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Erratum to "Locally unbounded topological fields with topological nilpotents"

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by

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The last part of the proof of Lemma 3.1 in [1] is not correct. The sentence "the coefficient b_n is a sum of less than n^2 terms, ..." is false. This has been pointed out by N. Shell. We rewrite the erroneous part of the proof; we assume that the characteristic of the field K is zero and $\mathbb{N} \subset K$, otherwise the proof is easier.

Let $m \geq 4$ be an integer having the 2-adic representation $m = \sum_{i=0}^{k} s_i 2^i$, with $s_i \in \{0,1\}$ and $s_k = 1$. Using properties (N1), (N2) and (N3) in Definition 2.1 of [1] we get

$$\begin{aligned} N(m) &\leq \sum_{i=0}^{k} s_i N(2^i) \leq N(2)\Big(1 + \sum_{i=1}^{k} s_i i\Big) \leq N(2)\bigg(1 + \frac{k(k+1)}{2}\bigg) \\ &\leq N(2)(\log_2(m))^2. \end{aligned}$$

We replace the above-mentioned wrong sentence with the following: The coefficient b_n is a sum

$$b_n = \sum m(j_1, \ldots, j_n) a_1^{j_1} \ldots a_n^{j_n}, \tag{1}$$

where $m(j_1, \ldots, j_n) \in \mathbb{Z}$ satisfies $|m(j_1, \ldots, j_n)| < 2^n$, and the product $a_1^{j_1} \ldots a_n^{j_n}$ satisfies

$$1j_1 + 2j_2 + \ldots + nj_n = n.$$

The number of solutions of this equation is equal to $p(n)$, the number of partitions of the integer n [2, Lemma 6.12, p. 233], which is a bound for the number of terms in (1). Consequently, we get the bound

$$\begin{aligned} N(b_n) &\leq p(n) \max(N(m(j_1, \ldots, j_n)) + N(a_1^{j_1} \ldots a_n^{j_n})) \\ &\leq p(n)(N(2)n^2 + n \max\{N(a_i) : i = 0, 1, \ldots, n\}). \end{aligned} \tag{2}$$

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The following bound for partitions is well known [2, Theorem 6.10, p. 235]:

$$\log p(n) < \pi\sqrt{2/3}\sqrt{n}.$$

Therefore $\lim \log(N(b_n))/n = 0$. We have proven that $\alpha^{-1} \in A$.

There is a completely analogous error at the end of the proof of Lemma 3.3 in [1]. The sentence "We choose $k \geq 2m$ such that $3\log(n)/n \leq 1/(2m)$ for all $n \geq k$" should be replaced with the following: We choose $k \geq 2m+2$ such that

$$\frac{\log(p(n)) + \log(N(2)) + 3\log(n)}{n} \leq \frac{1}{2m} \quad \text{for all } n \geq k.$$

Using the same reasoning as in the (corrected) proof of Lemma 3.1, we get the bound (2). Consequently,

$$\frac{\log(N(b_n))}{n} \leq \frac{\log(p(n)) + \log(N(2)) + 3\log(n)}{n} + \frac{\log(\max\{N(a_i) : i = 0, 1, \ldots, n\})}{n} \leq \frac{1}{2m} + \frac{1}{k} \leq \frac{1}{m}.$$

Moreover, to the properties (1)–(5) on page 22, we must add the following one:

(5′) For all $i, j \in I$ there exists $k \in I$ such that $U_k \subseteq U_i \cap U_j$.

In Definition 2.1, page 22, one should add the condition $N(a) = N(-a)$ for all $a \in K$.

Finally, a comment on Section 5: consider a series $\alpha = \sum_{n=0}^{\infty} f_n p^n \in A_p$ which can be rewritten, after rearranging and adding some of its terms, as $\alpha = \sum_{n=m}^{\infty} g_{n,m} p^n$ for each $m \in \mathbb{N}$. If $\alpha = \sum_{n=m}^{\infty} g_{n,m} p^n \in W_m$ for each m, we must understand that $\alpha = 0$ in the topological rings $(A_p, \mathcal{T}_W)$ and $(E_p, \mathcal{T}_W)$. We see that the representation of elements in A_p is far from being unique.

References

[1] J. E. Marcos, *Locally unbounded topological fields with topological nilpotents*, Fund. Math. 173 (2002), 21–32.

[2] W. Narkiewicz, *Number Theory*, World Sci., Singapore, 1983.

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