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IRREDUCIBILITY OF SOME REPRESENTATIONS OF THE GROUPS OF SYMPLECTOMORPHISMS AND CONTACTOMORPHISMS

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Abstract. We show the irreducibility of some unitary representations of the group of symplectomorphisms and the group of contactomorphisms.

1. Introduction. Any action of a group G on a measure space (X, μ), preserving the class of μ, induces an action of G on the space of measurable functions on X. Under suitable assumptions, this action can be normalized to yield a unitary representation of G on the Hilbert space of square-integrable functions $L^2(X, \mu)$. Many important and well-known irreducible representations can be obtained in this fashion. Examples of actions giving rise to such representations are:

- the action of the group of compactly supported volume-preserving diffeomorphisms of a manifold M with infinite volume μ on M [19],
- the action of Thompson's groups F and T on the unit interval and the unit circle, respectively [12],
- the action of a lattice of a Lie group on its Furstenberg boundary [6, 2],
- the action of the automorphism group of a regular tree on the boundary of the tree [8],
- the action of a free group on its boundary [9, 10],
- the action of the fundamental group of a compact strictly negatively curved Riemannian manifold M on the boundary of the universal cover of M [1].

The relationship between irreducibility of such representations and the properties of the group action is fully understood only in the case of the action of a discrete group on a discrete space [3, 4, 5]. The case of transitive actions is understood much better than the general one, via the notion of imprimitivity system [11, 17].

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In this paper we prove that the representations of subgroups of the group of compactly supported diffeomorphisms of a smooth manifold, preserving a symplectic structure (Theorem 4.2) or a contact structure (Theorem 5.5), with their natural actions on the said manifold, are irreducible. We enhance the argument of [19], which was used therein to establish irreducibility of representations of the group of volume-preserving diffeomorphisms. Representations of various subgroups of the group of diffeomorphisms were also studied in [16].

The article is organized as follows. Section 2 introduces some notation used throughout the paper, and comments on the case of volume-preserving diffeomorphisms. In Section 3 we show in an elementary way that the convolution algebra of compactly supported continuous functions on the Heisenberg group contains no zero divisors. We will need this later in the case of the group of contactomorphisms. Section 4 deals with representations of the group of symplectomorphisms of a symplectic manifold, while Section 5—with representations of the group of contactomorphisms of a contact manifold.

2. Preliminaries. Let M be a smooth second-countable manifold. There exists a natural diffeomorphism-invariant measure class on M, consisting of measures having positive density with respect to the Lebesgue measure in every coordinate chart. We will refer to them simply as *Lebesgue measures*.

Let μ be a Lebesgue measure on M. For a group G acting on M by diffeomorphisms we may consider a series π_μ^θ of unitary representations on $L^2(M,\mu)$ given by

$$\pi_\mu^\theta(\gamma)f = f\circ\gamma^{-1}\left(\frac{d\gamma_*\mu}{d\mu}\right)^{1/2+i\theta}, \tag{2.1}$$

where $\theta\in\mathbb{R}$.

If a measure ν is equivalent to μ, then the operator $T\colon L^2(M,\mu)\to L^2(M,\nu)$ defined by

$$Tf = f\left(\frac{d\mu}{d\nu}\right)^{1/2+i\theta} \tag{2.2}$$

gives an isomorphism of the representations π_μ^θ and π_ν^θ. In particular, if μ is equivalent to a G-invariant measure, the representations π_μ^θ are equivalent for all $\theta\in\mathbb{R}$.

For a diffeomorphism $\phi\colon M\to M$ we define its *support* $\operatorname{supp} f$ as the closure of the set $\{p\in M : \phi(p)\neq p\}$. Compactly supported diffeomorphisms of M form a group $\mathrm{Diff}_c(M)$. In [19] it was proved that for an infinite measure μ the representation π_μ^0 of the group $\mathrm{Diff}_c(M,\mu)$ of compactly supported, measure preserving diffeomorphisms of M is irreducible. It follows that the representations π_μ^θ of the groups $\mathrm{Diff}_c(M,\mu)$ and $\mathrm{Diff}_c(M)$ are irreducible for any $\theta \in \mathbb{R}$. The idea of the proof is to take two functions $f, g \in L^2(M,\mu)$ and explicitly find a measure preserving diffeomorphism ϕ such that $\langle f, \pi_\mu^0(\phi) g\rangle \neq 0$, thus showing that f and g cannot lie in two distinct orthogonal invariant subspaces. This implies irreducibility of all π_μ^θ representations, since on $\mathrm{Diff}(M,\mu)$ they are equal to π_μ^0.

3. Convolution on the Heisenberg group. In this section we are dealing with the problem of zero divisors in the convolution algebra of compactly supported continuous functions on Heisenberg groups. In our recent paper [13], it has been shown that this algebra contains no zero divisors for any simply connected supersolvable Lie group. Here we provide an elementary proof for the special case of Heisenberg groups.

3.1. Zero divisors for $\mathbb{R}^n$. On $\mathbb{R}^n$ the following well-known statement holds. We include its simple proof, which we will try to adapt to the case of the Heisenberg group. (It can also be immediately deduced from the much stronger Theorem 4.3.3 of [15], describing the convex hull of the support of the convolution of two functions in terms of their supports).

THEOREM 3.1. *If $f, g \in L^1(\mathbb{R}^n)$ are compactly supported and nonzero, then $f * g$ is nonzero.*

Proof. Let $\hat{h}(\xi) = \int h(x) e^{-ix\xi}\, dx$ denote the Fourier transform of $h \in L^1(\mathbb{R}^n)$. Suppose that $f * g = 0$. As f and g are compactly supported, their Fourier transforms extend to entire functions. Since $\hat{f}\hat{g} = \widehat{f * g} = 0$ on $\mathbb{R}^n$, it follows by holomorphy that $\hat{f}\hat{g} = 0$ on $\mathbb{C}^n$, and either $\hat{f}$ or $\hat{g}$ must vanish. This contradicts the assumption that f and g are nonzero. ∎

3.2. The Heisenberg group. Let n be a positive integer. The multiplicative group of all matrices of the form

$$\begin{pmatrix} 1 & \bar{x}^T & z \\ 0 & I_n & \bar{y} \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.1}$$

where $z \in \mathbb{R}$, $\bar{x}, \bar{y} \in \mathbb{R}^n$, and I_n denotes the $n \times n$ identity matrix, is called the *Heisenberg group* H_n. It is a unimodular Lie group diffeomorphic with $\mathbb{R}^{2n+1}$, and its Haar measure is the $(2n+1)$-dimensional Lebesgue measure. We will identify H_n and $\mathbb{R}^{2n+1}$ as manifolds. The convolution of functions $f, g \in L^1(H_n)$ will be denoted $f *_H g$.

3.3. Convolution of compactly supported functions on H_n. Let $f \in L^1(\mathbb{R})$. Define

$$Tf(x) = \int_{-\infty}^{x} f(t)\,dt. \tag{3.2}$$

If $f \in L^2(\mathbb{R})$ is supported in $[a,b]$, then it is integrable; furthermore, if $\int f(t)\,dt = 0$, then $\operatorname{supp} Tf \subseteq [a,b]$ and we may write

$$Tf(x) = \int f(t) K_{[a,b]}(t,x)\,dt, \tag{3.3}$$

where

$$K_{[a,b]}(t,x) = \begin{cases} 1 & \text{for } a \le t \le x \le b, \\ 0 & \text{otherwise.} \end{cases} \tag{3.4}$$

Hence, $Tf \in L^2(\mathbb{R})$ and $\|Tf\|_2 \le \|K_{[a,b]}\|_2 \|f\|_2$, where $\|K_{[a,b]}\|_2$ stands for the L^2-norm of $K_{[a,b]} \in L^2(\mathbb{R}^2)$. We may iterate the process of applying T to f as long as it yields a function integrating to 0. The next lemma shows that unless $f = 0$, this process terminates.

LEMMA 3.2. *If $f \in L^2(\mathbb{R})$ is nonzero and compactly supported, then there exists $k \ge 0$ such that $T^j f$ is compactly supported in $L^2(\mathbb{R})$ for $j \le k$, and $\int T^k f(x)\,dx \ne 0$.*

Proof. If there is no such k, then $T^k f \in L^2(\mathbb{R})$ and $\int T^k f(x)\,dx = 0$ for all k. Suppose this is the case. We may assume that $\operatorname{supp} f \subseteq [0,1]$, and replace T with a bounded operator of the form (3.3) with kernel $K_{[0,1]}$.

Since f is compactly supported, $\hat{f}$ extends to an entire function on $\mathbb{C}$. We now have

$$\widehat{T^k f}(\xi) = (i\xi)^{-k} \hat{f}(\xi), \tag{3.5}$$

and by the Plancherel theorem

$$4\pi^2 \|T^k f\|_2^2 = \|\widehat{T^k f}\|_2^2 \ge \int_{-1}^{1} \big|\hat{f}(\xi)\big|^2\,d\xi \tag{3.6}$$

But $\|T\| \le \|K_{[0,1]}\|_2 < 1$, so the left-hand side of the above inequality can be made arbitrarily small. Therefore $\hat{f} = 0$, as it is an entire function vanishing on $[-1,1]$. This contradicts the assumption that f is nonzero. ■

Let $f \in L^2(H_n)$ be compactly supported. Define $Sf \in L^1(\mathbb{R}^{2n})$ by

$$Sf(\bar{x},\bar{y}) = \int_{\mathbb{R}} f(\bar{x},\bar{y},z)\,dz. \tag{3.7}$$

This is just the pushforward of f, thought of as a measure on H_n, through the quotient map $H_n \to H_n/Z(H_n) \cong \mathbb{R}^{2n}$, and is still compactly supported.

If $Sf = 0$, we may also define $Tf \in L^2(H_n)$ by applying the previously defined operator T to each coset of the center of H_n, i.e.

$$Tf(\bar{x}, \bar{y}, z) = \int_{-\infty}^{z} f(\bar{x}, \bar{y}, t)\, dt \tag{3.8}$$

The proof of the next lemma consists of a straightforward application of the Fubini theorem. Part (1) is just the fact that the pushforward map corresponding to a group homomorphism is itself a homomorphism of convolution algebras.

Lemma 3.3. *If $f, g \in L^2(H_n)$ are compactly supported, then:*

(1) $S(f *_H g) = Sf * Sg$,
(2) *if* $Sf = 0$, *then* $(Tf) *_H g = T(f *_H g)$,
(3) *if* $Sg = 0$, *then* $f *_H (Tg) = T(f *_H g)$.

Theorem 3.4. *If $f, g \in L^2(H_n)$ are compactly supported and nonzero, then $f *_H g \neq 0$.*

Proof. By Lemma 3.2 there exist minimal k and l such that $T^i f$ and $T^j g$ are compactly supported in $L^2(H_n)$ for $i \leq k$ and $j \leq l$, and $ST^k f, ST^l g \in L^1(\mathbb{R}^{2n})$ are nonzero. From Lemma 3.3 and Theorem 3.1 we obtain (using the observations preceding Lemma 3.3)

$$ST^{k+l}(f *_H g) = S(T^k f *_H T^l g) = ST^k f * ST^l g \neq 0, \tag{3.9}$$

which implies that $f *_H g \neq 0$. ∎

4. Symplectic manifolds

4.1. Symplectic manifolds. Let M be a *symplectic manifold*, that is, a $2n$-dimensional manifold equipped with a nondegenerate closed 2-form ω. A *symplectomorphism* of (M, ω) is a diffeomorphism $\phi \in \mathrm{Diff}(M)$ satisfying $\phi^*\omega = \omega$. The group of all compactly supported symplectomorphisms will be denoted by $\mathrm{Sympl}_c(M, \omega)$. Since ω is nondegenerate, ω^n defines a positive measure μ on M, invariant under the action of $\mathrm{Sympl}_c(M, \omega)$.

A standard example of a symplectic manifold is $\mathbb{R}^{2n}$, with coordinates denoted by x^i, y^i for $i \leq n$, endowed with the symplectic form $\omega_0 = \sum_{i=1}^n dx^i \wedge dy^i$. It is a theorem of Darboux that any symplectic manifold is locally symplectomorphic to $(\mathbb{R}^{2n}, \omega_0)$:

Theorem 4.1 ([7, Theorem 8.1]). *For every $p \in M$ there exists a chart $\phi\colon U \to \mathbb{R}^{2n}$ centered at p such that $\omega|_U = \phi^*\omega_0$.*

The chart satisfying the conditions of Theorem 4.1 is called a *Darboux chart.* The pushforward of μ through a Darboux chart is the standard Lebesgue measure, up to a constant factor.

The flow Fl_t^X of a complete vector field $X \in \mathfrak{X}(M)$ consists of symplectomorphisms if and only if

$$\mathcal{L}_X \omega = 0. \tag{4.1}$$

There is an easy way to produce such vector fields. Namely, consider a compactly supported smooth function $f \in C^\infty(M)$. Since ω is nondegenerate, there exists a unique vector field $X_f \in \mathfrak{X}(M)$ such that $X_f \lrcorner\, \omega = df$. This field satisfies (4.1).

For more information on symplectic manifolds see [7, 18].

4.2. The representation π_μ^0 of $\mathrm{Sympl}_c(M,\omega)$. As μ is a $\mathrm{Sympl}_c(M,\omega)$-invariant measure, the only interesting representation is π_μ^0, taking the form

$$\pi_\mu^0(\gamma)f = f \circ \gamma^{-1}. \tag{4.2}$$

Notice that the space of constant square-integrable functions is π_μ^0-invariant. It is nontrivial when $\mu(M) < \infty$. Let us denote its orthogonal complement by $\mathcal{H}$.

THEOREM 4.2. *The representation π_μ^0 of the group $\mathrm{Sympl}_c(M,\omega)$ on the space $\mathcal{H}$ is irreducible.*

Denote by $B_r^n \subseteq \mathbb{R}^n$ the ball of radius r around 0.

LEMMA 4.3. *Let $p \in M$ and let $\phi\colon U \to \mathbb{R}^{2n}$ be a Darboux chart centered at p. Then there exist $r > 0$ and for every $x \in B_{2r}^{2n}$ a symplectomorphism $\tau_x \in \mathrm{Sympl}_c(U, \omega|_U) \subseteq \mathrm{Sympl}_c(M,\omega)$ such that*

(1) $\overline{B_{3r}^{2n}} \subseteq \phi[U]$,
(2) $\phi\tau_x\phi^{-1}(y) = y + x$ *for all* $y \in B_r^{2n}$.

Proof. Take $r > 0$ satisfying (1) and a bump function $h \in C^\infty(\mathbb{R}^{2n})$ supported in $\phi[U]$ and equal to 1 on $\overline{B_{3r}^{2n}}$. On $\mathbb{R}^{2n}$ there exists a linear function f such that $X_f = x$ is a constant field. Then $X_{fh} = x$ on $\overline{B_{3r}^{2n}}$ and $\operatorname{supp} X_{fh} \subseteq \phi[U]$. The desired symplectomorphism is $\tau_x = \phi^{-1}\mathrm{Fl}_1^{X_{fh}}\phi$. ∎

By using a standard local argument we obtain the following well-known corollary:

COROLLARY 4.4. *The action of $\mathrm{Sympl}_c(M,\omega)$ on M is k-transitive for all $k \geq 1$.*

LEMMA 4.5. *Let $\phi\colon U \to \mathbb{R}^{2n}$ be a Darboux chart. Then for every nontrivial π_μ^0-invariant subspace $\mathcal{H}_0$ of $\mathcal{H}$, there exists $f \in \mathcal{H}_0$ such that $f \neq 0$ and $\operatorname{supp} f \subseteq U$.*

Proof. We may assume that $0 \in U = \phi[U] \subseteq \mathbb{R}^{2n}$. Let $r > 0$ be as in Lemma 4.3. Take a nonzero $g \in \mathcal{H}_0$. The 2-transitivity of $\mathrm{Sympl}_c(M,\omega)$ allows us to assume without loss of generality that there exists $c \in \mathbb{R}$ such that the sets $A = \{p \in B^{2n}_r : \operatorname{Re} g(p) < c\}$ and $B = \{p \in B^{2n}_r : \operatorname{Re} g(p) > c\}$ both have positive measure. By the Lebesgue density theorem there exist $a \in A$ and $b \in B$ with the property that A (resp. B) has Lebesgue density 1 at a (resp. b). Lemma 4.3 asserts the existence of a symplectomorphism $\tau = \tau_{b-a}$ that takes a onto b and preserves the Lebesgue density on B^{2n}_{3r}. The function $f = g - \pi^0_\mu(\tau)g \in \mathcal{H}_0$ then satisfies the conclusion of the lemma. ■

Proof of Theorem 4.2. Suppose that $\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_0^\perp$ is a nontrivial decomposition into π^0_μ-invariant subspaces. Let $\phi\colon U \to \mathbb{R}^{2n}$ be a Darboux chart, and let $r > 0$ and $\tau_x \in \mathrm{Sympl}_c(M, \omega)$ be as in Lemma 4.3. Without loss of generality assume that $U = \phi[U] \subseteq \mathbb{R}^{2n}$. By Lemma 4.5 we may choose nonzero $f \in \mathcal{H}_0$ and $g \in \mathcal{H}_0^\perp$ supported in B^{2n}_r. We have

$$(4.3) \qquad \langle f, \pi^0_\mu(\tau_x)g\rangle = \int_{B^{2n}_r} f(y)\overline{g(\tau_x^{-1}(y))}\,dy = f * g^*(x),$$

where $g^*(y) = \overline{g(-y)}$. But from Theorem 3.1 we know that this is nonzero for some $x \in \operatorname{supp} f * g^* \subseteq B^{2n}_{2r}$. We obtain a contradiction, since $\pi^0_\mu(\tau_x)g \in \mathcal{H}_0^\perp$. ■

5. Contact manifolds

5.1. Contact manifolds. Let $\dim M = 2n + 1$. A *contact form* on M is a 1-form $\alpha \in \Omega^1(M)$ suct that $\alpha \wedge (d\alpha)^n$ is a volume form. Consider a $2n$-dimensional distribution $\xi \leq TM$. There exists an open cover $\mathcal{U} = \{U_i\}$ of M such that for every $U \in \mathcal{U}$ the restriction $\xi|_U$ is the kernel of a 1-form $\alpha_U \in \Omega^1(U)$. If the forms α_U are contact forms, we call (M, ξ) a *contact manifold.* Unless ξ is the kernel of a globally defined contact form, there is no distinguished measure on M.

Assume for the rest of this section that (M, ξ) is a contact manifold. A *contactomorphism* of (M, ξ) is a diffeomorphism $\phi \in \mathrm{Diff}(M)$ such that $\phi_*\xi = \xi$. The group of compactly supported contactomorphisms will be denoted by $\mathrm{Cont}_c(M, \xi)$.

An example of a contact manifold is the Heisenberg group H_n with the distribution $\xi = \ker \alpha_0$, where $\alpha_0 = dz - \sum_i y^i dx^i$ is a right-invariant form on H_n.

There is an analogue of Darboux theorem for contact manifolds:

THEOREM 5.1 ([14, Theorem 2.5.1]). *For every $p \in M$ there exists a chart $\phi\colon U \to H_n$ centered at p such that $\xi|_U = \ker \phi^*\alpha_0$.*

Let $U \subseteq M$ be such that $\xi|_U = \ker \alpha$ for some $\alpha \in \Omega^1(U)$. There exists a unique vector field $R \in \mathfrak{X}(U)$ such that $\alpha(R) = 1$ and $R \lrcorner\, d\alpha = 0$, called the *Reeb vector field.* If $X \in \mathfrak{X}(U)$ is a complete vector field, then its flow

Fl^X consists of contactomorphisms if and only if

$$\mathcal{L}_X\alpha = u\alpha \tag{5.1}$$

for some $u \in C^\infty(U)$. If we take any $f \in C^\infty(U)$, by nondegeneracy of $d\alpha$ there exists $X_f \in \mathfrak{X}(U)$ satisfying $\alpha(X_f) = f$ and $X_f \lrcorner\, d\alpha = df(R)\alpha - df$. These conditions imply (5.1). Conversely, if X satisfies (5.1), then it is of the form X_f for $f = \alpha(X)$.

For more information on contact manifolds see [14].

5.2. Representations of $\mathrm{Cont}_c(M,\xi)$

LEMMA 5.2. *Let $p \in M$ and let $\phi\colon U \to H_n$ be a Darboux chart centered at p. Then there exist an open set $V \subseteq H_n$, a convex open neighborhood W of 0 in the Lie algebra of H_n, and for every $x \in \exp[W]$ a contactomorphism $\rho_x \in \mathrm{Cont}_c(U, \xi|_U) \subseteq \mathrm{Cont}_c(M,\xi)$ such that*

(1) $0 \in V \subseteq VV \subseteq \exp[W] \subseteq \overline{V\exp[W]} \subseteq \phi[U]$,
(2) $\phi\rho_x\phi^{-1}(y) = yx$ *for all* $y \in V$.

Proof. Existence of V and W satisfying (1) is obvious. Let $x = \exp v$, where $v \in W$. Then v extends to a left-invariant vector field $X \in \mathfrak{X}(H_n)$, and $\mathrm{Fl}^X_t = R_{\exp tv}$, where R_y is right multiplication by y. If $f = h\alpha_0(X)$, where $h|_{V\exp[W]} = 1$ and $\operatorname{supp} h \subseteq \phi[U]$, then $X_f = X$ on $V\exp[W]$. The contactomorphism $\rho_x = \phi^{-1}\mathrm{Fl}^{X_f}_1\phi$ satisfies condition (2). ∎

COROLLARY 5.3. *The action of $\mathrm{Cont}_c(M,\xi)$ on M is k-transitive for all $k \geq 1$.*

Now, fix a Darboux chart $\phi\colon U \to H_n$ and a Lebesgue measure μ on M such that $0 \in \phi[U]$ and $\phi_*\mu$ is the standard Lebesgue measure on $\phi[U] \subseteq \mathbb{R}^{2n+1}$.

LEMMA 5.4. *Let $\phi\colon U \to H_n$ be a Darboux chart. Then for every nontrivial π^θ_μ-invariant subspace $\mathcal{H}_0 \subseteq L^2(M,\mu)$ there exists $f \in \mathcal{H}_0$ such that $f \neq 0$ and $\operatorname{supp} f \subseteq U$.*

Proof. Without loss of generality assume that $0 \in U \subseteq H_n$ and $\xi|_U = \ker\alpha_0$. Let $\delta_t(\bar{x},\bar{y},z) = (e^t\bar{x}, e^t\bar{y}, e^{2t}z)$ be the flow of the field $X = (\bar{x},\bar{y},2z)$. We have $\delta_t^*\alpha_0 = e^{2t}\alpha_0$, so $X = X_g$ for some function $g \in C^\infty(H_n)$.

There exist $V = B^{2n+1}_r \subseteq \overline{V} \subseteq U$ and a smooth function h supported in U such that $h|_V = g|_V$. Let $\psi_t = \mathrm{Fl}^{X_h}_t$. Then $\psi_t|_V = \delta_t|_V$ for $t < 0$. Now, by transitivity of $\mathrm{Cont}_c(M,\xi)$, we may take a nonzero $f \in \mathcal{H}_0$ such that $\operatorname{supp} f \cap V \neq \emptyset$. Since

$$\int_V |\pi^\theta_\mu(\psi_t)f|^2\,d\mu = \int_{\psi_{-t}[V]} |f|^2\,d\mu \xrightarrow[t\to\infty]{} 0, \tag{5.2}$$

there exists $t > 0$ such that $f - \pi_\mu^\theta(\psi_t)f$ satisfies the conclusion of the lemma. ■

THEOREM 5.5. *For every $\theta \in \mathbb{R}$ the representation π_μ^θ of $\mathrm{Cont}_c(M, \xi)$ on the space $L^2(M, \mu)$ is irreducible.*

Proof. The proof is analogous to the proof of Theorem 4.2. Lemma 5.2 gives us $V \subseteq U$ and contactomorphisms ρ_x, such that for f and g supported in V the matrix coefficient $\langle f, \pi_\mu^\theta(\rho_x)g\rangle$ is nonzero for some ρ_x because of Theorem 3.4. ■

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