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BLOW UP FOR A COMPLETELY COUPLED FUJITA TYPE REACTION-DIFFUSION SYSTEM

BY

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Abstract. This paper provides blow up results of Fujita type for a reaction-diffusion system of 3 equations in the form $u_t - \Delta(a_{11}u) = h(t,x)|v|^p$, $v_t - \Delta(a_{21}u) - \Delta(a_{22}v) = k(t,x)|w|^q$, $w_t - \Delta(a_{31}u) - \Delta(a_{32}v) - \Delta(a_{33}w) = l(t,x)|u|^r$, for $x \in \mathbb{R}^N$, $t > 0$, $p > 0$, $q > 0$, $r > 0$, $a_{ij} = a_{ij}(t,x,u,v)$, under initial conditions $u(0,x) = u_0(x), v(0,x) = v_0(x), w(0,x) = w_0(x)$ for $x \in \mathbb{R}^N$, where u_0, v_0, w_0 are nonnegative, continuous and bounded functions. Subject to conditions on dependence on the parameters p, q, r, N and the growth of the functions h, k, l at infinity, we prove finite blow up time for every solution of the above system, generalizing results of H. Fujita for the scalar Cauchy problem, of M. Escobedo and M. A. Herrero, of Fila, Levine and Uda, and of J. Rencławowicz for systems.

1. Introduction. The aim of this paper is to establish new results on blowing up solutions to systems of parabolic equations of Fujita type. We are mainly interested in critical exponents. Since the pioneering articles of Fujita [6], [7], critical exponents have attracted the attention of a sizable number of researchers. For valuable surveys of Fujita type theorems for equations as well as for systems of reaction-diffusion equations we refer to Levine [10], Samarskiĭ–Galaktionov–Kurdyumov–Mikhaĭlov [16], Bandle–Brunner [1] and Deng–Levine [3]. In [4], Escobedo and Herrero considered the system

$$\text{(EH)}\qquad \begin{cases} u_t = \delta\Delta u + u^p, \quad v_t = \Delta v + v^q, \quad t > 0,\ x \in \mathbb{R}^N, \\ u(0,x) = u_0 \ge 0, \quad v(0,x) = v_0 \ge 0, \quad x \in \mathbb{R}^N, \\ u_0, v_0 \in L^\infty(\mathbb{R}^N), \end{cases}$$

with $\delta = 1$, $p, q > 0$, and derived global existence and blow up results for (EH). They showed that all positive solutions of (EH) blow up in finite time for

$$1 < pq \quad \text{and} \quad \frac{N}{2} \le \frac{\max(p,q)+1}{pq-1},$$

while positive global solutions exist for $N/2 > (\max(p,q)+1)/(pq-1)$. They used the same technique as Fujita. Let us mention that they strongly use the fact that the two parabolic equations in (EH) have the same diffusion operators. Then in [5], Fila, Levine and Uda extended the results to the

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case where $0 \leq \delta \leq 1$. They use the same technique as in [4] and a property satisfied by the heat kernel. In a recent paper, Guedda and Kirane [8] considered, with respect to the nonexistence of global solutions, the more general system

$$\text{(GK)} \qquad \begin{cases} u_t = -a(-\Delta)^{\alpha/2}u + k(t,x)v^p, \\ v_t = -b(-\Delta)^{\beta/2}v + h(t,x)u^q, \end{cases}$$

for any $a, b, \alpha, \beta > 0$; for $0 < \alpha, \beta \leq 2$, the operators $(-\Delta)^{\alpha/2}$ and $(-\Delta)^{\beta/2}$ stand for diffusion in media with impurities. We define $(-\Delta)^{\alpha/2}$ by the formula $((-\Delta)^{\alpha/2}v)(x) = \mathcal{F}^{-1}(|\xi|^\alpha \widehat{v}(\xi))(x)$ where $\widehat{\ } = \mathcal{F}$ denotes the Fourier transform and $\mathcal{F}^{-1}$ its inverse.

The conditions on h, k required are

$$\begin{cases} 0 < k(R^2t, Ry) \simeq CR^{\lambda_k}, \\ 0 < h(R^2t, Ry) \simeq CR^{\lambda_h}, \end{cases}$$

for some $\lambda_k, \lambda_h \in \mathbb{R}$, $R \gg 1$ and (t, y) in a bounded domain in $Q := \mathbb{R}^+ \times \mathbb{R}^N$. Observe that the diffusive operators in (GK) are different and that the reaction terms are nonautonomous. One can take for example $k(t,x) = t^{\sigma_1}|x|^{\varrho_1}$ and $h(t,x) = t^{\sigma_2}|x|^{\varrho_2}$.

Let us note in passing that Kusano and Oharu [9] and Oharu [13] gave sufficient conditions for the existence of solutions to the Cauchy problem for the weakly coupled system

$$\begin{aligned} u_t &= \Delta u + f(x,u,v), \\ v_t &= \Delta v + g(x,u,v). \end{aligned}$$

In [14], Rencławowicz studied the completely coupled Fujita-type system

$$\text{(R)} \qquad \begin{cases} u_t = \Delta u + v^p, & (t,x) \in Q, \\ v_t = \Delta v + w^q, & (t,x) \in Q, \\ w_t = \Delta w + u^r, & (t,x) \in Q, \end{cases}$$

with $p, q, r > 0$, $N \geq 1$ and nonnegative bounded continuous initial values. She proved that, if $pqr \leq 1$, then any solution is global, while when $pqr > 1$ and $N/2 \leq \max(\alpha, \beta, \gamma)$ where

$$\alpha = \frac{1+p+pq}{pqr-1}, \qquad \beta = \frac{1+q+rq}{pqr-1}, \qquad \gamma = \frac{1+r+rp}{pqr-1},$$

then every nontrivial solution exhibits a finite blow up time. She also uses Fujita's method; that is why in (R), the equations have the same diffusion coefficient. In [15], she extended her study to a diagonal system of N equations.

Here, we present global nonexistence results for the triangular system

$$\text{(IK)}\qquad \begin{cases} u_t - \Delta(a_{11}u) = h(t,x)|v|^p & \text{in } Q,\\ v_t - \Delta(a_{21}u) - \Delta(a_{22}v) = k(t,x)|w|^q & \text{in } Q,\\ w_t - \Delta(a_{31}u) - \Delta(a_{32}v) - \Delta(a_{33}w) = l(t,x)|u|^r & \text{in } Q,\\ u_0 \ge 0, \quad v_0 \ge 0, \quad w_0 \ge 0, \end{cases}$$

where $0 < p,q,r$ are real, $a_{ij}(t,x,u,v)$ are measurable, positive bounded functions, and the nontrivial nonnegative functions h, k and l are assumed to satisfy

$$(1.1)\qquad \begin{cases} 0 < h(R^2\tau, Ry) \simeq CR^{\mu},\\ 0 < k(R^2\tau, Ry) \simeq CR^{\kappa},\\ 0 < l(R^2\tau, Ry) \simeq CR^{\lambda}, \end{cases}$$

for $\mu,\kappa,\lambda \in \mathbb{R}$, $R \ll 1$, and (τ, y) belonging to a bounded set in Q.

Observe that nonnegative initial data for (IK) do not necessarily lead to nonnegative solutions.

It is absolutely clear that our system (IK) is not only much more general than those cited above with respect to the reaction terms but also concerning the diffusion terms. Let us point out that the method of Fujita is here inoperative. We deal with nonlinear operators which generate propagators rather than semigroups and for which we have no comparison result.

Our method of proof is that introduced by Mitidieri, Pokhozhaev and Tesei in [11] and [12]; very close ideas were developed previously by Baras and Pierre [2]. Before setting our theorem concerning (IK), let us define the solutions we use.

Definition 1. The 3-tuple (u,v,w) such that $u \in C([0,T]; L^1_{\mathrm{loc}}(\mathbb{R}^N)) \cap C([0,T]; L^r_{\mathrm{loc}}((0,T)\times\mathbb{R}^N, l\,dtdx))$, $v \in C([0,T]; L^1_{\mathrm{loc}}(\mathbb{R}^N)) \cap L^p_{\mathrm{loc}}((0,T)\times\mathbb{R}^N, h\,dtdx))$ and $w \in C([0,T]; L^1_{\mathrm{loc}}(\mathbb{R}^N)) \cap L^q_{\mathrm{loc}}((0,T)\times\mathbb{R}^N, k\,dtdx))$ is called a *solution* to system (IK) if

$$(1.2)\qquad \begin{cases} -\int_{\mathbb{R}^N} u_0\xi(0) - \int_Q (u\xi_t - a_{11}u\Delta\xi) = \int_Q h|v|^p\xi,\\ -\int_{\mathbb{R}^N} v_0\xi(0) - \int_Q (v\xi_t - (a_{21}u + a_{22}v)\Delta\xi) = \int_Q k|w|^q\xi,\\ -\int_{\mathbb{R}^N} w_0\xi(0) - \int_Q (w\xi_t - (a_{31}u + a_{32}v + a_{33}w)\Delta\xi) = \int_Q l|u|^r\xi, \end{cases}$$

for any nonnegative test function $\xi \in C^2_0(\mathbb{R}^+ \times \mathbb{R}^N)$ with $\xi(T,x) = 0$. If $T = \infty$, we say that (u,v,w) is a *global weak solution*.

Here, we also require that the nonnegative initial data $(u_0(x), v_0(x), w_0(x))$ is such that a local solution exists.

NOTATION. We let $L^r_{\text{loc}}((0,T)\times\mathbb{R}^N, l\,dtdx))$ be the set of all functions $u:(0,T)\times\mathbb{R}^N\to\mathbb{R}$ such that $\int_K |u|^r l\,dt\,dx<\infty$ for any compact $K\subset(0,T)\times\mathbb{R}^N$.

Our result is

THEOREM 1. *Let (u,v,x) be a solution of* (IK) *such that $u_0,v_0,w_0\ge 0$. Let $pqr>1$ and*

$$N\le\min\left\{\frac{r(\mu+2+p(k+2))+\lambda+2}{pqr-1},\frac{p[r(k+2)+(\lambda+2)]}{pqr-p},\frac{\lambda+2}{r-1},\frac{q[r(\mu+2)+(\lambda+2)]}{pqr-q}\right\}. \tag{1.3}$$

Then every nontrivial solution of (IK) *blows up in finite time.*

Proof. The proof is by contradiction. Let (u,v,w) be a global solution of (IK) with $u_0,v_0,w_0\ge0$, $pqr>1$, and suppose (1.3) is satisfied. Let ξ be a nonnegative test function such that

$$\text{(H)}\qquad\begin{cases}\int_Q(l|\xi|)^{-\widetilde r/r}|\xi_t|^{\widetilde r}<\infty, & \int_Q(h|\xi|)^{-\widetilde p/p}|\xi_t|^{\widetilde p}<\infty,\\ \int_Q(k\,|\xi|)^{-\widetilde q/q}|\xi_t|^{\widetilde q}<\infty, & \int_Q(l\,|\xi|)^{-\widetilde r/r}|\Delta\xi|^{\widetilde r}<\infty,\\ \int_Q(h|\xi|)^{-\widetilde p/p}|\Delta\xi|^{\widetilde p}<\infty, & \int_Q(k|\xi|)^{-\widetilde q/q}|\Delta\xi|^{\widetilde q}<\infty.\end{cases}$$

As $u_0,v_0,w_0\ge0$, using (1.2) we have, for $\xi\ge0$,

$$\int_Q h|v|^p\xi\le\int_Q|u|\cdot|\xi_t|+\|a_{11}\|_\infty\int_Q|u|\cdot|\Delta\xi|, \tag{1.4}$$

$$\int_Q k|w|^q\xi\le\int_Q|v|\cdot|\xi_t|+\|a_{21}\|_\infty\int_Q|u|\cdot|\Delta\xi|+\|a_{22}\|_\infty\int_Q|v|\cdot|\Delta\xi|, \tag{1.5}$$

$$\int_Q l|u|^p\xi\le\int_Q|w||\xi_t|+\|a_{31}\|_\infty\int_Q|u|\cdot|\Delta\xi|+\|a_{32}\|_\infty\int_Q|v|\cdot|\Delta\xi| +\|a_{33}\|_\infty\int_Q|w|\cdot|\Delta\xi|, \tag{1.6}$$

where $\|a_{21}\|_\infty:=\max_{t,x}|a_{21}|$, etc. Writing

$$\int_Q|u|\cdot|\xi_t|=\int_Q|u|(l|\xi|)^{1/r}(l|\xi|)^{-1/r}|\xi_t|$$

and using the Hölder inequality, we obtain

$$\int_Q|u|\cdot|\xi_t|\le\Big(\int_Q|u|^r l|\xi|\Big)^{1/r}\Big(\int_Q(l|\xi|)^{-\widetilde r/r}|\xi_t|^{\widetilde r}\Big)^{1/\widetilde r}, \tag{1.7}$$

where $1/\widetilde{r}+1/r=1$. Similarly

$$\int_Q |u|\cdot|\Delta\xi| \le \Big(\int_Q |u|^r l|\xi|\Big)^{1/r}\Big(\int_Q (l|\xi|)^{-\widetilde{r}/r}|\Delta\xi|^{\widetilde{r}}\Big)^{1/\widetilde{r}}. \tag{1.8}$$

Next, we set

$$\begin{aligned}\mathcal{A}_{r,l} &= \Big(\int_Q (l\xi)^{-\widetilde{r}/r}|\xi_t|^{\widetilde{r}}\Big)^{1/\widetilde{r}} + C\Big(\int_Q (l\xi)^{-\widetilde{r}/r}|\Delta\xi|^{\widetilde{r}}\Big)^{1/\widetilde{r}}\\ &=: \mathcal{A}^{(1)}_{r,l} + C\mathcal{A}^{(2)}_{r,l}\end{aligned}$$

and

$$X := \Big(\int_Q |u|^r l\xi\Big)^{1/r} \quad\text{and}\quad \mathcal{Y} := \Big(\int_Q |v|^p h\xi\Big)^{1/p};$$

here C denotes a constant that may change in different occurrences. Then using (1.6) and (1.7) in (1.3), we obtain

$$\mathcal{Y}^p \le X\mathcal{A}_{r,l}. \tag{1.9}$$

We also have

$$\int k|w|^q\xi \le \Big(\int_Q h|v|^p\xi\Big)^{1/p}\mathcal{A}_{p,h} + C\Big(\int_Q |u|^r l\xi\Big)^{1/r}\mathcal{A}^{(2)}_{r,l},$$

$$\begin{aligned}\int_Q l|u|^p\xi &\le \Big(\int_Q k|w|^q\xi\Big)^{1/q}\mathcal{A}_{q,k} + C\Big(\int_Q |u|^r l\xi\Big)^{1/r}\mathcal{A}^{(2)}_{r,l}\\ &\quad + C\Big(\int_Q |v|^p h\xi\Big)^{1/p}\mathcal{A}_{p,h}.\end{aligned}$$

If we set

$$\mathcal{Z} := \Big(\int_Q |w|^q k\xi\Big)^{1/q},$$

then we can write

$$\mathcal{Z}^q \le \mathcal{Y}\mathcal{A}_{p,h} + CX\mathcal{A}^{(2)}_{r,l}, \tag{1.10}$$

$$X^r \le \mathcal{Z}\mathcal{A}_{q,k} + CX\mathcal{A}^{(2)}_{r,l} + C\mathcal{Y}\mathcal{A}_{p,h}. \tag{1.11}$$

So,

$$\mathcal{Z}^{pq} \le C\mathcal{Y}^p\mathcal{A}^p_{p,h} + CX^p(\mathcal{A}^{(2)}_{r,l})^p, \tag{1.12}$$

$$X^{rpq} \le C\mathcal{Z}^{pq}\mathcal{A}^{pq}_{q,k} + CX^{pq}(\mathcal{A}^{(2)}_{r,l})^{pq} + C\mathcal{Y}^{pq}\mathcal{A}^{pq}_{p,h}. \tag{1.13}$$

Inserting (1.12) in (1.13), we get

$$\begin{aligned}X^{pqr} &\le C\mathcal{Y}^p\mathcal{A}^p_{p,h}\mathcal{A}^{pq}_{q,k} + CX^p(\mathcal{A}^{(2)}_{r,l})^p\mathcal{A}^{pq}_{q,k}\\ &\quad + CX^{pq}(\mathcal{A}^{(2)}_{r,l})^{pq} + C\mathcal{Y}^{pq}\mathcal{A}^{pq}_{p,h}.\end{aligned} \tag{1.14}$$

Using now (1.9) in (1.14), we obtain

$$X^{pqr} \le CX\mathcal{A}_{r,l}\mathcal{A}_{p,h}^{p}\mathcal{A}_{q,k}^{pq} + CX^{p}(\mathcal{A}_{r,l}^{(2)})^{p}\mathcal{A}_{q,k}^{pq} + CX^{pq}(\mathcal{A}_{r,l}^{(2)})^{pq} + CX^{q}\mathcal{A}_{p,h}^{pq}\mathcal{A}_{r,l}^{q},$$

which we write as

$$X^{pqr-1} \le a + bX^{p-1} + cX^{pq-1} + dX^{q-1} \tag{1.15}$$

with

$$a = C\mathcal{A}_{r,l}\mathcal{A}_{p,h}^{p}\mathcal{A}_{q,k}^{pq}, \quad b = C\mathcal{A}_{q,k}^{pq}(\mathcal{A}_{r,l}^{(2)})^{p},$$
$$c = C(\mathcal{A}_{r,l}^{(2)})^{pq}, \quad d = C\mathcal{A}_{p,h}^{pq}\mathcal{A}_{r,l}^{q}.$$

Now, by using the ε-Young inequality, we obtain

$$bX^{p-1} \le \varepsilon X^{pqr-1} + C_\varepsilon b^{\widetilde{\alpha}} \tag{1.16}$$

where $\alpha := (pqr-1)/(p-1)$ and $1/\widetilde{\alpha} + 1/\alpha = 1$; similarly

$$cX^{pq-1} \le \varepsilon X^{pqr-1} + C_\varepsilon c^{\widetilde{\beta}} \tag{1.17}$$

where $\beta = (pqr-1)/(pq-1)$ and $1/\widetilde{\beta} + 1/\beta = 1$; and

$$dX^{q-1} \le \varepsilon X^{pqr-1} + C_\varepsilon d^{\widetilde{\gamma}} \tag{1.18}$$

where $\gamma = (pqr-1)/(q-1)$, $1/\widetilde{\gamma} + 1/\gamma = 1$. C_ε has a different meaning in (1.16), (1.17) and (1.18). Taking ε small enough and using (1.16)–(1.18) in (1.15), we obtain

$$(1-3\varepsilon)X^{pqr-1} \le a + C_\varepsilon(b^{\widetilde{\alpha}} + c^{\widetilde{\beta}} + d^{\widetilde{\gamma}}). \tag{1.19}$$

Next, we consider $\phi \in C^2(\mathbb{R};\mathbb{R}^+)$ such that

$$\phi(r) = \begin{cases} 1 & \text{for } r \le 1, \\ 0 & \text{for } r \ge 2, \end{cases}$$

and $0 \le \phi \le 1$ for any $r > 0$. If we set

$$\xi(t,x) = \phi^{\lambda}\left(\frac{t^2 + |x|^2}{R^2}\right), \quad R > 0,$$

and take λ large enough, we ensure the validity of the requirement (H) at the beginning of the proof.

At this stage, we introduce the scaled variables

$$\tau = tR^{-2}, \quad y = xR^{-1}.$$

We have the estimates

$$\mathcal{A}_{p,h} \le CR^{[-(\mu+2)+N(p-1)]/p},$$
$$\mathcal{A}_{p,h}^{p} \le CR^{-(\mu+2)+N(p-1)},$$
$$\mathcal{A}_{q,k}^{pq} \le CR^{p[-(k+2)+N(q-1)]},$$

$$(\mathcal{A}^{(2)}_{r,l})^p \leq R^{(p/r)[N(r-1)-(\lambda+2)]}, \quad \mathcal{A}^q_{r,l} \leq R^{(q/r)[N(r-1)-(\lambda+2)]}.$$

So

$$a \leq CR^{s_a}, \quad b \leq CR^{s_b}, \quad c \leq CR^{s_c}, \quad d \leq CR^{s_d},$$

where

$$\begin{aligned} s_a &= N(pq-p) - p(k+2) - (\mu+2) + N(p-1) + \frac{-(\lambda+2)+N(r-1)}{r}, \\ s_b &= -p(k+2) + Np(q-1) + \frac{p}{r}[N(r-1)-(\lambda+2)] \\ rs_c &= pq[-(\lambda+2)+N(r-1)], \\ s_d &= -q(\mu+2) + qN(p-1) + \frac{q}{r}[N(r-1)-(\lambda+2)]. \end{aligned}$$

Now, we require

$$s_a \leq 0, \quad s_b \leq 0, \quad s_c \leq 0, \quad s_d \leq 0,$$

which are, respectively, equivalent to

$$\begin{aligned} N &\leq \frac{r[\mu+2+p(k+2)]+\lambda+2}{pqr-1}, \\ N &\leq \frac{p[r(k+2)+\lambda+2]}{pqr-p}, \\ N &\leq \frac{\lambda+2}{r-1}, \\ N &\leq \frac{q[r(\mu+2)+\lambda+2]}{pqr-q}; \end{aligned}$$

in other words,

$$N \leq \min\left\{\frac{r[\mu+2+p(k+2)]+\lambda+2}{pqr-1}, \frac{r(k+2)+(\lambda+2)}{qr-1}, \frac{\lambda+2}{r-1}, \frac{r(\mu+2)+(\lambda+2)}{pr-1}\right\}.$$

We have two cases:

• Either $s_a < 0$, $s_b < 0$, $s_c < 0$, and $s_d < 0$. In this case, we let $R \to \infty$ in (1.19) to obtain

$$\lim_{R\to\infty} X^{pqr-1} = 0,$$

hence $u \equiv 0$; this in turn implies $v \equiv 0$ via (1.9); and finally $w \equiv 0$ from (1.10)—a contradiction.

• Or $s_a < 0$ or $s_b < 0$ or $s_c < 0$ or $s_d < 0$, i.e. at least one of the exponents is zero. In this case, we get

$$\lim_{R\to\infty} X^{pqr-1} \leq C < \infty.$$

So

$$\lim_{R\to\infty}\int_{\Omega_R}|u|^r l\xi = 0$$

where $\Omega_R := \{(t,x) : R^2 \le t + |x|^2 \le 2R^2\}$. Now we write (1.9) in the form

$$\int |v|^p h\xi \le \Big(\int_{\Omega_R}|u|^r l\xi\Big)\mathcal{A}_{r,l}$$

and let $R \to \infty$. The right-hand side goes to zero while the left-hand side is assumed to be positive—a contradiction.

Remark 1. When the system (IK) is diagonal ($a_{21} = a_{31} = a_{32} = 0$), the inequalities (1.9)–(1.11) become

$$\mathcal{Y}^p \le \mathcal{A}_{r,l}X, \quad \mathcal{Z}^q \le \mathcal{A}_{p,h}\,\mathcal{Y}, \quad X^r \le \mathcal{A}_{q,k}\,\mathcal{Z},$$

which combined leads to

$$\begin{aligned} X^{pqr-1} &\le \mathcal{A}_{r,l}\mathcal{A}_{p,h}^{p}\mathcal{A}_{q,k}^{pq},\\ \mathcal{Y}^{pqr-1} &\le \mathcal{A}_{r,l}^{rq}\mathcal{A}_{p,h}\mathcal{A}_{q,k}^{q},\\ \mathcal{Z}^{pqr-1} &\le \mathcal{A}_{r,l}^{r}\mathcal{A}_{p,h}^{pr}\mathcal{A}_{q,k}. \end{aligned}$$

Now, if we use the scaled variables, we obtain

$$X^{pqr-1} \le R^{s_x}, \quad \mathcal{Y}^{pqr-1} \le R^{s_y}, \quad \mathcal{Z}^{pqr-1} \le R^{s_z},$$

where

$$\begin{aligned} s_x &= \frac{-(\lambda+2)+N(r-1)}{r} + N(p-1)\\ &\quad - (\mu+2) - p(k+2) + Np(q-1),\\ s_y &= [-(\lambda+2)+N(r-1)]q - (k+2) + N(q-1)\\ &\quad + \frac{N(p-1)-(\mu+2)}{p},\\ s_z &= -r(\mu+2) + rN(p-1) - (\lambda+2) + N(r-1)\\ &\quad + \frac{N(q-1)-(k+2)}{q}. \end{aligned}$$

The choice of $s_x \le 0, s_y \le 0$ and $s_z \le 0$ leads to

$$\begin{aligned} N &\le \frac{\lambda+2+r[\mu+2+p(k+2)]}{pqr-1},\\ N &\le \frac{p[(\lambda+2)q+(k+2)]+\mu+2}{pqr-1},\\ N &\le \frac{q[r(\mu+2)+\lambda+2)]+k+2}{pqr-1}. \end{aligned}$$

Now, if we take the case studied by Rencławowicz: $\lambda = k = \mu = 0$, we obtain the same result as she did:

$$\frac{N}{2} \leq \frac{1}{pqr-1} \min\{pq+p+1, rp+r+1, qr+q+1\}.$$

REMARK 2. Our results can be generalized to the more general system

$$\begin{cases} u_t + |x|^\alpha (-\Delta)^{\alpha_1/2}(a_{11}u) = h(t,x)|v|^p & \text{in } Q, \\ v_t + |x|^\beta \{(-\Delta)^{\alpha_2/2}(a_{21}u) + (-\Delta)^{\alpha_3/2}(a_{22}v)\} = k(t,x)|w|^q & \text{in } Q, \\ w_t + |x|^\gamma \{(-\Delta)^{\alpha_4/2}(a_{31}u) & \\ \quad + (-\Delta)^{\alpha_5/2}(a_{32}v) + (-\Delta)^{\alpha_6/2}(a_{33}w)\} = l(t,x)|u|^r & \text{in } Q, \\ u_0 \geq 0, \quad v_0 \geq 0, \quad w_0 \geq 0. & \end{cases}$$

REFERENCES

[1] C. Bandle and H. Brunner, *Blowup in diffusion equations: a survey*, J. Comput. Appl. Math. 97 (1998), 3–22.

[2] P. Baras et M. Pierre, *Critère d'existence de solutions positives pour des équations semi-linéaires non monotones*, Ann. Inst. H. Poincaré Anal. Non Linéaire 2 (1985), 185–212.

[3] K. Deng and H. A. Levine, *The role of critical exponents in blow-up theorems: the sequel*, J. Math. Anal. Appl. 243 (2000), 85–126.

[4] M. Escobedo and M. A. Herrero, *Boundedness and blow up for a semilinear reaction-diffusion system*, J. Differential Equations 89 (1991), 176–202.

[5] M. Fila, H. A. Levine and Y. Uda, *A Fujita-type global existence–global non-existence theorem for a system of reaction diffusion equations with differing diffusivities*, Math. Methods Appl. Sci. 17 (1994), 807–835.

[6] H. Fujita, *On the blowing up solutions of the Cauchy problem for* $u_t = \Delta u + u^{1+\alpha}$, J. Fac. Sci. Univ. Tokyo Sect. I 13 (1966), 109–124.

[7] —, *On some existence and uniqueness theorems for nonlinear parabolic equations*, in: Proc. Sympos. Pure Math. 18, Part I, Amer. Math. Soc., 1968, 138–161.

[8] M. Guedda and M. Kirane, *Criticality for some evolution equations*, Differential Equations 37 (2001), to appear.

[9] T. Kusano and S. Oharu, *Bounded entire solutions of second order semilinear elliptic equations with application to a parabolic initial value problem*, Indiana Univ. Math. J. 34 (1985), 85–95.

[10] H. A. Levine, *The role of critical exponents in blowup theorems*, SIAM Rev. 32 (1990), 262–288.

[11] E. Mitidieri and S. I. Pokhozhaev, *Absence of global positive solutions of quasilinear elliptic inequalities*, Dokl. Akad. Nauk 359 (1998), 456–460 (in Russian).

[12] —, —, *Absence of positive solutions for systems of quasilinear elliptic equations and inequalities in* $\mathbb{R}^N$, ibid. 366 (1999), 13–17 (in Russian).

[13] S. Oharu, *Semilinear evolution equations in Fréchet spaces*, in: Lecture Notes in Math., 1223, Springer, 1986, 186–207.

[14] J. Rencławowicz, *Global existence and blow-up of solutions for a completely coupled Fujita type system of reaction-diffusion equations*, Appl. Math. (Warsaw) 25 (1998), 313–326.

[15] —, *Global existence and blow-up for a completely coupled Fujita type system*, ibid. 27 (2000), 203–218.

[16] A. Samarskii, V. Galaktionov, S. Kurdyumov and A. Mikhailov, *Blow-up in Quasilinear Parabolic Equations*, de Gruyter Expositions in Math. 19, de Gruyter, Berlin, 1995.

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