\mathbb{N}\times\mathbb{N}} : \forall m\ (f(m,n) = 0 \text{ for all but a finite number of } n)\},$$

which is known to be a true $F_{\sigma\delta}$-subset of the Cantor set $\{0,1\}^{\mathbb{N}\times\mathbb{N}}$ (see [11, p. 179]), and find its continuous reduction to $\mathcal{M}(X)$ or $\mathcal{A}(X)$ (see [11, p. 156] for a definition). To this end we construct an auxiliary continuum $\widetilde{B}$ which is a $(k-1)$-dimensional subset of $\mathbb{R}^k$ in the case of $M \underset{\text{top}}{=} M^k_{k-1}$ or it is k-dimensional in the case of $M \underset{\text{top}}{=} M^{2k+1}_k$.

For each pair $(m,n) \in \mathbb{N} \times \mathbb{N}$ choose a number $0 < x(m,n) < 1$ such that

- $x(m,n) \neq x(m',n')$ if $(m,n) \neq (m',n')$;
- for each m, the sequence $(x(m,n))_n$ is decreasing and converges to 0;
- $x(m,1) < x(1,m)$ for $m > 1$.

Surround each point $x(m,n)$ by an interval

$$I(m,n) = [x(m,n) - \varepsilon_{(m,n)}, x(m,n) + \varepsilon_{(m,n)}] \subset I$$

such that $I(m,n) \cap I(m',n') = \emptyset$ if $(m,n) \neq (m',n')$.

In the first case ($M \underset{\text{top}}{=} M^k_{k-1}$), let $B_{-1} = [-1,1] \times I^{k-2} \times [-1,0]$, $B_0 = [-1,0] \times I^{k-2} \times I$ and $B(m,n) = I(m,n) \times I^{k-2} \times [0,1/m]$. In each k-cell $D \in \{B_{-1}, B_0\} \cup \{B(m,n) : (m,n) \in \mathbb{N} \times \mathbb{N}\}$, consider a standard model $\widetilde{D}$ of M^k_{k-1} as constructed, e.g., in [9, p. 122] (remove a null sequence of open k-cells from the interior of D so that the union of the sequence is dense in D, the closures of the removed open k-cells are mutually disjoint and their boundaries are locally flat $(k-1)$-spheres). We thus obtain copies $\widetilde{B_{-1}}, \widetilde{B_0}, \widetilde{B(m,n)}$, where $(m,n) \in \mathbb{N} \times \mathbb{N}$, of M^k_{k-1}.

In the second case ($M \underset{\text{top}}{=} M^{2k+1}_k$), take $(2k+1)$-cells $B_{-1} = [-1,1] \times I^{2k-1} \times [-1,0]$, $B_0 = [-1,0] \times I^{2k-1} \times I$, $B(m,n) = I(m,n) \times I^{2k-1} \times [0,1/m]$ and standard models $\widetilde{B_{-1}}, \widetilde{B_0}, \widetilde{B(m,n)}$, where $(m,n) \in \mathbb{N} \times \mathbb{N}$, of M^{2k+1}_k constructed in them [9].

In either case, put

$$\widetilde{B} = \widetilde{B_{-1}} \cup \widetilde{B_0} \cup \bigcup_{(m,n) \in \mathbb{N} \times \mathbb{N}} \widetilde{B(m,n)}.$$

Observe that if, for each $m \in \mathbb{N}$, $J(m)$ is a finite subset of $\mathbb{N}$, then the set

$$\widetilde{B_{-1}} \cup \widetilde{B_0} \cup \bigcup \{\widetilde{B(m,n)} : m \in \mathbb{N}, n \in J(m)\}$$

is homeomorphic to M (see appropriate characterizations of M: [4], [6, p. 74] in the first case, and [3] in the second); it is not locally connected if $J(m)$ is infinite for some m.

Define a mapping $F : \{0,1\}^{\mathbb{N} \times \mathbb{N}} \to \mathcal{C}(\widetilde{B})$ as follows. If $f(m,n) = 0$, then put $F(f)(m,n) = \widetilde{B_{-1}} \cup \widetilde{B_0}$; if $f(m,n) = 1$, then put $F(f)(m,n) = \widetilde{B_{-1}} \cup \widetilde{B_0} \cup \widetilde{B(m,n)}$. Set

$$F(f) = \bigcup_{(m,n) \in \mathbb{N} \times \mathbb{N}} F(f)(m,n).$$

The mapping F homeomorphically embeds the Cantor set $\{0,1\}^{\mathbb{N} \times \mathbb{N}}$ in $\mathcal{C}(\widetilde{B})$. We can assume $\widetilde{B}$ is contained in M.

Observe that

$$F(f) \text{ is homeomorphic to } M \quad \text{iff} \quad f \in P \quad \text{iff} \quad F(f) \in \mathcal{A}(X).$$

This means that F homeomorphically reduces P to $\mathcal{A}(X)$ as well as to the space $\mathcal{M}(X)$. Hence, we have

PROPOSITION 2. *If a Polish space X contains a topological copy of M, where M is the Sierpiński continuum M^k_{k-1} or M is the Menger continuum M^{2k+1}_k, then neither of the sets $\mathcal{A}(X)$ and $\mathcal{M}(X)$ is $G_{\delta\sigma}$.*

REMARK 1. If X contains a k-cell, then $\mathcal{M}^k_{k-1}(X)$ contains a copy of the $F_{\sigma\delta}$-absorber $\widehat{c}_0$ as a closed subset. An analogous conclusion holds for $\mathcal{M}^{2k+1}_k(X)$ if X contains a $(2k+1)$-cell [12, 13].

THEOREM 1. *If X is a locally compact space containing $\mathcal{M}^2_1$, then $\mathcal{A}(X)$ is a true absolute $F_{\sigma\delta}$-set.*

Sierpiński plane universal curves. It is known that the subspace $\mathrm{DIM}_1(X) \subset 2^X$ of 1-dimensional compacta in an arbitrary space X is G_δ (see, e.g., [7]). If X is a 2-dimensional sphere or a plane, then Whyburn's [16] topological characterization of M^2_1 can be expressed in the following form:

$$\mathcal{M}^2_1(X) = \mathcal{A}(X) \cap \mathrm{DIM}_1(X).$$

Hence, by Proposition 2 and Theorem 1, $\mathcal{M}^2_1(X)$ is a true absolute $F_{\sigma\delta}$-set.

In order to establish the Borel class of $\mathcal{M}^2_1(X)$ for more general spaces X one has to deal with planability properties. Recall that if C is a locally connected continuum with no local cut points, then C is non-planar if and only if C contains a complete five-point graph K_5 [15, pp. 23–24]. Graphs K_5, however, are not convenient for exact Borel class evaluation. We are going to replace K_5 by K_5-like continua.

A continuum $K \subset X$ is said to be K_5-*like* if there exist five mutually disjoint continua $V_1, \ldots, V_5 \subset X$, called *vertices* of K, and ten mutually disjoint continua $K_1, \ldots, K_{10} \subset X$, called *edges* of K, such that

(1) $K = V_1 \cup \ldots \cup V_5 \cup K_1 \cup \ldots \cup K_{10}$;
(2) any two distinct vertices are both intersected by exactly one edge;
(3) each edge intersects exactly two distinct vertices.

LEMMA 3. *No K_5-like continuum is planable.*

Proof. Suppose K is a K_5-like continuum in $\mathbb{R}^2$ with vertices $V_1, \ldots, V_5$ and edges $K\{i,j\}$, $i \neq j, i, j \in \{1, \ldots, 5\}$, such that $K\{i,j\}$ is the unique edge joining V_i and V_j. For each $\{i,j\}$, one can find an arc $a\{i,j\} \subset \mathbb{R}^2$ in a neighborhood of $K\{i,j\}$ so that conditions (1)–(3) are satisfied for edges being replaced by the arcs. We can also assume that the arcs meet vertices only at their endpoints. Let $W_1, \ldots, W_5$ be mutually disjoint connected neighborhoods of $V_1, \ldots, V_5$, respectively, which satisfy (1)–(3) if substituted for vertices with arcs $a\{i,j\}$ as new edges. Let $a\{i,j_1\}, \ldots, a\{i,j_4\}$ denote the arcs that meet V_i at their end-points $e\{i,j_1\}, \ldots, e\{i,j_4\}$, respectively. For every $i = 1, \ldots, 5$, one can easily find a finite tree $T_i \subset W_i$ such that

$$T_i \cap (a\{i,j_1\} \cup \ldots \cup a\{i,j_4\}) = \{e\{i,j_1\}, \ldots, e\{i,j_4\}\}.$$

It follows from the Moore decomposition theorem [14, p. 533] that shrinking each tree T_i to a point t_i yields a K_5 graph in the plane with vertices t_i and edges $a\{i,j\}$, $i,j = 1, \ldots, 5$, a contradiction.

Proposition 3. *A locally connected continuum $C \subset X$ with no local cut point is planable if and only if C contains no K_5-like subcontinuum.*

Assume now X is compact. Denote by $\mathcal{K}_5(X)$ the subspace of $\mathcal{C}(X)$ consisting of all continua in X which contain a K_5-like continuum in X. We have the following formula:

$$\begin{aligned}\mathcal{K}_5(X) = \{C \in \mathcal{C}(X) : {} & \exists V_1, \ldots, V_5 \in \mathcal{C}(X)\ \exists K_1, \ldots, K_{10} \in \mathcal{C}(X) \\ & (V_1 \cup \ldots V_5 \cup K_1 \cup \ldots \cup K_{10} \text{ is a } K_5\text{-like} \\ & \text{continuum contained in } C \text{ with vertices} \\ & V_1, \ldots, V_5 \text{ and edges } K_1, \ldots, K_{10})\}.\end{aligned}$$

Let A be the set of all 16-tuples $(C, V_1, \ldots, V_5, K_1, \ldots, K_{10}) \in \mathcal{C}(X)^{16}$ such that the union of all V_i's and K_j's is a subcontinuum of C and appropriate non-empty intersections occur between K_j's and V_i's (as the definition of a K_5-continuum requires). Then A is a closed subset of $\mathcal{C}(X)^{16}$. The set B of all 16-tuples $(C, V_1, \ldots, V_5, K_1, \ldots, K_{10}) \in \mathcal{C}(X)^{16}$ such that appropriate empty intersections occur between K_j's and V_i's (according to the same definition) is an open subset of $\mathcal{C}(X)^{16}$. Finally, $\mathcal{K}_5(X)$ is the projection of $A \cap B$ on the first coordinate space, whence $\mathcal{K}_5(X)$ is an absolute F_σ. Using the one-point compactification trick (as in the paragraph preceding Proposition 1) we get

Proposition 4. *If X is a locally compact space, then $\mathcal{K}_5(X)$ is an absolute F_σ-set.*

It follows from Proposition 3 that $\mathcal{M}_1^2(X) = \mathcal{A}(X) \cap \mathrm{DIM}_1(X) \setminus \mathcal{K}_5(X)$. Thus, by Propositions 1, 2 and 4, we obtain

Theorem 2. *If X is a locally compact space, then $\mathcal{M}_1^2(X)$ is an absolute $F_{\sigma\delta}$-set. If, moreover, X contains a copy of M_1^2, then $\mathcal{M}_1^2(X)$ is a true $F_{\sigma\delta}$-set.*

Menger universal curves. The Menger universal curve M_1^3 was topologically characterized by R. D. Anderson as a locally connected, one-dimensional continuum with no local cut points and no non-empty open planar subsets [1, 2]. Recall that a locally connected continuum C with no local cut points has no non-empty open planar subset if and only if each open non-empty subset of C contains K_5 (see [15, p. 24]). In view of Lemma 3, such a continuum C has no open non-empty planar subsets if and only if each open non-empty subset of C contains a K_5-like subcontinuum.

Theorem 3. (1) *If X is a locally compact space, then $\mathcal{M}_1^3(X)$ is an absolute $F_{\sigma\delta}$-set.*

(2) *If X is a Polish space containing a copy of M_1^3, then $\mathcal{M}_1^3(X)$ is not $G_{\delta\sigma}$.*

Proof. Assume X is a compact space with an open base $\{U_1, U_2, \ldots\}$. Let $Z(i) = \{C \in \mathcal{C}(X) : C \cap U_i = \emptyset\}$ and

$$K(i) = \{C \in \mathcal{C}(X) : \exists K \in \mathcal{C}(X)\ (K \subset C \cap U_i\ \&\ K \in \mathcal{K}_5(X))\}.$$

It is easy to see that $Z(i)$ is closed in $\mathcal{C}(X)$. The set $K(i)$ is a projection on the first coordinate space of the set

$$\{(C, K) \in \mathcal{C}(X) \times \mathcal{C}(X) : K \subset C \cap U_i\ \&\ K \in \mathcal{K}_5(X)\},$$

which is F_σ by Proposition 4, so $K(i)$ is F_σ as well. The formula

$$\mathcal{M}_1^3(X) = \mathcal{A}(X) \cap \mathrm{DIM}_1(X) \cap \bigcap_{i \in \mathbb{N}} (Z(i) \cup (C(X) \setminus K(i)))$$

implies the first part of the theorem (the case of a locally compact X is handled by a one-point compactification of X).

The second part follows from Proposition 2.

2-cells. Denote by $\mathcal{D}(X)$ the subspace of $\mathcal{C}(X)$ of all topological 2-cells in a space X, by $\mathrm{AR}(X)$ the subspace of $\mathcal{C}(X)$ of all absolute retracts in X and by $F_1(X)$ the subspace of $\mathcal{C}(X)$ of all singletons.

LEMMA 4. $\mathcal{D}(X) = \mathrm{AR}(X) \cap \mathcal{A}(X) \setminus (F_1(X) \cup \mathcal{K}_5(X))$.

Proof. Suppose $C \in \mathrm{AR}(X) \cap \mathcal{A}(X) \setminus (F_1(X) \cup \mathcal{K}_5(X))$. We can assume, by Proposition 3, that $C \subset \mathbb{R}^2$. Then C is a 2-cell by [14, Theorem 11, p. 534]. The reverse inclusion is evident.

THEOREM 4. *$\mathcal{D}(\mathbb{R}^2)$ is a true absolute $F_{\sigma\delta}$-set containing a copy of $\widehat{c}_0$ as a closed subset.*

Proof. That $\mathcal{D}(\mathbb{R}^2)$ is $F_{\sigma\delta}$ is a consequence of Lemma 4, Propositions 1 and 4, [5] and the fact that $F_1(\mathbb{R}^2)$ is closed in $\mathcal{C}(\mathbb{R}^2)$. It is shown in [12, 13] that $\mathcal{D}(\mathbb{R}^2)$ contains $\widehat{c}_0$ as a closed subset, hence it is not $G_{\delta\sigma}$.

REMARK 2. It is proved in [8] that $\mathrm{AR}(\mathbb{R}^k)$, where $k > 2$, is a true $G_{\delta\sigma\delta}$-set. Thus $\mathcal{D}(\mathbb{R}^k)$ also is $G_{\delta\sigma\delta}$. The question is whether $\mathcal{D}(X)$ is $F_{\sigma\delta}$ for $X = \mathbb{R}^k$, $k > 2$, or, more generally, for locally compact spaces X (it is not $G_{\delta\sigma}$ if X contains a 2-cell by [12, 13]).

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