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A NOTE ON p-ADIC VALUATIONS OF SCHENKER SUMS

BY

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Abstract. A prime number p is called a Schenker prime if there exists $n \in \mathbb{N}_+$ such that $p \nmid n$ and $p \,|\, a_n$, where $a_n = \sum_{j=0}^{n} (n!/j!) n^j$ is a so-called Schenker sum. T. Amdeberhan, D. Callan and V. Moll formulated two conjectures concerning p-adic valuations of a_n when p is a Schenker prime. In particular, they conjectured that for each $k \in \mathbb{N}_+$ there exists a unique positive integer $n_k < 5^k$ such that $v_5(a_{m\cdot 5^k + n_k}) \geq k$ for each nonnegative integer m. We prove that for every $k \in \mathbb{N}_+$ the inequality $v_5(a_n) \geq k$ has exactly one solution modulo 5^k. This confirms the above conjecture. Moreover, we show that if $37 \nmid n$ then $v_{37}(a_n) \leq 1$, which disproves the other conjecture of the above mentioned authors.

1. Introduction. Questions concerning the behavior of p-adic valuations of elements of integer sequences are interesting subjects of research in number theory. The knowledge of all p-adic valuations of a given number is equivalent to its factorization. Papers [1]–[3], [6], [5] present interesting results concerning the behavior of p-adic valuations in some integer sequences.

Fix a prime number p. Every nonzero rational number x can be written in the form $x = (a/b)p^t$, where $a \in \mathbb{Z}$, $b \in \mathbb{N}_+$, $\gcd(a, b) = 1$, $p \nmid ab$ and $t \in \mathbb{Z}$. Such a representation of x is unique, so the number t is well defined. We call it the *p-adic valuation* of x and denote it by $v_p(x)$. By convention, $v_p(0) = \infty$. In particular, if $x \in \mathbb{Q} \setminus \{0\}$ then $|x| = \prod_{p \text{ prime}} p^{v_p(x)}$, where $v_p(x) \neq 0$ for finitely many prime numbers p. We denote by $s_d(n)$ the sum of digits of a positive integer n in base d, i.e. if $n = \sum_{i=0}^{m} c_i d^i$ is an expansion of n in base d, then $s_d(n) = \sum_{i=0}^{m} c_i$.

In a recent paper T. Amdeberhan, D. Callan and V. Moll [1] introduced the sequence of *Schenker sums*, defined by

$$a_n = \sum_{j=0}^{n} \frac{n!}{j!} n^j, \qquad n \in \mathbb{N}_+.$$

They obtained an exact expression for the 2-adic valuation of a Schenker sum:

$$v_2(a_n) = \begin{cases} 1 & \text{when } 2 \nmid n, \\ n - s_2(n) & \text{when } 2 \,|\, n. \end{cases}$$

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Moreover, they proved two results concerning the p-adic valuations of elements of the sequence $(a_n)_{n\in\mathbb{N}_+}$ when p is an odd prime number:

Proposition 1 ([1, Proposition 3.1]). *Let p be an odd prime number dividing a positive integer n. Then*

$$v_p(a_n) = \frac{n - s_p(n)}{p-1} = v_p(n!).$$

Proposition 2 ([1, Proposition 3.2]). *Let p be an odd prime number and $n = pm + r$, where $0 < r < p$. Then $p \mid a_n$ if and only if $p \mid a_r$.*

These propositions allow us to compute the p-adic valuation of a_n when $p \mid n$ or $p \nmid a_{n \bmod p}$. This gives a complete description of the p-adic valuations of a_n for some prime numbers ($3, 7, 11, 17$, for example). However, the case when $p \nmid n$ and $p \mid a_n$ for some positive integer n is much more difficult. The first prime p such that $p \nmid n$ and $p \mid a_n$ for some $n \in \mathbb{N}_+$ is $p = 5$. We have $5 \mid a_{5m+2}$ for every $m \in \mathbb{N}$. Observe that if $n \not\equiv 0, 2 \pmod 5$, then $5 \nmid a_n$. From numerical experiments, the authors of [1] formulated a conjecture, an equivalent version of which is as follows:

Conjecture 1 ([1, Conjecture 4.6]). *Assume that there exists a unique positive integer n_k less than 5^k such that $5^k \mid a_{m\cdot 5^k+n_k}$, $m \in \mathbb{N}$. Then there exists a unique $n_{k+1} \in \{n_k, 5^k+n_k, 2\cdot 5^k+n_k, 3\cdot 5^k+n_k, 4\cdot 5^k+n_k\}$ such that $5^{k+1} \mid a_{m\cdot 5^{k+1}+n_{k+1}}$, $m \in \mathbb{N}$. In other words, for every $k \in \mathbb{N}$ the inequality $v_5(a_n) \geq k$ has a unique solution $n \pmod{5^k}$ with $5 \nmid n$.*

5 is not the only prime number p such that $p \nmid n$ and $p \mid a_n$ for some $n \in \mathbb{N}_+$. The prime numbers which satisfy this condition are called *Schenker primes.*

The first question which comes to mind is: what is the cardinality of the set of Schenker primes? We will prove the following proposition using a modification of Euclid's proof of the infinitude of the set of prime numbers:

Proposition 3. *There are infinitely many Schenker primes.*

Proof. Assume that there are only finitely many Schenker primes, and let $p_1, \dots, p_s$ be all the odd Schenker primes in ascending order. Since $a_1 = 2$, Proposition 2 implies that $p_1, \dots, p_s \nmid a_{p_1\dots p_s+1}$. Set $t := p_1 \dots p_s + 1$, obviously an even number. By Proposition 1 we have

$$2t! \leq t! \sum_{j=0}^{t} \frac{t^j}{j!} = a_t = \prod_{p\,\mathrm{prime},\, p \mid t} p^{v_p(t!)} \leq t!,$$

a contradiction. ∎

The main result of this paper is the following theorem:

THEOREM 1. *Let p be a prime number, let $n_k \in \mathbb{N}$ be such that $p \nmid n_k$, $p^k \,|\, a_{n_k}$ and set*

$$q_{n_k,p} := a_{n_k+p} - a_{n_k}(n_k+p)^{n_k+2} n_k^{p-n_k-2}.$$

Then:

- *if $q_{n_k,p} \not\equiv 0 \pmod{p^2}$, then there exists a unique n_{k+1} modulo p^{k+1} for which $p^{k+1} \,|\, a_{n_{k+1}}$ and $n_{k+1} \equiv n_k \pmod{p^k}$;*
- *if $q_{n_k,p} \equiv 0 \pmod{p^2}$ and $p^{k+1} \,|\, a_{n_k}$, then $p^{k+1} \,|\, a_{n_{k+1}}$ for any n_{k+1} satisfying $n_{k+1} \equiv n_k \pmod{p^k}$;*
- *if $q_{n_k,p} \equiv 0 \pmod{p^2}$ and $p^{k+1} \nmid a_{n_k}$, then $p^{k+1} \nmid a_{n_{k+1}}$ for any n_{k+1} satisfying $n_{k+1} \equiv n_k \pmod{p^k}$.*

Moreover, if $p \nmid n_1$, $p \,|\, a_{n_1}$ and $q_{n_1,p} \not\equiv 0 \pmod{p^2}$, then for any $k \in \mathbb{N}_+$ the inequality $v_p(a_n) \geq k$ has a unique solution n_k modulo p^k satisfying the congruence $n_k \equiv n_1 \pmod{p}$.

The proof of this theorem is given in Section 2.

The authors of [1] stated another, more general conjecture concerning the p-adic valuations of the numbers a_n when p is an odd Schenker prime. An equivalent version of this conjecture is as follows:

CONJECTURE 2 ([1, Conjecture 4.12]). *Let p be an odd Schenker prime. Then for every k there exists a unique solution modulo p^k of the inequality $v_p(a_n) \geq k$ which is not congruent to 0 modulo p.*

Using Theorem 1 we will show that for $p = 37$ Conjecture 2 is not satisfied.

Convention. We assume that $x \equiv y \pmod{p^k}$ means $v_p(x-y) \geq k$ for p a prime number and k an integer. This convention extends the relation of equivalence modulo p^k to all rational numbers x, y and integers k. Moreover, we set by convention $\prod_{i=0}^{-1} = 1$.

2. Proof of the main theorem. Theorem 1 resembles a well known fact concerning the p-adic valuation of a value of a polynomial with integer coefficients (see [4, p. 44]):

THEOREM 2 (Hensel's lemma). *Let f be a polynomial with integer coefficients, p be a prime number and k be a positive integer. Assume that $f(n_0) \equiv 0 \pmod{p^k}$ for some integer n_0. Then the number of solutions n of the congruence $f(n) \equiv 0 \pmod{p^{k+1}}$, satisfying the condition $n \equiv n_0 \pmod{p^k}$, is equal to:*

- *1 when $f'(n_0) \not\equiv 0 \pmod{p}$;*
- *0 when $f'(n_0) \equiv 0 \pmod{p}$ and $f(n_0) \not\equiv 0 \pmod{p^{k+1}}$;*
- *p when $f'(n_0) \equiv 0 \pmod{p}$ and $f(n_0) \equiv 0 \pmod{p^{k+1}}$.*

Similarity of these theorems is not accidental. Namely, we will show that checking the p-adic valuation of values of some polynomials is sufficient for computation of the p-adic valuation of a Schenker sum. Firstly, note that for any coprime positive integers d, n the divisibility of a_n by d is equivalent to the divisibility of $a_{n \bmod p}$ by d:

$$
\begin{aligned}
(2.1) \qquad a_n &= \sum_{j=0}^{n} \frac{n!}{j!} n^j = \sum_{j=0}^{n} n^{n-j} \prod_{i=0}^{j-1} (n-i) \\
&\overset{(*)}{\equiv} \sum_{j=0}^{d-1} n^{n-j} \prod_{i=0}^{j-1} (n-i) = n^{n-d+2} \sum_{j=0}^{d-1} n^{d-j-2} \prod_{i=0}^{j-1} (n-i) \pmod{d},
\end{aligned}
$$

where the equivalence $(*)$ follows from the fact that the product of at least d consecutive integers contains an integer divisible by d, hence it is equal to $0 \bmod d$. Thus for every $d \in \mathbb{N}_+$ we define the polynomial

$$f_d(X) := \sum_{j=0}^{d-1} X^{d-j-2} \prod_{i=0}^{j-1} (X-i).$$

With this notation the formula (2.1) can be rewritten in the following way:

$$a_n \equiv n^{n-d+2} f_d(n) \pmod{d}. \tag{2.2}$$

Let $r = n \pmod{d}$. If d, n are coprime, then

$$
\begin{aligned}
a_n \equiv n^{n-d+2} f_d(n) \equiv 0 \pmod{d} \;&\Leftrightarrow\; f_d(n) \equiv 0 \pmod{d} \\
&\Leftrightarrow\; f_d(r) \equiv 0 \pmod{d} \;\Leftrightarrow\; a_r \equiv r^{r-d+2} f_d(r) \equiv 0 \pmod{d}.
\end{aligned}
$$

If $d = p^k$ for some prime number p and positive integer k, then (2.2) takes the form

$$a_n \equiv n^{n-p^k+2} f_{p^k}(n) \pmod{p^k}. \tag{2.3}$$

We thus see that if $p \nmid n$, then $v_p(a_n) \geq k$ if and only if $v_p(f_{p^k}(n)) \geq k$. Moreover, for any $k_1, k_2 \in \mathbb{N}$ satisfying $k_1 \leq k_2$,

$$n^{n-p^{k_1}+2} f_{p^{k_1}}(n) \equiv n^{n-p^{k_2}+2} f_{p^{k_2}}(n) \pmod{p^{k_1}}.$$

Hence, if $p \nmid n$ and $k_1 \leq k_2$, then

$$p^{k_1} \mid f_{p^{k_2}}(n) \;\Leftrightarrow\; p^{k_1} \mid f_{p^{k_1}}(n).$$

If we assume now that $k > 1$, then by Fermat's little theorem (in the form $n^{p^k} \equiv n \pmod{p}$) and the fact that the product of at least p consecutive integers is divisible by p, we obtain

$$f'_{p^k}(n) = \sum_{j=0}^{p^k-1}\Big[(p^k-j-2)n^{p^k-j-3}\prod_{i=0}^{j-1}(n-i) + n^{p^k-j-2}\sum_{h=0}^{j-1}\prod_{i=0,\, i\neq h}^{j-1}(n-i)\Big]$$
$$\equiv \sum_{j=0}^{2p-1}\Big[(-j-2)n^{-j-2}\prod_{i=0}^{j-1}(n-i)+n^{-j-1}\sum_{h=0}^{j-1}\prod_{i=0,\, i\neq h}^{j-1}(n-i)\Big] \pmod p.$$

The formula above implies

$$f'_{p^{k_1}}(n) \equiv f'_{p^{k_2}}(n) \pmod p \tag{2.4}$$

for $k_1, k_2 > 1$ and $p \nmid n$. Recall that if $f \in \mathbb{Z}[X]$, then for any $x_0 \in \mathbb{Z}$ there exists a $g \in \mathbb{Z}[X]$ such that

$$f(X) = f(x_0) + (X - x_0)f'(x_0) + (X - x_0)^2 g(X).$$

Using (2.3) and the equality above for $f = f_{p^2}$, $x_0 = n$ and $X = n + p$, we obtain

$$\frac{a_{n+p}}{(n+p)^{n+p-p^2+2}} - \frac{a_n}{n^{n-p^2+2}} \equiv f_{p^2}(n+p) - f_{p^2}(n) \equiv p f'_{p^2}(n) \pmod{p^2}. \tag{2.5}$$

If $p \nmid x$, then by Euler's theorem $x^{p^2-p} \equiv 1 \pmod{p^2}$ we can simplify (2.5) to get

$$\frac{a_{n+p}}{(n+p)^{n+2}} - \frac{a_n}{n^{n-p+2}} \equiv p f'_{p^2}(n) \pmod{p^2},$$

which leads to

$$\frac{1}{p}\left(\frac{a_{n+p}}{(n+p)^{n+2}} - \frac{a_n}{n^{n-p+2}}\right) \equiv f'_{p^2}(n) \pmod p.$$

Our discussion shows that

$$\begin{aligned} f'_{p^2}(n) \not\equiv 0 \pmod p \quad &\Leftrightarrow \quad \frac{1}{p}\left(\frac{a_{n+p}}{(n+p)^{n+2}} - \frac{a_n}{n^{n-p+2}}\right) \not\equiv 0 \pmod p \\ &\Leftrightarrow \quad \frac{a_{n+p}}{(n+p)^{n+2}} - \frac{a_n}{n^{n-p+2}} \not\equiv 0 \pmod{p^2} \\ &\Leftrightarrow \quad a_{n+p} - \frac{a_n(n+p)^{n+2}}{n^{n-p+2}} \not\equiv 0 \pmod{p^2} \\ &\Leftrightarrow \quad a_{n+p} - a_n(n+p)^{n+2}n^{p-n-2} \not\equiv 0 \pmod{p^2}. \end{aligned} \tag{2.6}$$

Assume now that $p^k \mid a_{n_k}$ for some $n_k \in \mathbb{N}$ not divisible by p and

$$a_{n_k+p} - a_{n_k}(n_k+p)^{n_k+2} n_k^{p-n_k-2} \not\equiv 0 \pmod{p^2}.$$

Then $p \nmid f'_{p^2}(n_k)$, and by (2.5), $p \nmid f'_{p^{k+1}}(n_k)$. Using now Theorem 2 for $f = f_{p^{k+1}}$, we conclude that there exists a unique $n_{k+1} \in \mathbb{Z}$ modulo p^{k+1} satisfying $p^{k+1} \mid a_{n_{k+1}}$ and $n_{k+1} \equiv n_k \pmod{p^k}$.

By simple induction on k we will show that if $p \nmid n_1$ and $p \mid a_{n_1}$ together with

$$a_{n_1+p} - a_{n_1}(n_1+p)^{n_1+2} n_1^{p-n_1-2} \not\equiv 0 \pmod{p^2},$$

then there exists a unique n_k modulo p^k such that $p^k \mid a_{n_k}$, $n_k \equiv n_1 \pmod{p}$ and

$$\frac{1}{p}\left(\frac{a_{n_k+p}}{(n_k+p)^{n_k+2}} - \frac{a_{n_k}}{n_k^{n_k-p+2}}\right) \equiv \frac{1}{p}\left(\frac{a_{n_1+p}}{(n_1+p)^{n_1+2}} - \frac{a_{n_1}}{n_1^{n_1-p+2}}\right) \pmod{p}.$$

Certainly this statement is true for $k=1$. Now, if we assume that there exists a unique n_k modulo p^k satisfying the conditions in the statement, then there exists a unique n_{k+1} modulo p^{k+1} such that $p^{k+1} \mid a_{n_{k+1}}$, $n_{k+1} \equiv n_k \pmod{p^k}$. Using (2.6) we conclude that

$$\begin{aligned}\frac{1}{p}\left(\frac{a_{n_{k+1}+p}}{(n_{k+1}+p)^{n_{k+1}+2}} - \frac{a_{n_{k+1}}}{n_{k+1}^{n_{k+1}-p+2}}\right) &\equiv f'_{p^{k+1}}(n_{k+1}) \equiv f'_{p^{k+1}}(n_k) \\ \equiv \frac{1}{p}\left(\frac{a_{n_k+p}}{(n_k+p)^{n_k+2}} - \frac{a_{n_k}}{n_k^{n_k-p+2}}\right) &\equiv \frac{1}{p}\left(\frac{a_{n_1+p}}{(n_1+p)^{n_1+2}} - \frac{a_{n_1}}{n_1^{n_1-p+2}}\right) \pmod{p}.\end{aligned}$$

Summing up, we see that the first case in the statement of Theorem 1 is proved. We prove the rest of the statement now.

Let $p \nmid n_k$ and $p^k \mid a_{n_k}$ and

$$a_{n_k+p} - a_{n_k}(n_k+p)^{n_k+2} n_k^{p-n_k-2} \equiv 0 \pmod{p^2}.$$

Since the last condition above is equivalent to divisibility of $f'_{p^k}(n_k)$ by p, Theorem 2 allows us to conclude that:

- if $p^{k+1} \mid a_{n_k}$, then $p^{k+1} \mid a_n$ for any $n \equiv n_k \pmod{p^k}$;
- if $p^{k+1} \nmid a_{n_k}$, then $p^{k+1} \nmid a_n$ for any $n \equiv n_k \pmod{p^k}$.

We have obtained a useful criterion for the behavior of the p-adic valuation of a_n. In particular, the condition

$$a_{n_1+p} - a_{n_1}(n_1+p)^{n_1+2} n_1^{p-n_1-2} \not\equiv 0 \pmod{p^2}$$

is not only sufficient, but also necessary for existence of a unique solution modulo p^k of the inequality $v_p(a_n) \geq k$ such that $n \equiv n_1 \pmod{p}$.

3. Solution of conjectures. First of all note that Theorem 1 implies that $v_2(a_n) = 1$ for every odd positive integer n. Indeed,

$$q_{1,2} = a_3 - a_1 \cdot 3^{1+2} \cdot 1^{2-1-2} = 78 - 2 \cdot 27 = 24 \equiv 0 \pmod{4}$$

and $a_1 = 2$, and thus if $2 \nmid n$, then $v_2(a_n) = 1$. This gives an alternative proof of Amdeberhan, Callan and Moll's result.

Theorem 1 allows us to prove that Conjecture 1 is true by verifying the condition $a_7 - a_2 \cdot 7^{2+2} \cdot 2^{5-2-2} \not\equiv 0 \pmod{5^2}$: indeed,

$$a_7 - a_2 \cdot 7^{2+2} \cdot 2^{5-2-2} = 3309110 - 10 \cdot 2401 \cdot 2 = 3261090 \equiv 15 \not\equiv 0 \pmod{25}.$$

Let us take the next Schenker prime $p = 13$. If $13 \nmid n$, then $13 \mid a_n$ if and only if $n \equiv 3 \pmod{13}$. Using Theorem 1 for $p = 13$ and $n_1 = 3$:

$$\begin{aligned} a_{16} - a_3 \cdot 16^{3+2} \cdot 3^{13-3-2} &= 105224992014096760832 - 78 \cdot 1048576 \cdot 6561 \\ &= 117 - 78 \cdot 100 \cdot 139 = -1084083 \equiv 52 \pmod{169}, \end{aligned}$$

we conclude that for every positive natural k there exists a unique solution modulo 13^k of the inequality $v_{13}(a_n) \geq k$ which is not divisible by p, and we know that it is congruent to 3 modulo 13. This implies that if $p = 13$, then Conjecture 2 is true.

Conjecture 2 states that for every odd Schenker prime p, there exists a unique $n_1 \in \mathbb{N}_+$ less than p such that $p \mid a_{n_1}$ and for this n_1 we have

$$a_{n_1+p} - a_{n_1}(n_1 + p)^{n_1+2} n_1^{p-n_1-2} \not\equiv 0 \pmod{p^2}.$$

However, it is easy to see that Conjecture 2 is not true in general. Indeed, let $p = 37$. If $37 \nmid n$, then $37 \mid a_n$ if and only if $n \equiv 25 \pmod{37}$. However, $37^2 \mid a_{62} - a_{25} \cdot 62^{27} \cdot 25^{10}$. Moreover, $a_{25} \equiv 851 = 23 \cdot 37 \pmod{37^2}$, thus $v_{37}(a_n) = 1$ for any $n \equiv 25 \pmod{37}$. Hence the 37-adic valuation of Schenker sums is bounded by one on the set of positive integers not divisible by 37. We can describe it by a simple formula:

$$v_{37}(a_n) = \begin{cases} \frac{n - s_{37}(n)}{36} & \text{when } n \equiv 0 \pmod{37}, \\ 1 & \text{when } n \equiv 25 \pmod{37}, \\ 0 & \text{when } n \not\equiv 0, 25 \pmod{37}. \end{cases}$$

Our result shows that Conjecture 2 is false for $p = 37$. Moreover, there exist prime numbers p for which the number of solutions modulo p of the

Table 1

$\lambda(p)$	p
1	5, 13, 23, 31, 37, 43, 47, 53, 59, 61, 71, 79, 101, 103, 107, 109, 127, 137, 157, 163, 173, 229, 241, 251, 257, 263, 317, 337, 349, 353, 359, 397, 421, 431, 487, 499, 503, 521, 547, 571, 577, 587, 617, 619, 641, 653, 661, 727, 733, 757, 797, 811, 821, 829, 881, 883, 937, 947, 967, 977, 991, 1013, 1031, 1039, 1091, 1097, 1123, 1163, 1181, 1213
2	41, 149, 181, 191, 199, 211, 271, 283, 293, 311, 367, 383, 401, 409, 419, 439, 461, 523, 541, 563, 569, 607, 613, 647, 673, 691, 709, 761, 787, 827, 929, 941, 983, 997, 1021, 1051, 1061, 1087, 1117, 1151, 1153, 1223
3	179, 197, 223, 277, 509, 601, 683, 743, 887, 1201
4	—
5	593

congruence $a_n \equiv 0 \pmod{p}$, where $p \nmid n$, is greater than 1. Denote this number by $\lambda(p)$ (note that p is a Schenker prime if and only if $\lambda(p) > 0$). By using Mathematica [7], we find that there are 126 Schenker primes among the 200 first prime numbers. In Table 1 we present the solutions of the equation $\lambda(p) = k$ for $k \leq 5$.

4. Questions. Although Conjecture 2 is not true, we do not know if 2 and 37 are the only primes p such that $p \mid a_n$ and

$$q_{n,p} = a_{n+p} - a_n(n+p)^{n+2}n^{p-n-2} \equiv 0 \pmod{p^2}$$

for some $n \in \mathbb{N}_+$ not divisible by p. Numerical computations show that they are unique such prime numbers among all primes less than 30000. The results above suggest the following questions:

QUESTION 1. *Is there any Schenker prime greater than* 37 *for which there exists* $n \in \mathbb{N}_+$ *such that* $p \nmid n$, $p \mid a_n$ *and* $q_{n,p} \equiv 0 \pmod{p^2}$*?*

QUESTION 2. *Are there infinitely many Schenker primes* p *for which there exists* $n \in \mathbb{N}_+$ *such that* $p \nmid n$, $p \mid a_n$ *and* $q_{n,p} \equiv 0 \pmod{p^2}$*?*

In the light of the results presented in the table some natural questions arise:

QUESTION 3. *Are there infinitely many Schenker primes* p *for which* $\lambda(p) > 1$*?*

QUESTION 4. *Let* m *be a positive integer. Is there any Schenker prime* p *such that* $\lambda(p) \geq m$*?*

In Section 1 we gave a short proof of the infinitude of the set of Schenker primes. It is natural to ask:

QUESTION 5. *Are there infinitely many primes which are not Schenker primes?*

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