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Remainders of metrizable and close to metrizable spaces

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Abstract. We continue the study of remainders of metrizable spaces, expanding and applying results obtained in [Fund. Math. 215 (2011)]. Some new facts are established. In particular, the closure of any countable subset in the remainder of a metrizable space is a Lindelöf p-space. Hence, if a remainder of a metrizable space is separable, then this remainder is a Lindelöf p-space. If the density of a remainder Y of a metrizable space does not exceed 2^ω, then Y is a Lindelöf Σ-space. We also show that many of the theorems on remainders of metrizable spaces can be extended to paracompact p-spaces or to spaces with a σ-disjoint base. We also extend to remainders of metrizable spaces the well known theorem on metrizability of compacta with a point-countable base.

1. Introduction. In this article, a "space" is a Tikhonov topological space. A *compactification* of a space X is any compact space bX such that X is a subspace of bX and X is dense in bX. A *remainder* Y of a space X is the subspace $Y = bX \setminus X$ of a compactification bX of X. One of the major tasks in the theory of compactifications is to investigate how the properties of a space X are related to the properties of some or all of the remainders of X.

We are especially interested in the *invariant* properties of the remainders of X, that is, in the properties that do not depend on the choice of a compactification bX of X. In this connection, we recall a concept introduced in [19]. A topological property is *perfect* if it is preserved by perfect mappings in both directions. The importance of perfect properties for us is due to the fact that if some remainder of a space X has a perfect property, then every remainder of X has this property. For example, being a paracompact p-space is a perfect property [17]. Being a Lindelöf p-space and being a Lindelöf Σ-space are also perfect properties. But metrizability is not a perfect property.

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If every remainder of a space X has a certain property $\mathcal{P}$, then we say that X has the property $\mathcal{P}'$. The next statement from [19, Theorem 2.7] is obvious:

Proposition 1.1. *If $\mathcal{P}$ is a perfect property, then so is $\mathcal{P}'$.*

Recall that a space X is of *countable type* if every compact subspace of X is contained in a compact subspace of X which has a countable base of open neighbourhoods in X [1]. All metrizable spaces and all locally compact spaces, as well as all Čech-complete spaces, are of countable type [1].

Recently, several investigations on remainders of spaces have been conducted (see [7], [6], [9], [8], [20]). See also [11], where it has been observed that remainders of metrizable spaces are quite close in their properties to compacta.

This article is a continuation of [10], where several new results on remainders of metrizable spaces have been presented. We apply results from [10] and develop further the techniques from [10] to establish some new facts on remainders of metrizable and close to metrizable spaces.

It is proved that the closure of any countable subset in the remainder of a metrizable space is a Lindelöf p-space. Hence, if a remainder of a metrizable space is separable, then this remainder is a Lindelöf p-space. If the density of a remainder of a metrizable space does not exceed 2^ω, then this remainder is a Lindelöf Σ-space. We also prove that if a remainder of a metrizable space is symmetrizable, then this remainder is separable and metrizable. Several new results in the article concern some generalizations of metrizable spaces, such as paracompact p-spaces and spaces with a σ-disjoint base.

A famous classical result in the theory of compactifications is the following theorem of M. Henriksen and J. Isbell [19]:

Theorem 1.2. *A space X is of countable type if and only if the remainder in any* (*or some*) *compactification of X is Lindelöf.*

It follows from this theorem that every remainder of a metrizable space is Lindelöf. A much stronger property is enjoyed by remainders of separable metrizable spaces. It was observed in [6] that every remainder of a separable metrizable space is a Lindelöf p-space. Recall that *paracompact p-spaces* introduced in [1] can be characterized as preimages of metrizable spaces under perfect mappings. A *Lindelöf p-space* is the preimage of a separable metrizable space under a perfect mapping. However, it is not true that every remainder of any metrizable space is a paracompact p-space (see [10]). A strong necessary condition for a space to be a remainder of some metrizable space has been obtained in [10]. An attractive condition of this kind has been established in [10] for metrizable spaces of weight not greater than 2^ω. Here it is:

THEOREM 1.3. *If X is a metrizable space of weight not greater than 2^ω, then every remainder of X is a Lindelöf Σ-space.*

Recall that a space is a *Lindelöf Σ-space* if it is the image of a Lindelöf p-space under a continuous mapping. This important class has been introduced by K. Nagami [23]. There is a very useful characterization of Lindelöf Σ-spaces in terms of their location in compactifications. We recall it now. Let X and Y be some subspaces of a space Z, and γ be a family of subsets of Z such that for any $x \in X$ and any $y \in Y$, where $x \neq y$, there exists $P \in \gamma$ such that $x \in P$ and $y \notin P$. Then we will say that γ is a *T_0-separator* in Z for the pair (X, Y). A T_0-separator γ is called *closed* (or *open*) if every member of γ is closed (respectively, open) in Z. The following fact is well-known:

THEOREM 1.4. *A space X is a Lindelöf Σ-space if and only if for every* (*or equivalently, for some*) *compact space B containing X there exists a countable closed T_0-separator in B for the pair $(X, B \setminus X)$.*

In [10], it was shown that one of the main results of [10], Theorem 1.3, cannot be extended to the class of all metrizable spaces.

In Section 2, we find a new sufficient condition for a remainder of a space to be a Lindelöf Σ-space, considerably extending [10, Theorem 1.3]. The proof of this result is based on a new technique, and provides an alternative and simpler proof for [10, Theorem 1.3]. In Section 3 we provide new results on remainders of spaces close to metrizable without strong restrictions on the weight.

2. Remainders of "small" spaces with a σ-disjoint base. We call a Tikhonov space X an *s-space* if, for some compactification bX of X, there exists a countable open T_0-separator in bX for the pair $(X, bX \setminus X)$. See [16, 3.9.E] for references to some early appearances of s-spaces in the literature and for comments.

Every Lindelöf p-space is an s-space. However, an s-space need not be a p-space [12]. Our next statement obviously follows from Theorem 1.4. It shows how s-spaces are related to Lindelöf Σ-spaces.

PROPOSITION 2.1. *A space X is an s-space if and only if any* (*or some*) *remainder of X is a Lindelöf Σ-space.*

Here is a curious application of the above statement:

PROPOSITION 2.2. *Being an s-space is a perfect property.*

Proof. Being a Lindelöf Σ-space is a perfect property. Therefore, Proposition 1.1 implies that being an s-space is a perfect property. ∎

A space X is said to be *perfect* if every closed subset of X is a G_δ-set in X. We need the following result from [12]:

Theorem 2.3. *If a perfect space is an s-space, then it is a p-space.*

The next result immediately follows from Proposition 2.1 and Theorem 2.3.

Corollary 2.4. *If a perfect space X has a remainder which is a Lindelöf Σ-space, then X is a p-space.*

Now we can show that Theorem 1.3 cannot be extended to spaces X with a point-countable base such that $|X| \leq 2^\omega$.

Theorem 2.5. *Under the Continuum Hypothesis* [*CH*], *there exists a Lindelöf space X with a point-countable base such that no remainder of X is a Lindelöf Σ-space.*

Proof. Assuming [CH], E. K. van Douwen, F. D. Tall, and W. Weiss have constructed a non-metrizable hereditarily Lindelöf space X with a point-countable base [15].

We claim that no remainder of X is a Lindelöf Σ-space. Assume the contrary. Then, by Proposition 2.1, X is an s-space. It is also perfect, being hereditarily Lindelöf. Therefore, it follows from Theorem 2.3 that X is a p-space. Since it is a Lindelöf p-space with a point-countable base, we conclude that it is metrizable, a contradiction. ∎

Clearly, the following statement also holds:

Proposition 2.6. *For any hereditarily Lindelöf space X, the following three conditions are pairwise equivalent:*

(1) *X is a p-space.*
(2) *Some* (*or every*) *remainder of X is a Lindelöf Σ-space.*
(3) *Some* (*or every*) *remainder of X is a Lindelöf p-space.*

The class of spaces with a σ-disjoint base lies between the classes of metrizable spaces and of spaces with a point-countable base. We show below that here the situation of remainders is closer to that of remainders of metrizable spaces.

Theorem 2.7. *Suppose that X is a space with a σ-disjoint base $\mathcal{B}$ such that $|\mathcal{B}| \leq 2^\omega$. Then every remainder of X is a Lindelöf Σ-space.*

To prove this theorem, we need the following lemma:

Lemma 2.8. *Suppose that γ is a disjoint family of open subsets of a topological space X such that $|\gamma| \leq 2^\omega$. Then there exists a countable family $\mathcal{W}$ of open subsets of X satisfying the following condition:*

(s) *For each $V \in \gamma$ and any $x, y \in X$ such that $x \in V$ and $y \notin V$, there exists $W \in \mathcal{W}$ such that $x \in W$ and $y \notin W$.*

Proof. Since $|\gamma| \leq 2^\omega$, there exists a countable family $\mathcal{E}$ of subfamilies of γ satisfying the following condition:

(c) For any distinct $V_1, V_2 \in \gamma$, there exists $\eta \in \mathcal{E}$ such that $V_1 \in \eta$ and $V_2 \notin \eta$.

We can also assume that $\gamma \in \mathcal{E}$. For each $\eta \in \mathcal{E}$, put $W_\eta = \bigcup \eta$, and let $\mathcal{W} = \{W_\eta : \eta \in \mathcal{E}\}$.

Clearly, $\mathcal{W}$ is a countable family of open subsets of X. Let us show that $\mathcal{W}$ satisfies condition (s).

Fix $V \in \gamma$ and take any $x, y \in X$ such that $x \in V$ and $y \notin V$.

Case 1: *There exists $U \in \gamma$ such that $y \in U$.* Clearly, $U \neq V$. By condition (c), there exists $\eta \in \mathcal{E}$ such that $V \in \eta$ and $U \notin \eta$. Then, obviously, $x \in W_\eta = \bigcup \eta$ and $y \notin W_\eta$. Thus, in this case condition (s) holds.

Case 2: $y \notin \bigcup \gamma$. Then $y \notin W_\gamma \in \mathcal{W}$ and $x \in W_\gamma$. Again, condition (s) holds. ∎

Proof of Theorem 2.7. Let bX be any compactification of X. We have a base $\mathcal{B} = \bigcup\{\mu_n : n \in \omega\}$ of X such that $|\mathcal{B}| \leq 2^\omega$ and each family μ_n is disjoint. Since X is dense in bX, we can find a disjoint family γ_n of open subsets of bX such that $\mu_n = \{W \cap X : W \in \gamma_n\}$ and $|\gamma_n| = |\mu_n| \leq 2^\omega$ for $n \in \omega$.

By Lemma 2.8, for each $n \in \omega$, we can fix a countable family $\mathcal{W}_n$ of open subsets of bX satisfying the following condition:

(s_n) For each $V \in \gamma_n$ and any $x, y \in bX$ such that $x \in V$ and $y \notin V$, there exists $W \in \mathcal{W}_n$ such that $x \in W$ and $y \notin W$.

Put $\mathcal{S} = \bigcup\{\mathcal{W}_n : n \in \omega\}$. Clearly, $\mathcal{S}$ is a countable family of open subsets of bX.

Claim 1. *For each $x \in X$ and each $y \in Y = bX \setminus X$, there exists $W \in \mathcal{S}$ such that $x \in W$ and $y \notin W$.*

Clearly, $x \neq y$. Since $\mathcal{B}$ is a base for X, we can find $V_0 \in \mathcal{B}$ such that $x \in V_0$ and y is not in the closure of V_0 in bX. Now we can find $k \in \omega$ such that $V_0 \in \mu_k$. By the definition of the family γ_k, $V_0 = W \cap X$ for some $W \in \gamma_k$. Then $x \in W$ and $y \notin W$, since $W \subset \overline{V_0}$ and $y \notin \overline{V_0}$. By condition (s_k), there exists $W \in \mathcal{W}_k$ such that $x \in W$ and $y \notin W$. Then $W \in \mathcal{S}$, so that Claim 1 is established.

It follows from Claim 1 that X is an s-space. Hence, by Proposition 2.1, Y is a Lindelöf Σ-space. ∎

Corollary 2.9. *Suppose that a space X is the union of a countable family η of dense metrizable subspaces and that $|X| \leq 2^\omega$. Then every remainder of X is a Lindelöf Σ-space.*

Proof. Fix a compactification bX of X. Every member of η has a σ-disjoint base. Therefore, the space X also has a σ-disjoint base $\mathcal{B}$. Clearly, $|\mathcal{B}| \le |X| \le 2^\omega$. It remains to apply Theorem 2.7. ∎

Example 2.10. The assumption in Corollary 2.9 that every member of η is dense in X is essential. Take for X the countable Fréchet–Urysohn fan $V(\omega)$. Then $V(\omega)$ is the union of two discrete (hence, metrizable) subspaces Y and Z one of which is a singleton and consists of the unique non-isolated point of $V(\omega)$. The space $V(\omega)$ is countable but not first-countable. Therefore, it is not of countable type. Hence, by the Henriksen–Isbell Theorem, no remainder of $V(\omega)$ is Lindelöf.

The restriction on the cardinality of the base $\mathcal{B}$ in Theorem 2.7 is also essential, as we have seen in the Introduction. However, we have the following two results related to Theorem 2.7.

Theorem 2.11. *Suppose that X is a space with a σ-disjoint base $\mathcal{B}$. Then the remainder of X in each homogeneous compactification bX of X is a Lindelöf Σ-space.*

Proof. Clearly, bX is first-countable at every point of X. Therefore, bX is first-countable at every point, since it is homogeneous. Since bX is compact, it follows that $|bX| \le 2^\omega$ ([3]). It remains to apply Theorem 2.7. ∎

Theorem 2.12. *Suppose that X is a Lindelöf space with a σ-disjoint base $\mathcal{B}$. Then every remainder of X is a Lindelöf Σ-space.*

Proof. The cardinality of X does not exceed 2^ω, since X is Lindelöf and first-countable [3]. It remains to apply Theorem 2.7. ∎

3. Remainders of paracompact p-spaces. Results in this section are presented in a rather general form. However, their main, and still new, corollary concerns remainders of metrizable spaces. Besides, in the proofs metrizability will be used in a decisive way. We also rely upon the concept of an Eberlein compactum [4, Chapter 4, Section1], [5]. See also [21], [22].

We start by formulating our main result in the simplest case.

Proposition 3.1. *Suppose that X is a metrizable space, and that Y is an arbitrary remainder of X. Then Y has the following property:*

$\mathcal{P}_0$: *For any countable subset C of Y, the closure of C in Y is a Lindelöf p-space.*

This result obviously follows from the following general statement:

Theorem 3.2. *Suppose that X is a paracompact p-space, and that Y is an arbitrary remainder of X. Then Y has the following property:*

$\mathcal{P}_c$: *Any closed subspace P of Y such that the Suslin number $c(P)$ of P is countable is a Lindelöf p-space.*

To prove Theorem 3.2, we need the following fact:

PROPOSITION 3.3. *$\mathcal{P}_c$ is a perfect property.*

Proof. Suppose that f is a perfect mapping of a space Z onto a space Y.

a) Assume that Y has $\mathcal{P}_c$. Take any closed subspace F of Z with $c(F)$ countable, and put $P = f(F)$. Clearly, P is closed in Y, and $c(P) \leq \omega$. Therefore, P is a Lindelöf p-space, since Y has $\mathcal{P}_c$. It follows that $f^{-1}(P)$ is a Lindelöf p-space, since f is a perfect mapping. Note that F is a closed subspace of $f^{-1}(P)$. Hence, F is also a Lindelöf p-space.

b) Assume that Z has $\mathcal{P}_c$. Take any closed subspace P of Y with $c(P)$ countable. Since f is perfect and $f(Z) = Y$, there exists a closed subspace F of Z such that the restriction of f to F is an irreducible perfect mapping of F onto P. Thus $c(F)$ is countable. Hence, F is a Lindelöf p-space, since Z has $\mathcal{P}_c$. It follows that $P = f(F)$ is a Lindelöf p-space. ∎

Proof of Theorem 3.2. Since $\mathcal{P}_c$ is a perfect property, the property

$\mathcal{P}'_c$: every remainder of X has $\mathcal{P}_c$

is also a perfect property, by Proposition 1.1. Therefore, since there exists a perfect mapping of X onto a metrizable space, we can assume that X itself is metrizable.

Since $\mathcal{P}_c$ is a perfect property, it suffices to show that some remainder of X has $\mathcal{P}_c$. By [4, Theorem 4.1.25], we can fix a compactification bX of X such that bX is an Eberlein compactum. The remainder Y of X has $\mathcal{P}_c$, since every subspace M of an Eberlein compactum with $c(M)$ countable is metrizable (see [5] or [4, Chapter 4, Section 1]). ∎

COROLLARY 3.4. *Every remainder of a paracompact p-space has the property $\mathcal{P}_0$.*

COROLLARY 3.5. *If a paracompact p-space X has a separable remainder, then every remainder of X is a Lindelöf p-space.*

The last statement shows that *there are Lindelöf Σ-spaces that cannot be represented as remainders of paracompact p-spaces* (just take any non-metrizable countable Tikhonov space).

COROLLARY 3.6. *Suppose that X is a paracompact p-space, and Y is a remainder of X. Then every closed subspace P of Y with a countable network has a countable base.*

Proof. This follows from Corollary 3.4, since every p-space with a countable network has a countable base, and hence is metrizable [1]. ∎

The next statement and its proof are similar to Theorem 3.2 and its proof, but we use one more technical tool in the argument.

THEOREM 3.7. *Suppose that X is a paracompact p-space. Then any remainder Y of X has the following property:*

$\mathcal{P}_s$: *For any subset L of Y such that $|L| \le 2^\omega$, the closure of L in Y is a Lindelöf Σ-space.*

Proof. A standard argument shows that $\mathcal{P}_s$ is a perfect property (see the proof of Proposition 3.3). Therefore, so is $\mathcal{P}'_s$. Hence, it suffices to consider the case when X is metrizable, and to prove the statement for some remainder Y of X. Again, we can fix an Eberlein compactification bX of X. Put $Y = bX \setminus X$, and fix any subset L of Y such that $|L| \le 2^\omega$. Let K be the closure of L in Y.

It is known (see [5] or [4, Chapter 4, Section 1]) that *the tightness of any Eberlein compactum is countable.* Thus, for each $h \in L$, we can fix a countable subset A_h of X such that $h \in \overline{A_h}$. Put $A = \bigcup\{A_h : h \in L\}$. Clearly, $|A| \le 2^\omega$, and $L \subset \overline{A}$. Let B be the closure of A in X. Since X is a metric space, we have $|B| \le 2^\omega$. The closure of B in bX is a compactification bB of the metric space B. By Theorem 2.7, $S = bB \setminus B$ is a Lindelöf Σ-space. Consequently, K is a Lindelöf Σ-space, as $K \subset S$ and K is closed in S. ∎

4. Applications to remainders of some "large" spaces. It was shown in [10] that not for every metrizable space are all its remainders Lindelöf Σ-spaces. We do not know yet how to characterize remainders of metrizable spaces. However, a partial result has been obtained in [10]. It is based on the concept of a charming space [10]. A space X is called *charming* if there exists a Lindelöf Σ-subspace Y of X such that, for each open neighbourhood U of Y in X, the subspace $X \setminus U$ is also a Lindelöf Σ-space. The idea of this definition can be used to construct other interesting new classes of spaces (see [13]).

Every Lindelöf Σ-space is charming. Hence, all separable metrizable spaces and all Lindelöf p-spaces are charming as well. A motivation for the study of charming spaces is provided by the following statement from [10]:

THEOREM 4.1. *Every remainder of a paracompact p-space is a charming space.*

We need yet another theorem from [10]:

THEOREM 4.2. *The cardinality of any charming space X such that every $x \in X$ is a G_δ-point in X does not exceed 2^ω.*

THEOREM 4.3. *If Y is a remainder of a paracompact p-space such that every $y \in Y$ is a G_δ-point in Y, then Y is a Lindelöf Σ-space.*

Proof. Indeed, it follows from Theorem 4.1 that Y is charming. Therefore, $|Y| \leq 2^\omega$, by Theorem 4.2. Now Theorem 3.7 implies that Y is a Lindelöf Σ-space. ■

Some applications of the techniques developed above are summed up in our next theorem. For the basic properties of symmetrizable spaces, see [2], [24], or [18].

THEOREM 4.4. *Suppose that X is a paracompact p-space, and Y is a remainder of X. Then any one of the following conditions is sufficient for Y to be separable and metrizable:*

(i) *Y has a G_δ-diagonal;*
(ii) *Y has a point-countable base;*
(iii) *Y is a subspace of a symmetrizable space.*

Proof. (i) Every $y \in Y$ is a G_δ-point in Y. Therefore, Y is a Lindelöf Σ-space by Theorem 4.3. By a result of K. Nagami [23], Y has a countable network. Now Corollary 3.6 implies that Y is separable and metrizable.

(ii) The remainder Y is a Lindelöf Σ-space, by Theorem 4.3. But every Lindelöf Σ-space with a point-countable base is metrizable (see [18, Theorem 7.9 (ii)]).

(iii) The tightness of Y is countable, since Y is a subspace of a symmetrizable space.

CLAIM. *The space Y is Fréchet–Urysohn.*

Take any non-closed subset A of Y, and fix $y \in \overline{A} \setminus A$. There exists a countable subset B of A such that $y \in \overline{B} \setminus B$. The subspace $F = \overline{B}$ is a Lindelöf p-space, by Proposition 3.1. Every compact subspace of F is symmetrizable, and therefore metrizable [2]. It follows that F is first-countable, since it is of point-countable type. Hence, there exists a sequence in B converging to y. The Claim is verified.

Since Y is Fréchet–Urysohn, and Y is a subspace of a symmetrizable space, we conclude that Y is symmetrizable and first-countable [2], [18], that is, Y is semi-metrizable. Hence, $Y \times Y$ is also semi-metrizable, which implies that Y has a G_δ-diagonal. It remains to apply the sufficiency of (i). ■

Every compact space can be easily represented as the remainder of a discrete (hence metrizable) space. Therefore, the sufficiency of (ii) in the above theorem can be interpreted as a generalization of the well known theorem of A. S. Mischenko on metrizability of every compact space with a point-countable base [16].

THEOREM 4.5. *Suppose that X is a metrizable space and Y is a remainder of X in a compactification bX such that Y satisfies at least one of the following conditions:*

(i) *Y has a G_δ-diagonal;*
(ii) *Y has a point-countable base;*
(iii) *Y is a subspace of a symmetrizable space.*

Then bX is an Eberlein compactum and the closure bY of Y in bX is metrizable.

Proof. By Theorem 4.4, the space Y is separable and metrizable. Hence, by a result in [22], bX is an Eberlein compactum. The remainder $Z = bY \setminus Y$ is Lindelöf [19]. Therefore, Z has a countable base, since $Z \subset X$ and X is metrizable. Hence, bY has a countable network and is metrizable. ■

Observe that if the metrizable space X in 4.5 is nowhere locally compact, then the compactification $bX = bY$ is metrizable.

Theorem 4.4 can be used to show that certain Lindelöf spaces cannot be represented as remainders of metrizable spaces. For example, we see that no Lindelöf version of the Michael line is a remainder of a paracompact p-space, since it has a point-countable base but is not metrizable. Any version of the Michael line has a metrizable remainder. Thus, having a metrizable remainder and being a remainder of a metrizable space are not equivalent properties. We also see that a charming space with a point-countable base need not be metrizable.

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