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A QUASI-DICHOTOMY FOR $C(\alpha, X)$ SPACES, $\alpha < \omega_1$

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Abstract. We prove the following quasi-dichotomy involving the Banach spaces $C(\alpha, X)$ of all X-valued continuous functions defined on the interval $[0, \alpha]$ of ordinals and endowed with the supremum norm.

Suppose that X and Y are arbitrary Banach spaces of finite cotype. Then at least one of the following statements is true.

(1) There exists a finite ordinal n such that either $C(n, X)$ contains a copy of Y, or $C(n, Y)$ contains a copy of X.

(2) For any infinite countable ordinals α, β, ξ, η, the following are equivalent:

(a) $C(\alpha, X) \oplus C(\xi, Y)$ is isomorphic to $C(\beta, X) \oplus C(\eta, Y)$.

(b) $C(\alpha)$ is isomorphic to $C(\beta)$, and $C(\xi)$ is isomorphic to $C(\eta)$.

This result is optimal in the sense that it cannot be extended to uncountable ordinals.

1. Introduction. We follow the standard notation and terminology for Banach space theory, as can be found in [5]. Let K be a compact Hausdorff space and X a Banach space. We denote by $C(K, X)$ the Banach space of all X-valued continuous functions defined on K endowed with the supremum norm. If X is the scalar field, this space will be denoted by $C(K)$. When K is the interval $[0, \alpha]$ of ordinals endowed with the order topology, these spaces will be denoted respectively by $C(\alpha, X)$ and $C(\alpha)$. Let ω denote the first infinite ordinal, and ω_1 the first uncountable ordinal. Given Banach spaces X and Y, we write $X \sim Y$ whenever X and Y are isomorphic, and $Y \hookrightarrow X$ when X contains a copy of Y, that is, a subspace isomorphic to Y.

The notion of cotype of a Banach space emerged from the works of Hoffmann-Jorgensen, S. Kwapień, B. Maurey and G. Pisier in the early 1970's [4], [6], [7] and [9], and has since found frequent use in the geometry of Banach spaces; see for instance [8]. A Banach space $X \neq \{0\}$ is *of finite cotype* if there exist $2 \leq q < \infty$ and a constant $K > 0$ such that no matter how we select finitely many vectors $v_1, \ldots, v_n$ from X, we have

$$(1) \qquad \Big(\sum_{i=1}^{n} \|v_i\|^q\Big)^{1/q} \leq K\Big(\int_0^1 \Big\|\sum_{i=1}^{n} r_i(t) v_i\Big\|^2 \, dt\Big)^{1/2},$$

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where $r_i : [0,1] \to \mathbb{R}$ denote the *Rademacher functions*, defined by setting

$$r_i(t) = \mathrm{sign}(\sin 2^i \pi t).$$

It is well known that the classical l_p spaces, $1 \leq p < \infty$, are of finite cotype, and no infinite-dimensional $C(K)$ space is.

Moreover, in the setting of the local theory of Banach spaces, it was proved in [9, Corollary 1.2] that a Banach space X is of finite cotype if and only if $C(\omega)$ is not finitely representable in X, that is, there exists $\epsilon > 0$ and a finite-dimensional subspace F of $C(\omega)$ such that for every linear isomorphism T of F onto $T(F) \subset X$ we have $\|T\| \cdot \|T^{-1}\| \geq 1 + \epsilon$.

So, it seems natural to also investigate the interplay between the geometry of finite cotype spaces with the geometry of infinite-dimensional $C(K)$ spaces.

In this direction, the main purpose of this paper is to show that for any two Banach spaces of finite cotype, either one of them is isomorphic to a subspace of some finite sum of the other, or their respective spaces of vector-valued continuous functions defined on $[0, \alpha]$ for α an infinite countable ordinal are in some sense very far from each other. More precisely, our main result is the following theorem.

THEOREM 1.1. *Suppose that X and Y are Banach spaces of finite cotype such that neither X embeds into Y^n nor Y embeds into X^n for any finite ordinal n. Then for any infinite countable ordinals α, β, ξ, η, the following are equivalent:*

(a) $C(\alpha, X) \oplus C(\xi, Y) \sim C(\beta, X) \oplus C(\eta, Y)$.
(b) $C(\alpha) \sim C(\beta)$ *and* $C(\xi) \sim C(\eta)$.

REMARK 1.2. It is interesting to notice that Theorem 1.1 cannot be extended to uncountable ordinals. Indeed, take $1 < p \neq q < \infty$, $X = l_p$ and $Y = l_q$. It is well known that X and Y satisfy the hypotheses of Theorem 1.1. Moreover, since X is isomorphic to its square $X^2 = X \oplus X$, it follows that

$$C(\omega_1, X) \sim C(\omega_1, X^2) \sim C(\omega_1, X) \oplus C(\omega_1, X) \sim C(\omega_1 2, X).$$

Thus, for every ordinal ξ we have

$$C(\omega_1, X) \oplus C(\xi, Y) \sim C(\omega_1 2, X) \oplus C(\xi, Y).$$

However, according to [11], $C(\omega_1)$ is not isomorphic to $C(\omega_1 2)$.

REMARK 1.3. Without the assumption of finite cotype of X and Y, the statement of Theorem 1.1, may be false. Indeed, it is easy to check that for any Banach space $Y \neq \{0\}$ we have

$$C(\omega, C(\omega)) \oplus C(\omega^\omega, Y) \sim C(\omega^\omega, C(\omega)) \oplus C(\omega^\omega, Y),$$

but by [1] we know that $C(\omega)$ is not isomorphic to $C(\omega^\omega)$.

The paper is organized as follows. In order to prove Theorem 1.1, in Section 2, we first extend [3, Theorem 1.3.a] by proving:

THEOREM 1.4. *Let X be a Banach space of finite cotype, Y a Banach space, and α, β ordinals with $\omega \le \alpha \le \beta < \omega_1$. Then*

$$C(\alpha, X \oplus Y) \sim C(\beta, X \oplus Y) \Rightarrow \text{ either } X \hookrightarrow Y^n \text{ for some } 1 \le n < \omega, \text{ or } C(\alpha) \sim C(\beta).$$

REMARK 1.5. We stress that Theorem 1.4 in the case $X = l_1$ solves [3, Problem 5.2.a]. However, it remains an open problem whether the conclusion of Theorem 1.4 is also true when $X = l_\infty$ (see [3, Problem 5.2.b]).

Finally, in Section 3, we give the proof of Theorem 1.1. It was inspired by the proof of [3, Theorem 4.1], where the particular case $X = l_p$ and $Y = l_q$ with $1 < p \neq q < \infty$ was considered.

2. $C(\alpha, X \oplus Y)$ spaces for X of finite cotype. Before proving Theorem 1.4, we state some preliminary results. We set $C_0(\alpha, X) = \{f \in C(\alpha, X) : f(\alpha) = 0\}$. In what follows, we will often make use without explicit mention of [1, Lemma 1.2.1] which states that $C(\alpha, X)$ is isomorphic to $C_0(\alpha, X)$ whenever $\alpha \ge \omega$.

PROPOSITION 2.1. *Let X be a Banach space of finite cotype, Y a Banach space containing no copy of X, and ξ an ordinal with $\omega \le \xi < \omega_1$. Then*

$$C(\xi^\omega, X) \hookrightarrow C(\xi, X) \oplus Y \Rightarrow C(\xi, X) \hookrightarrow C(\gamma, X) \oplus Y \text{ for some } \omega \le \gamma < \xi.$$

Proof. Let $2 \le q < \infty$, and $K > 0$ be a constant satisfying (1). Suppose for contradiction that

$$C(\xi, X) \not\hookrightarrow C_0(\gamma, X) \oplus Y, \quad \forall \gamma < \xi.$$

By hypothesis there exist bounded linear operators $T_1 : C(\xi^\omega, X) \to C_0(\xi, X)$ and $T_2 : C(\xi^\omega, X) \to Y$ and a constant $M > 0$ such that for every f in $C(\xi^\omega, X)$ we have

$$M\|f\| \le \sup(\|T_1(f)\|, \|T_2(f)\|) \le \|f\|. \tag{2}$$

Fix $m \in \mathbb{N}$ such that

$$M\sqrt[q]{m}/K > 1,$$

and $\epsilon > 0$ such that

$$1 + \epsilon < M\sqrt[q]{m}/K. \tag{3}$$

For every $\eta \in [0, \xi)$, denote

$$\varDelta_\eta^1 = [\xi^m\eta + 1, \xi^m(\eta + 1)],$$

and let X_m be the subspace of all $f \in C(\xi^\omega, X)$ satisfying

$$\forall \eta \in [0, \xi), f \text{ is constant in } \varDelta_\eta^1 \quad \text{and} \quad f(\gamma) = 0, \forall \gamma \in [\xi^{m+1}, \xi^\omega].$$

As we can easily see, X_m is isometric to $C_0(\xi, X)$. Since Y contains no copy of X, the restriction of T_2 to X_m is not an isomorphism onto its image. Thus, there exists $f_1 \in X_m$ with

$$\|f_1\| = 1 \quad \text{and} \quad \|T_2(f_1)\| \leq \epsilon/2.$$

Let $0 \leq \eta_1 < \xi$ be such that there exists $x_1 \in X$ with

$$\|x_1\| = 1 \quad \text{and} \quad f_1(\gamma) = x_1, \forall \gamma \in \Delta^1_{\eta_1}.$$

Since $T_1(f_1) \in C_0(\xi, X)$, it follows that there exists $\gamma_1 < \xi$ such that

$$\|T_1(f_1(\gamma))\| \leq \epsilon/2, \qquad \forall \gamma \in [\gamma_1 + 1, \xi).$$

For every $\eta \in [0, \xi)$, denote

$$\Delta^2_\eta = [\xi^m \eta_1 + \xi^{m-1}\eta + 1, \xi^m \eta_1 + \xi^{m-1}(\eta + 1)],$$

and let X_{m-1} the subspace of all $f \in C(\xi^\omega, X)$ satisfying

$$\forall \eta \in [0, \xi), f \text{ is constant in } \Delta^2_\eta \quad \text{and} \quad f(\gamma) = 0, \forall \gamma \notin [\xi^m \eta_1, \xi^m(\eta_1 + 1)].$$

We can again easily see that X_{m-1} is isometric to $C_0(\xi, X)$. Let P_{γ_1} be the canonical projection from $C(\xi, X)$ onto $C(\gamma_1, X)$. By our hypothesis

$$C(\xi, X) \not\hookrightarrow C(\gamma_1, X) \oplus Y.$$

Thus, the restriction to X_{m-1} of the bounded linear operator $T_2 + P_{\gamma_1} T_1$ defined by

$$(T_2 + P_{\gamma_1} T_1)(g) = (T_2(g), P_{\gamma_1} T_1(g))$$

cannot be an isomorphism onto the image. So, there exists $f_2 \in X_{m-1}$ such that

$$\|f_2\| = 1, \quad \|T_2(f_2)\| \leq \epsilon/2^2, \quad \|T_1(f_2)(\gamma)\| \leq \epsilon/2^2, \forall \gamma \in [0, \gamma_1].$$

Fix $\gamma_2 \in [\gamma_1 + 1, \xi)$ such that

$$\|T_1(f_2)(\gamma)\| \leq \epsilon/2^2, \qquad \forall \gamma \in [\gamma_2 + 1, \xi).$$

Let $0 \leq \eta_2 < \xi$ be such that there exists $x_2 \in X$ with

$$\|x_2\| = 1 \quad \text{and} \quad f_2(\gamma) = x_2, \forall \gamma \in \Delta^2_{\eta_2}.$$

Repeating this procedure m times we can find ordinals $\eta_1 < \cdots < \eta_m < \xi$ and $\gamma_1 < \cdots < \gamma_m < \xi$, functions $f_1, \ldots, f_m$ and elements $x_1, \ldots, x_m \in X$ such that:

(i) $\|T_2(f_i)\| \leq \epsilon/2^i$, $1 \leq i \leq m$.
(ii) $\|T_1(f_i)(\gamma)\| \leq \epsilon/2^i$, $\forall \gamma \in [0, \gamma_{i-1}]$ and $2 \leq i \leq m$.
(iii) $\|T_1(f_i)(\gamma)\| \leq \epsilon/2^i$, $\forall \gamma \in [\gamma_i + 1, \xi]$ and $1 \leq i < m$.
(iv) $\|x_i\| = 1$, $1 \leq i \leq m$.
(v) $f_i(\gamma) = x_i$, $\forall \gamma \in \Delta^i_{\eta_i}$ and $1 \leq i \leq m$, where

$$\Delta^i_{\eta_i} = [\xi^m \eta_1 + \xi^{m-1}\eta_2 + \cdots + \xi^{m-(i-1)}\eta_i + 1, \\ \xi^m \eta_1 + \xi^{m-1}\eta_2 + \cdots + \xi^{m-(i-1)}(\eta_i + 1)].$$

Since $\|x_i\| = 1$, $1 \leq i \leq m$, it follows from (1) that

$$\sqrt[q]{m}/K \leq \left(\frac{\|x_1 + x_2 + \cdots + x_m\|^2}{2^m} + \frac{\|x_1 - x_2 + \cdots + x_m\|^2}{2^m} + \cdots + \frac{\|-x_1 - x_2 - \cdots - x_m\|^2}{2^m} \right)^{1/2}.$$

Thus, for an appropriate choice of scalars $r_i = \pm 1$, we have

$$\Big\| \sum_{i=1}^{m} r_i x_i \Big\| \geq \sqrt[q]{m}/K.$$

Let $f = \sum_{i=1}^{m} r_i f_i$. It is clear that:

(vi) $\|f\| \geq \sqrt[q]{m}/K$.
(vii) $\|T_2(f)\| \leq \epsilon$.
(viii) $\|T_1(f)\| \leq 1 + \epsilon$.

Consequently, by (2) we see that

$$M \sqrt[q]{m}/K \leq 1 + \epsilon.$$

This contradicts (3), and the proposition is proved. ■

Since Banach spaces of finite cotype contain no copy of c_0, we can use Proposition 2.1 and follow step by step the proof of [3, Proposition 2.3] to prove the following proposition.

PROPOSITION 2.2. *Let X be a Banach space of finite cotype, and Y a Banach space containing no copy of X. Then, for every $\omega \leq \xi < \omega_1$,*

$$C(\xi^\omega, X) \not\hookrightarrow C(\xi, X) \oplus Y.$$

Finally, we need to recall [2, Theorem 2.3].

THEOREM 2.3. *Let ξ be any ordinal, Y a Banach space, and X a closed subspace of $C(\xi, Y)$. Then either X is isomorphic to a subspace of Y^n for some $1 \leq n < \omega$, or X contains a copy of c_0 complemented in $C(\xi, Y)$.*

Now we are ready to prove Theorem 1.4.

Proof of Theorem 1.4. Suppose that $\omega \leq \alpha \leq \beta < \omega_1$,

$$C(\alpha, X \oplus Y) \sim C(\beta, X \oplus Y),$$

and

$$X \not\hookrightarrow Y^n, \qquad \forall 1 \leq n < \omega. \tag{4}$$

We will prove that $C(\alpha)$ is isomorphic to $C(\beta)$. For the classical Bessaga and Pełczyński Theorem [1] it suffices to show that $\beta < \alpha^\omega$. For contradiction assume that $\alpha^\omega \leq \beta$. Then by our hypothesis,

$$C(\alpha^\omega, X) \hookrightarrow C(\alpha^\omega, X \oplus Y) \hookrightarrow C(\beta, X \oplus Y) \sim C(\alpha, X \oplus Y).$$

That is,

$$C(\alpha^{\omega}, X) \hookrightarrow C(\alpha, X) \oplus C(\alpha, Y). \tag{5}$$

On the other hand, since X contains no copy of c_0 and (4) holds, it follows from Theorem 2.3 that

$$X \not\hookrightarrow C(\alpha, Y).$$

Hence according to Proposition 2.2, (5) cannot hold. Thus, the theorem is proved. ■

3. The proof of the quasi-dichotomy Theorem 1.1. In this section we prove Theorem 1.1 which provides a quasi-dichotomy for spaces of finite cotype involving the $C(\alpha, X)$ spaces, $\alpha < \omega_1$.

Proof of Theorem 1.1. First assume that statement (b) is true. Then according to [12, Corollary 21.5.2], we see that $C(\alpha, X)$ is isomorphic to $C(\beta, X)$, and $C(\xi, Y)$ is isomorphic to $C(\eta, Y)$. Consequently, (a) is also true.

Now, we will prove that (a) implies (b). Take ordinals $\omega \leq \alpha, \beta, \xi, \eta < \omega_1$ satisfying

$$C(\alpha, X) \oplus C(\xi, Y) \sim C(\beta, X) \oplus C(\eta, Y). \tag{6}$$

We will prove that $C(\alpha)$ is isomorphic to $C(\beta)$. Without loss of generality, we may assume that $\alpha \leq \beta$. By the Bessaga and Pełczyński Theorem [1], $C(\omega^{\omega^{\gamma}})$ for $0 \leq \gamma < \omega_1$ are a complete set of representatives of the isomorphism classes of $C(\alpha)$ spaces, $\omega \leq \alpha < \omega_1$. Thus, without loss of generality we can suppose that $\alpha = \omega^{\omega^{\delta_1}}$, $\xi = \omega^{\omega^{\delta_2}}$, $\beta = \omega^{\omega^{\delta_3}}$ and $\eta = \omega^{\omega^{\delta_4}}$ for some ordinals δ_i, $1 \leq i \leq 4$.

Pick $\omega < \theta < \omega_1$ such that $\theta > \delta_i$ for every $1 \leq i \leq 4$. For every $1 \leq i \leq 4$ consider the compact metric spaces

$$F_i = [0, \omega^{\omega^{\delta_i}}] \times [0, \omega^{\omega^{\theta}}] \quad \text{and} \quad K = [0, \omega^{\omega^{\theta}}].$$

It is easy to check that

$$C(K) \oplus C(\omega^{\omega^{\delta_i}}) \sim C(K), \quad \forall 1 \leq i \leq 4, \tag{7}$$

$$C([0, \omega^{\omega^{\delta_i}}] \times K) \sim C(K), \quad \forall 1 \leq i \leq 4, \tag{8}$$

and

$$C(K, X) \sim C(K, X)^n, \quad \forall 1 \leq n < \omega. \tag{9}$$

Moreover, since X contains no copy of c_0, and by hypothesis no Y^n contains a copy of X for any $1 \leq n < \omega$, Theorem 2.3 implies that

$$X \not\hookrightarrow C(K, Y). \tag{10}$$

Now, by (6) we deduce

$$C(K,Y) \oplus C(\alpha,X) \oplus C(\xi,Y) \sim C(K,Y) \oplus C(\beta,X) \oplus C(\eta,Y).$$

Thus, from (7) we infer that

$$C(\alpha,X) \oplus C(K,Y) \sim C(\beta,X) \oplus C(K,Y).$$

Next, in view of (8) we see that

$$C(\alpha,X) \oplus C([0,\alpha] \times K,Y) \sim C(\beta,X) \oplus C([0,\beta] \times K,Y),$$

that is,

$$C(\alpha,X) \oplus C(\alpha,C(K,Y)) \sim C(\beta,X) \oplus C(\beta,C(K,Y)).$$

Consequently,

$$C(\alpha,X \oplus C(K,Y)) \sim C(\beta,X \oplus C(K,Y)). \tag{11}$$

Hence from (9)–(11) and Theorem 1.4 we conclude that $C(\alpha)$ is isomorphic to $C(\beta)$.

Analogously, we prove that $C(\xi)$ is isomorphic to $C(\eta)$, and the proof is complete. ■

4. Final remarks and open problems on $C(\alpha, X)$ spaces

REMARK 4.1. Concerning extensions of Theorem 1.1, recall that if K is a countable compact metric space, then by the classical Mazurkiewicz and Sierpiński Theorem [10] K is homeomorphic to an ordinal interval $[0,\alpha]$ with $\alpha < \omega_1$. Thus, the equivalence of assertions (a) and (b) of Theorem 1.1 is a cancellation law for infinite countable compact metric spaces. We do not know whether this cancellation law can be extended to uncountable compact metric spaces, whenever the Banach spaces X and Y satisfy the hypotheses of Theorem 1.1.

However, notice that if $[0,1]$ is the interval of real numbers, then we have:

COROLLARY 4.2. *Let X and Y be Banach spaces of finite cotype, and let α and β be infinite countable ordinals. Suppose that*

$$C([0,1],X) \oplus C([0,1],Y) \sim C(\alpha,X) \oplus C(\beta,Y).$$

Then there exist $1 \le m,n < \omega$ such that

$$X \hookrightarrow Y^m \quad \text{and} \quad Y \hookrightarrow X^n.$$

Proof. Towards a contradiction, by symmetry we can assume that

$$X \not\hookrightarrow Y^m \tag{12}$$

for every $1 \le m < \omega$. Since $C([0,1])$ contains a copy of $C(\alpha^\omega)$, we would have

$$C(\alpha^\omega,X) \hookrightarrow C([0,1],X) \hookrightarrow C(\alpha,X) \oplus C(\beta,Y). \tag{13}$$

Thus, fron (12) and Theorem 2.3, we infer that $C(\beta, Y)$ contains no copy of X. Therefore, by Proposition 2.2, (13) cannot hold. ∎

Remark 4.3. Observe that Theorem 1.4 is a vector-valued extension of the classical isomorphic classification of $C(\alpha)$ spaces, $\omega \leq \alpha < \omega_1$, due to Bessaga and Pełczyński [1]. Of course, Theorem 1.4 can be used to provide the isomorphic classification of many spaces involving finite cotype spaces. For example, let c denote the cardinality of the continuum. Then it is well known that l_∞ contains no copy of $l_p(c)$, $1 < p < \infty$ [13, p. 48]. Hence it follows directly from Theorem 1.4 that if $\omega \leq \alpha \leq \beta < \omega_1$, then

$$C(\alpha, l_p(c) \oplus l_\infty) \sim C(\beta, l_p(c) \oplus l_\infty) \;\Leftrightarrow\; C(\alpha) \sim C(\beta). \tag{14}$$

However, we have not been able to solve the following problem closely related to Theorem 1.1 and (14).

Problem 4.4. *Let $1 < p < \infty$ and let α, β, ξ, η be countable ordinals. Is $C(\alpha)$ isomorphic to $C(\beta)$, and $C(\xi)$ isomorphic to $C(\eta)$, whenever*

$$C(\alpha, l_p(c)) \oplus C(\xi, l_\infty) \sim C(\beta, l_p(c)) \oplus C(\eta, l_\infty)?$$

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