FUNDAMENTA
MATHEMATICAE
175 (2002)

Some combinatorics involving ξ-large sets

by

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Abstract. We prove a version of the Ramsey theorem for partitions of (increasing) n-tuples. We derive this result from a version of König's infinity lemma for ξ-large trees. Here $\xi < \varepsilon_0$ and the notion of largeness is in the sense of Hardy hierarchy.

In this paper we prove some Ramsey style results for partitions of n-tuples of finite sets. This paper is a continuation of [1], and, in fact, in order to avoid repetitions we assume that the reader has a copy of [1] at hand.

The ideas are taken from Ketonen–Solovay [3] (they were interested only in the existence of ω-large homogeneous sets, see [3, Theorem 5.6]). We believe that our approach, using the Hardy hierarchy, is much simpler than that of [3]. On a more personal level our work was influenced by Z. Ratajczyk's work (see [7, 4, 5, 8]).

The main result of this paper is a version of the Ramsey theorem involving partitions of increasing n-tuples of elements of large (in the Hardy sense) sets of natural numbers, where n is an arbitrary fixed positive integer. We begin with the case $n = 2$ for clarity.

In both cases ($n = 2$ and arbitrary n) we have no idea how to get lower bounds. Ketonen and Solovay work merely with ω-large homogeneous sets and their ideas do not seem to generalize. The only results in this direction we know about are: the lower bound from [1, p. 36], which concerns the case $n = 1$, and the result due to Erdős and Mills (see [2, Theorem 2, p. 171]). But this last result also concerns only the existence of ω-large homogeneous sets, as in [3].

Let $\langle A, \prec\rangle$ be a tree. Thus, A is a finite subset of $\mathbb{N}$, the relation $\prec$ is a tree (in the usual set-theoretic sense) on A and $x \prec y$ implies $x < y$ for all $x, y \in A$. Trees in this sense were studied in greater depth by G. Mills [6]. Let $\gamma < \varepsilon_0$. We say that the tree $\langle A, \prec\rangle$ is γ-*large* if its underlying set A

2000 *Mathematics Subject Classification*: Primary 05A18.

is γ-large. We say that the tree $\langle A, \prec\rangle$ is γ-*unbranching* if for every $a \in A$, $\{a\} \cup \{b \in A : b$ is an immediate successor of $a\}$ is not γ-large. In particular, an at most binary tree is 3-unbranching in this terminology.

THEOREM 1. *If the tree* $\langle A, \prec\rangle$ *is* ω^α*-large*, ω*-unbranching and* $\min A > 1$, *then it has a branch* G *such that* $G \setminus \{\max G\}$ *is* α*-large.*

In order to prove Theorem 1 we shall prove the following:

LEMMA 2. *For every* α *we have*: *for every* $\beta \gg \alpha$ *and every tree* $\langle A, \prec\rangle$, *if* A *is* $\omega^{\beta+\alpha}$*-large*, *the tree* $\langle A, \prec\rangle$ *is* ω*-unbranching*, *and* $\min A > 1$, *then there exists* $c \in A$ *such that* $\{a \in A : a \preceq c\}$ *is* α*-large and* $\{a \in A : c \preceq a\}$ *is* ω^β*-large.*

Lemma 2 implies Theorem 1 immediately (just substitute $\beta = 0$).

Proof of Lemma 2. By induction on α. If $\alpha = 0$ then $c = \min A$, i.e., the root of A satisfies our demand.

Assuming the assertion holds for α, we prove it for $\alpha+1$. So let $\beta \gg \alpha+1$, hence $\beta \gg \alpha$. Let the tree $\langle A, \prec\rangle$ be $\omega^{\beta+\alpha+1}$-large. Let $a = \min A$ be its root. Let U_1 denote its first level, i.e., the set of all immediate successors of a. By the assumption, $\{a\} \cup U_1$ is not ω-large, so it has at most a elements. It follows that U_1 has strictly less than a elements. But A itself is $\omega^{\beta+\alpha+1}$-large, so $\omega^{\beta+\alpha} \cdot a$-large. It follows that $A \setminus \{a\}$ is $\omega^{\beta+\alpha} \cdot (a-1)$-large. Moreover, we have a partition $A \setminus \{a\} = \cup_{u \in U_1} B_u$, where $B_u = \{x \in A : u \preceq x\}$. By the result of [1], at least one of these parts, say B_{u_0}, is $\omega^{\beta+\alpha}$-large. By the inductive assumption, there exists $c \in B_{u_0}$ such that $\{x \in B_{u_0} : x \preceq c\}$ is α-large and $\{x \in B_{u_0} : c \preceq x\}$ is ω^β-large. This c satisfies our demand.

Assume the lemma for all $\alpha < \lambda$, λ limit. Let $\beta \gg \lambda$. Then $\beta \gg \{\lambda\}(a)$ for all a, in particular for $a = \min A$. The tree $\langle A, \prec\rangle$ is $\omega^{\beta+\{\lambda\}(a)}$-large, so by the inductive assumption there exists $c \in A$ such that $\{x \in A : x \preceq c\}$ is $\{\lambda\}(a)$-large and $\{x \in A : c \preceq x\}$ is ω^β-large; this c satisfies our demand. ∎

Our next goal is a result analogous to the classical Ramsey theorem for ξ-large sets. We adapt one of the usual proofs of the Ramsey theorem.

Let $P : [A]^2 \to c$ be a partition of (increasing) pairs of elements of A; we shall also use the notation $[A]^2 = \cup_{i<c} B_i$, where $B_i = \{(x, y) \in [A]^2 : P(x, y) = i\}$ for $i < c$. This partition determines an ordering $\prec$ on A so that $\langle A, \prec\rangle$ is a tree. We write $A = \{a_0, \ldots, a_{s-1}\}$ in increasing order and define a sequence $\prec_m$ of relations on $\{a_0, \ldots, a_m\}$. We let $\prec_0 = \emptyset$, $\prec_1 = \{(a_0, a_1)\}$. Further steps are inductive. We let $a_i \prec_{m+1} a_j$ iff

$$\begin{aligned}&(i < j \le m \,\&\, a_i \prec_m a_j) \vee \\ &\{i \le m \,\&\, j = m+1 \,\&\, \forall t\, [a_t \prec_m a_i \Rightarrow P(a_t, a_i) = P(a_t, a_j)]\}.\end{aligned}$$

It is well known (and easy to verify) that the relation $\prec \overset{\text{def}}{=} \prec_{s-1}$ is a tree on A. Its main property is:

$$(1)\qquad \forall x, y \in A\ [x \prec y \equiv (x < y \,\&\, \forall z \prec x\ P(z,x) = P(z,y))].$$

We claim that every $b \in A$ has at most c immediate successors in the tree $\langle A, \prec\rangle$. So let $b = a_m \in A$. Put $f(i) = \min\{j > m : P(a_m, a_j) = i\}$. This function is defined on a subset of $(< c)$ and every $u \succ b$ is $\prec$-greater than some $f(i)$. It follows that if $\min A > c$ then this tree is ω-unbranching. By Theorem 1 we obtain:

LEMMA 3. *If* $\min A > c$, A *is* ω^α*-large and* $[A]^2 = \cup_{i<c} B_i$ *is a partition of* $[A]^2$ *into* c *parts*, *then the tree* $\langle A, \prec\rangle$ *has a branch* G *such that* $G \setminus \{\max G\}$ *is* α*-large.* ∎

Let $G = \{a_{i_0}, \ldots, a_{i_r}\}$ be a branch in the tree $\langle A, \prec\rangle$. Thus, by (1), we have

$$\forall x, y \in G\ [x \prec y \ \Rightarrow\ \forall z \prec x\ P(z,x) = P(z,y)].$$

It follows that for $z \in G \setminus \{\max G\}$ the following function, F, is well defined: $F(z)$ = the unique i such that $P(z,x) = i$ for every $x \in G$ with $x \succ z$. The function F determines a partition of $G \setminus \{\max G\}$ into c parts. By the results of [1] we obtain:

LEMMA 4. *Under the notation introduced above*, *if the branch* G *is such that* $G \setminus \{\max G\}$ *is* $\omega^\alpha \cdot c$*-large then there exists an* ω^α*-large set which is homogeneous for* F.

Let us sum up:

THEOREM 5. *Let* A *be an* $\omega^{\omega^\alpha \cdot c}$*-large set and let* $P : [A]^2 \to (< c)$ *be a partition of* $[A]^2$ *into* c *parts as indicated. Assume also* $\min A > c$. *Then there exists an* ω^α*-large homogeneous set for this partition.*

Proof. If A is $\omega^{\omega^\alpha \cdot c}$-large and P is a partition of $[A]^2$ into at most c parts, then there exists a branch G in the tree $\langle A, \prec\rangle$ such that $G \setminus \{\max G\}$ is $\omega^\alpha \cdot c$-large, by Lemma 3. By Lemma 4, the partition F of this branch (without its maximum) as described above has an ω^α-large homogeneous set. It is easy to check that such a set is homogeneous for the original partition P: this follows from (1). ∎

Let us use the following notation, taken from Ramsey theory (cf. [2]):

> $\alpha \to (\beta)^n_c$ iff for every α-large set A with $\min A > c$ and every partition $P : [A]^n \to (< c)$ there exists a β-large homogeneous set.

Theorem 5 may be stated as the following partition property:

$$(2)\qquad \omega^{\omega^\alpha \cdot c} \to (\omega^\alpha)^2_c.$$

Let us go to a proof of a version of the Ramsey theorem for n-tuples. As usual at first we work out a lemma about trees. We need one more notation. Let $\alpha < \varepsilon_0$. Let

$$\alpha = \omega^{\alpha_0} \cdot a_0 + \ldots + \omega^{\alpha_s} \cdot a_s$$

be the Cantor normal form expansion of α, i.e., $\alpha > \alpha_0 > \ldots > \alpha_s$. For every $n \in \mathbb{N}$ we define $\alpha(\cdot)n = \omega^{\alpha_0} \cdot (a_0 \cdot n) + \ldots + \omega^{\alpha_s} \cdot (a_s \cdot n)$.

THEOREM 6. *For every $\alpha < \varepsilon_0$, every $n \in \mathbb{N} \setminus \{0\}$ and every tree $\langle A, \prec\rangle$ which is ω^n-unbranching and A is $\omega^{\alpha(\cdot)n}$-large and* $\min A > 1$, *there exists a branch B in $\langle A, \prec\rangle$ such that $B \setminus \{\max B\}$ is α-large.*

Theorem 6 is a corollary to the following lemma.

LEMMA 7. *For every $\alpha < \varepsilon_0$, every $n \in \mathbb{N} \setminus \{0\}$, every $\beta < \varepsilon_0$ and every tree $\langle A, \prec\rangle$ such that*

(i) $\{\beta\}(\min A) \gg \alpha$,
(ii) $\min A > 1$,
(iii) *the tree $\langle A, \prec\rangle$ is ω^n-unbranching*,
(iv) *the set A is $\omega^{\beta+\alpha(\cdot)n}$-large*,

there exists $c \in A$ such that $\{x \in A : x \preceq c\}$ is α-large and $\{x \in A : c \preceq x\}$ is ω^β-large.

The proof of Lemma 7 will be inductive on α. In the limit step we shall need the following lemma.

LEMMA 8. *For every $\alpha < \varepsilon_0$, every $\beta < \varepsilon_0$ and every set A such that*

(i) $\{\beta\}(\min A) \gg \mathrm{LM}(\alpha)$,
(ii) $\min A > 1$,
(iii) *A is $\omega^\beta \cdot \alpha$-large*,

the set A is $\omega^{\{\beta\}(\min A)} \cdot \alpha$-large.

Lemma 8 is a particular case of the following observation.

LEMMA 9. *For every $\alpha, \beta < \varepsilon_0$, every A such that* $\{\beta\}(\min A) \gg \mathrm{LM}(\alpha)$ *and every $x \in A$, if $h_{\omega^\beta \cdot \alpha}(x)\downarrow$, then $h_{\omega^{\{\beta\}(\min A)} \cdot \alpha}(x)\downarrow$.*

Proof of Lemma 9 (by induction on α). If $\alpha = 0$ then the conclusion is obvious. Assume the conclusion holds for α. Pick β and A such that $\{\beta\}(a) \gg \mathrm{LM}(\alpha + 1) = \mathrm{LM}(\alpha)$, where $a = \min A$. By the results of [3] (see [1, Lemma 2(iv), (vii)]) $\omega^\beta \Rightarrow_x \omega^{\{\beta\}(a)}$ for every $x > a$. It follows that

(3) for every $x \in A$, if $h_{\omega^\beta}(x)\downarrow$, then $h_{\omega^{\{\beta\}(a)}}(x)\downarrow$ and $h_{\omega^{\{\beta\}(a)}}(x) \le h_{\omega^\beta}(x)$

(see [1, Lemma 4(i)]). Let $x \in A$ and $h_{\omega^\beta \cdot (\alpha+1)}(x)\downarrow$. Then $h_{\omega^\beta \cdot \alpha}(h_{\omega^\beta}(x))\downarrow$. By (3) and the inductive assumption we get $h_{\omega^{\{\beta\}(a)} \cdot \alpha}(h_{\omega^{\{\beta\}(a)}}(x))\downarrow$, so the conclusion for $\alpha + 1$ holds.

Let λ be limit and assume $\{\beta\}(a) \gg \mathrm{LM}(\lambda)$. Assume the conclusion holds for all $\alpha < \lambda$. If $x \in A$ and $h_{\omega^\beta\cdot\lambda}(x)\!\downarrow$ then $h_{\{\omega^\beta\cdot\lambda\}(x)}(x)\!\downarrow$. But for every $\gamma \gg \mathrm{LM}(\lambda)$ and every x, $\{\omega^\gamma\cdot\lambda\}(x) = \omega^\gamma\{\lambda\}(x)$ (see [1, Lemma 3]). By the inductive assumption we get the conclusion for λ. ∎

Proof of Lemma 7 (by induction on α). Let $\alpha = 0$. Let n, β and the tree $\langle A, \prec\rangle$ satisfy the hypothesis. Then $c = \min A$ has the desired property.

Assume the conclusion holds for α. Let the set A be $\omega^{\beta+(\alpha+1)(\cdot)n}$-large. Let $a = \min A > 1$, $\{\beta\}(a) \gg \alpha+1$ and let the tree $\langle A, \prec\rangle$ be ω^n-unbranching. Let $W_1 = \{a_0, \ldots, a_k\}$ be the set consisting of a and all its immediate successors in the tree $\langle A, \prec\rangle$. Assume also that $a_0 = a$. Consider the partition $A = \cup_{i\le k} B_i$, where $B_0 = \{a_0\}$, $B_i = \{x \in A : a_i \preceq x\}$ for $i > 0$. This partition is ω^n-small [1] (because $\langle A, \prec\rangle$ is ω^n-unbranching), so by the main result of [1] there exists $i_0 \le k$ such that B_{i_0} is $\omega^{\beta+\alpha(\cdot)n}$-large. Obviously, $i_0 \ne 0$. By the inductive assumption applied to the tree $\langle B_{i_0}, \prec\rangle$ there exists c such that $\{x \in B_{i_0} : x \preceq c\}$ is α-large and $\{x \in B_{i_0} : c \preceq x\}$ is ω^β-large. This c has the desired property in the original tree $\langle A, \prec\rangle$.

Assume the conclusion holds for all $\alpha < \lambda$, where λ is limit. Let

$$\lambda = \omega^{\alpha_1}\cdot a_1 + \ldots + \omega^{\alpha_s}\cdot a_s$$

be the Cantor expansion of λ, i.e. $\alpha_1 > \ldots > \alpha_s$. Define

$$\gamma = \omega^{\alpha_1}\cdot a_1 + \ldots + \omega^{\alpha_s}\cdot (a_s - 1),$$

so that $\lambda = \gamma + \omega^{\alpha_s}$ and $\gamma \gg \omega^{\alpha_s}$. Let A be an $\omega^{\beta+\lambda(\cdot)n}$-large set, where $\beta \gg \lambda$. Hence A is $\omega^{\beta+\gamma(\cdot)n+\omega^{\alpha_s}\cdot n}$-large. We apply Lemma 8 to the ordinals

$$\begin{array}{lcr} \beta+\gamma(\cdot)n+\omega^{\alpha_s}\cdot(n-1) & \text{and} & \omega^{\{\omega^{\alpha_s}\}(a)} \\ \beta+\gamma(\cdot)n+\omega^{\alpha_s}\cdot(n-2) & \text{and} & \omega^{\{\omega^{\alpha_s}\}(a)\cdot 2} \\ \dots\dots\dots\dots & \dots & \dots\dots \\ \beta+\gamma(\cdot)n+\omega^{\alpha_s} & \text{and} & \omega^{\{\omega^{\alpha_s}\}(a)\cdot(n-1)} \end{array}$$

and infer that A is $\omega^{\beta+\gamma(\cdot)n+\{\omega^{\alpha_s}\}(a)\cdot n}$-large. Hence A is $\omega^{\beta+\{\lambda\}(a)(\cdot)n}$-large. By the inductive assumption there exists c with the desired properties. ∎

Our next goal is one more version of a result analogous to the classical Ramsey theorem for ξ-large sets. We adapt the same proof of the Ramsey theorem as before.

For every $\alpha < \varepsilon_0$ and every $c \in \mathbb{N}\setminus\{0\}$ we define $\omega_{(0)}(\alpha, c) = 1$, $\omega_{(1)}(\alpha, c) = \omega^\alpha\cdot c$, $\omega_{(2)}(\alpha, c) = \omega^{\omega_{(1)}(\alpha,c)}$, $\omega_{(n+1)}(\alpha, c) = \omega^{\omega_{(n)}(\alpha,c)\cdot 3}$.

THEOREM 10. *Let $n \in \mathbb{N}\setminus\{0\}$ and let A be an $\omega_{(n)}(\alpha, c)$-large set, where $\alpha < \varepsilon_0$, $c < \min A$. If $P : [A]^n \to (< c)$ is a partition of the set $[A]^n$ into at most c parts then there exists an ω^α-large homogeneous set.*

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[1] Recall from [1] that a set or partition is γ-*small* if it is not γ-large.

Proof. By induction on n. The case $n = 1$ is the main result of [1]. The case $n = 2$ was proved above. Assuming the conclusion holds for n, we derive it for $n+1$. Let A be an $\omega_{(n+1)}(\alpha, c)$-large set, where $\alpha < \varepsilon_0$ and $c < \min A$. Let $P : [A]^{n+1} \to (< c)$ be a partition of (increasing) $n+1$-tuples of elements of A; we shall also use the notation $[A]^{n+1} = \cup_{i<c} B_i$, where $B_i = \{(x_0, \ldots, x_n) \in [A]^{n+1} : P(x, y) = i\}$ for $i < c$. This partition determines an ordering $\prec$ on A so that $\langle A, \prec\rangle$ is a tree. We write $A = \{a_0, \ldots, a_{s-1}\}$ in increasing order and define a sequence $\prec_m$ of relations on $\{a_0, \ldots, a_m\}$. We let $\prec_0 = \emptyset$, $\prec_1 = \{(a_0, a_1)\}$. Further steps are inductive. We let $a_i \prec_{m+1} a_j$ iff

$$\begin{aligned}
&(i, j \le m \,\&\, a_i \prec_m a_j) \vee \\
&\qquad \{i \le m \,\&\, j = m+1 \,\&\, \forall t_0, \ldots, t_{n-1}\, [a_{t_0} \prec_m \ldots \prec_m a_{t_{n-1}} \prec_m a_i \Rightarrow \\
&\qquad\qquad\qquad P(a_{t_0}, \ldots, a_{t_{n-1}}, a_i) = P(a_{t_0}, \ldots, a_{t_{n-1}}, a_j)]\}.
\end{aligned}$$

Once again, it is well known (and easy to verify) that the relation $\prec \stackrel{\text{def}}{=} \prec_{s-1}$ is a tree on A. Its main property is:

$$\begin{aligned}
(4)\quad &\forall x, y \in A\, [x \prec y \equiv (x < y \,\&\, \forall z_0 \prec \ldots \prec z_{n-1} \prec x \\
&\qquad\qquad P(z_0, \ldots, z_{n-1}, x) = P(z_0, \ldots, z_{n-1}, y))].
\end{aligned}$$

Let, for each $x \in A$, $\operatorname{rank}(x) = \operatorname{Card}(\{y \in A : y \prec x\})$, the *rank* of x in the tree $\langle A, \prec\rangle$, and let W_x denote the set consisting of x and all its immediate successors. It is easy to see that for every $x \in A$ if $x < a_{n-1}$ then $\operatorname{Card}(W_x) = 2$, while if $x \ge a_{n-1}$ then $\operatorname{Card}(W_x) \le c^{\binom{\operatorname{rank}(x)+1}{n}} + 1$. The binomial coefficient in the exponent is just the cardinality of the set of all n-tuples of elements of the set $\{z \in A : z \preceq x\}$. Each set W_x is ω^3-small because $c < a_0 \le x$ and $\operatorname{rank}(x) \le x$. By Theorem 6 the tree $\langle A, \prec\rangle$ has a branch G such that $G \setminus \{\max G\}$ is $\omega_{(n)}(\alpha, c)$-large. Let $G = \{a_{i_0}, \ldots, a_{i_r}\}$ be such a branch in the tree $\langle A, \prec\rangle$. Thus, by (4), we have

$$\begin{aligned}
&\forall x, y \in G\, [x \prec y \;\Rightarrow \\
&\qquad \forall z_0 \prec \ldots \prec z_{n-1} \prec x\; P(z_0, \ldots, z_{n-1}, x) = P(z_0, \ldots, z_{n-1}, y)].
\end{aligned}$$

It follows that for an increasing n-tuple $z_0, \ldots, z_{n-1} \in [G \setminus \{\max G\}]^n$ the following function is well defined: $F(z_0, \ldots, z_{n-1}) =$ the i such that $P(z_0, \ldots z_{n-1}, x) = i$ for every $x \in G$ with $x \succ z_{n-1}$. The function F determines a partition of $[G \setminus \{\max G\}]^n$ into c parts. Clearly, every set homogeneous for F is homogeneous for P. By the inductive assumption we infer the conclusion for $n+1$. ∎

Still using the notation taken from Ramsey theory, Theorem 10 may be stated as the following partition property:

$$\omega_{(n)}(\alpha, c) \to (\omega^\alpha)^n_c. \tag{5}$$

We remark that Ketonen and Solovay [3], working with a slightly different notion of largeness, show that if

$$\theta = \omega^\alpha + \omega^3 + \max(c, \|\alpha\|) + 3$$

then for every θ-large set X and every $F : [X]^n \to c$ there exists an α-large set $Y \subseteq X$ and a map $G : [Y]^{n-1} \to c$ such that

$$F(z_0, \ldots, z_{n-1}) = G(z_0, \ldots, z_{n-2})$$

(see [3, Theorem 5.6]). From this they derive the following fact:

> Let $\omega_0(\alpha) = \alpha$ and let $\omega_{n+1}(\alpha) = \omega^{\omega_n(\alpha)}$ (they write $W_n(\alpha)$ rather than $\omega_n(\alpha)$). Then whenever X is $\omega_{n-1}(\omega\cdot(c+3))$-large then every partition $F : [X]^n \to c$ has an ω-large homogeneous set.

Of course, as pointed out above, our main result (i.e., Theorem 10) is similar and was suggested by theirs.

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Received 31 May 2001;
in revised form 22 July 2002