COLLOQUIUM MATHEMATICUM

VOL. 118 2010 NO. 1

MASTER

BY

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Dedicated to the memory of Andrzej Hulanicki (1933–2008)

Andrzej Hulanicki was one of the most distinguished Polish mathematicians of the second half of the XXth century. He was a member of the Polish Academy of Sciences, as well as an author of over eighty original research papers, which have brought him recognition both in Poland and abroad.

In the 1930s the Polish mathematical school was one of the top in the world. World War II, the post war isolation, the death or emigration of many mathematicians has changed this status. As soon as it became possible, in the late fifties, Wrocław mathematicians and other scientists started to travel both to the West and the Soviet Union, in order to undo the isolation of Polish science.

The British Council offered stipends that allowed many to do this. With the aid of such a stipend Andrzej Hulanicki spent the years 1959–60 in Manchester, where there was a very good school of infinite groups. This was while Andrzej was pursuing his Ph.D. in the area straddling algebra, set theory, topology and measure theory. The experience from Manchester was most valuable. Andrzej was one of Wrocław's best young mathematicians and found himself in a place where nobody was interested at all in Poland's mathematics. Andrzej quickly realized what it means to be from a mathematical backyard. Since then he devoted his whole life to pull Wrocław out of that backyard. To do that he constantly pursued topics that were in the center of mathematical interest, and kept bringing to Wrocław mathematicians who talked about these subjects.

The sixties. In Manchester, Andrzej learned the theory of infinite nonabelian groups and the basics of harmonic analysis. In 1963 he spent three months in Moscow, which was then one of the most respected mathematical centers in the world. The theory of representations of Lie groups was a

The author is grateful to Maciej Paluszyński and Guido Weiss for language corrections and, in particular, for translating a part of the text from the Polish original.

DOI: 10.4064/cm118-1-1

leading research subject there at the time. In this atmosphere Andrzej established one of the basic results concerning amenable groups: *every unitary representation of a locally compact group is weakly contained in the left regular representation if and only if the group is amenable.* This theorem has become classic and is probably Andrzej's most often cited result.

In the late sixties Andrzej met people who did harmonic analysis on locally compact groups: E. Kaniuth, H. Leptin and their collaborators. This was then a very active field of research. E. Kaniuth and H. Leptin started to organize conferences in Oberwolfach. The conferences took place every three years until 2000 when E. Kaniuth retired. Only very poor mail and phone communication from communist Poland prevented Andrzej from being on the organizing committee. Still, he often took part in setting up the list of participants.

During this period Andrzej studied the symmetry of the group algebra $L^1(G)$. The theorem saying that a discrete nilpotent group G has a symmetric algebra $L^1(G)$ is contained in [32]. This was quite an important result in the field. Andrzej's original complicated proof was later simplified by J. Ludwig, H. Leptin and D. Poguntke.

The first PhD theses were written under Andrzej's supervision: by Z. Anusiak, A. Czuba, E. Płonka and T. Pytlik.

Nilpotent groups and real analysis. In 1970 Andrzej Hulanicki went to the ICM congress in Nice, where he heard a talk by Elias Stein who was laying the foundations of analysis on nilpotent groups. This analysis, which was initiated with the representations of semisimple Lie groups in mind, has developed into a powerful theory, which has been supporting researchers for more than thirty years. Applying Andrzej's experience in the area of harmonic analysis on groups to these new topics was one of the best moves in his career: much functional calculus has emerged ([35], [38]). In [38] Andrzej proved that every sublaplacian on a Lie group of polynomial growth has the same spectrum on all L^p spaces. After the paper had been accepted Andrzej received from J. Dixmier his paper on similar matters. Later on, both contributed essentially to the development of functional calculus on groups of polynomial growth (see [40], [41]).

In 1981 Andrzej Hulanicki traveled to Chicago and St. Louis. In St. Louis he learned real analysis and in Chicago he met Elias Stein again, and showed him his results on functional calculus. Stein embarked on the subject enthusiastically. Their joint result was never published separately, but entered into Folland and Stein's book "Hardy Spaces on Homogeneous Groups" as a separate chapter [54]. This result has inspired numerous mathematicians, some of whom have obtained results Andrzej could not have even imagined. It was always like that with him: having a good idea which is a starting point for interesting investigations, which by far outreached what seemed possible initially.

One should mention here paper [44], one of the most often cited papers of Andrzej. For some time Andrzej had been trying to understand the heat kernel on the Heisenberg group. It turned out that not the kernel itself but its Fourier transform was the right thing to study: Andrzej obtained an explicit formula for it. Later on, and independently, this formula was shown by B. Gaveau and it is often called the Gaveau–Hulanicki formula.

New subject, new PhD students: J. Cygan, E. Porada, P. Głowacki, J. Długosz.

Analysis on nilpotent Lie groups was applied by Andrzej to study eigenfunctions of the Schrödinger operator with a polynomial potential ([57], [59], [60], [61], [63], [66]). The basic observation was that such an operator is the image in the representation of a sublaplacian on a Lie group. This was an original and fruitful idea. Applying it Andrzej Hulanicki, with Joe Jenkins, obtained a number of theorems on eigenvalues of the Schrödinger operator and summability of related expansions, in particular, a new result concerning Riesz summability of Hermite functions. It was soon improved by S. Thangavelu in his PhD thesis written at Princeton under the supervision of E. Stein. Combination of this research and functional calculus gave rise to PhD theses of W. Cupała, W. Hebisch, J. Dziubański, A. Sikora, J. Zienkiewicz and M. Letachowicz (completed after Andrzej's passing away, under the supervision of J. Dziubański).

NA groups, 1983–2003. In the early eighties, for the benefit of my PhD, Andrzej undertook the study of solvable groups of type NA. An NA group is a semi-direct product of a nilpotent group N and an Abelian group A acting on N by automorphisms. It is an important class of groups acting on classical objects such as *symmetric spaces, homogeneous spaces with negative curvature, homogeneous cones, bounded homogeneous domains in* $\mathbb{C}^n$. It was a very natural effort. Analysis on nilpotent groups N was slowly becoming a mature theory, and the NA groups were the next step. At that time harmonic functions with respect to the Laplace–Beltrami operator were studied using the NA model. Andrzej suggested researching a wide class of left-invariant Hörmander type operators L in the context of general NA groups. This brought about a completely new viewpoint—for most of the NA groups the older methods of symmetric spaces do not apply.

It turned out that bounded functions harmonic with respect to L can be reproduced as Poisson integrals from a "maximal boundary" [68]. Existence of a positive moment of the L-Poisson kernel as well as the almost everywhere convergence of Poisson integrals to their boundary values were proved in my PhD thesis. The convergence was the right subject at the moment: for symmetric spaces it was proved by E. Stein (Princeton) in 1983 and P. Sjögren (Göteborg) in 1986. In January 1988 Andrzej gave a talk about that during a harmonic analysis semester at Berkeley. After the talk E. Stein told him that he needed existence of the moment for his proof, but he was not able to prove it and he had to proceed differently.

In the nineties we concentrated on NA groups with one-dimensional A acting via dilating automorphisms (not necessarily diagonal). Such groups can be identified with homogeneous manifolds of negative curvature. When A is one-dimensional, the Poisson kernel may be described much more precisely. We found a probability approach to Hörmander type operators that allowed us to treat operators uniformly, regardless of whether the bottom of their spectrum was strictly positive or zero. As a result we obtained a precise description at infinity of the Poisson kernel and its derivatives ([80], [84], [89], [91]) as well as a description of the Martin boundary for L.

This research area proved to be very rich, and it gave me a chance to stay in the midst of interesting topics. My first PhD student R. Urban wrote his PhD thesis and a few other papers about Poisson kernels on NA groups with one-dimensional A.

Siegel domains, 1991–2003. In the early nineties we thought that it would be a good idea to find an application to our NA groups somewhere, and homogeneous Siegel domains fit ideally. R. Penney, whom Andrzej invited to Wrocław, proposed investigating a rather natural topic in this context: Hua operators naturally defined in terms of geometry of these domains. We wrote a joint paper with Penney [81], but soon it became clear that we need to leave the rigid framework of these operators. Andrzej suggested studying a wider class of NA operators which fit wonderfully our previously constructed theory of Poisson integrals on NA groups: real, left-invariant, second order, elliptic degenerate operators annihilating holomorphic functions. We called them admissible. Applying what we knew about Poisson integrals on NA groups we obtained a striking characterization of bounded pluriharmonic functions: they are zeros of two (on tube domains) or three (type two domains) admissible operators [83], [85], [86].

Next we challenged the classical problem of describing the zeros of the Hua system on type two symmetric Siegel domains. Andrzej's idea of admissible operators proved to be another hit: by widening the class of operators we made observations thanks to which D. Buraczewski in 2003 solved the problem for second type domains proving that the zeros must be pluriharmonic. That was his PhD thesis written under my supervision and distinguished by the Prime Minister's prize in 2003.

The original problem was formulated way back in 1958 and solved for tube domains in 1980 by K. Johnson and A. Korányi who showed that then the Hua harmonic functions are exactly Poisson integrals. The papers due to N. Berline and M. Vergne (1981) and M. Lassalle (1984) followed but the case of second type domains was left open. Pluriharmonicity did not ring any bells because a semisimple group does not see it, in contrast to an NA group. It was discovered in Wrocław, because we were studying NA groups for more than twenty years and because Andrzej asked a good question about more general admissible operators.

Stochastic recursions. When the subject of Siegel domains slowly came to an end, in 2002 Andrzej invited Y. Guivarc'h, and immediately after this the NA groups reappeared. We started practicing affine recursions on them, and as a result, Dariusz Buraczewski gathered material for his habilitation. The joint paper of five of us: Buraczewski, Guivarc'h, Hulanicki, Urban and myself [92] was accepted for publication in the fall 2007. Andrzej's last Ph.D. student, W. Bąk, graduated in 2007.

Constructing a research center (1)**.** In his pursuit of best research topics Andrzej Hulanicki not only traveled to good centers, but also methodically brought to us the best experts from abroad. Regular harmonic analysis conferences commenced in 1972 and were organized every two years, with the exception of the martial law period. The last conference, organized by Andrzej, took place after his death, in April 2008. Anyone aged at least 45-50 years can imagine what it meant to organize such conferences in communist times, when communication and telephones all worked poorly. Andrzej assigned tasks among his students and infected all of us with his unequalled enthusiasm. These conferences were very warm; warmth was the one thing we could offer our guests with no problem. So they kept coming, even the best: we once had a Fields medallist.

Thanks to his contacts abroad Andrzej was able, as early as in the seventies, to arrange for his collaborators to travel abroad for extended stays. These were of paramount importance for our mathematical development, but also helped to put away money, for example for a flat. From among a few dozen who were dispatched abroad, all have returned, except for two. We returned, because we had a place to return to: a good, lively center developing harmonic analysis, and its leader, always full of new ideas, full of energy, which we, the youngsters, could only envy.

He usually set the crossbar a little higher than we thought we could clear, but he knew what he was doing. We managed, and we saw that we can achieve more and more. He was a great authority to us. In the Institute people referred to him as the "superchief".

(1) This section is part of a longer article www.math.uni.wroc.pl/~harm.

He knew how to formulate good questions, which always stimulated new, interesting research. He explained to us that it is not only what mathematicians in Princeton do that matters. A good second league also counts. He repeatedly changed the area of his research trying to keep up with the development of mathematics, passing from mature theories to new ideas, which he generously disseminated among his students. He was the advisor of seventeen PhD's, he has several mathematical grandchildren, and four grand-grandchildren (A. Hulanicki — T. Pytlik — K. Stempak — R. Kapelko, A. Nowak, and A. Hulanicki — T. Pytlik — R. Szwarc — M. Zygmunt, A. Kazun). From among his mathematical descendants Płonka, Pytlik, Głowacki, Stempak, Szwarc, Hebisch and myself have the title of professor, and Dziubański, Zienkiewicz and Żuk have habilitations.

Of primary importance is a good seminar. Since I remember, Andrzej Hulanicki's harmonic analysis seminar was not only lively, but also mandatory. As any other responsibility, it generated the sense of meaningfulness. No one of us could count on the Master's letter of recommendation if he or she had skipped the seminar. The fact that you do not understand was not a good excuse. Firstly, after a few years of not understanding one starts to understand some, and secondly we went a long way so that the young people could, eventually, understand. We were not afraid to ask silly questions, which relaxed the audience wonderfully.

The abilities to collaborate and to extract necessary information are important to mathematics. Success depends on this. Both require certain psychological qualities, like openness, ability to act unschematically, to listen, and to keep one's ambitions under control. We do not learn these from books, we learn from our colleagues. I learned from Andrzej.

In the nineties Andrzej Hulanicki was the chief creator of the European programs in our Institute. These were the two Tempus programs, three Harmonic Analysis networks, and in 2004 we were awarded the Transfer of Knowledge program. In the early nineties our friends in the West offered us access to the networks as soon as it became possible for the Eastern block countries. Undoubtedly, this was the fruit of Andrzej Hulanicki's activities throughout the previous thirty years. In 2002 the Université d'Orléans awarded Andrzej an honorary doctorate—a symbolic recognition by the international community of his contribution to mathematics.

In the past fifteen years Andrzej Hulanicki supervised a substantial remodeling of our Institute's building, including, among other things, the addition of guest rooms. They brought an entirely new quality: we can now invite specialists for extended stays and fund positions of post-doc type, both of which wonderfully enable us to extend our scientific horizons. There are eight rooms, always in demand and they have to be booked well in advance.

He gave us—new generations—a very good start.

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