Erratum to
“On rings with a unique proper essential right ideal”
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by

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Let $Q = \text{End}(V)$, where $V$ is an infinite-dimensional vector space over a field $F$. Put $R = S + F$, where $S$ is the socle of $Q$. In [1, Example 11], we observed that $R$ is a right ue-ring (i.e., it has a unique proper essential right ideal). Imitating the proof of this observation word for word, one can easily see that $R$ is a left ue-ring too. But unfortunately this trivial fact was overlooked by us and we wrongly claimed otherwise. Moreover, we were led on by this to claim in [1, Theorem 18] that every semiprime right ue-ring is a right V-ring (note that the above ring $R$ is well-known to be a right V-ring which is not a left V-ring, see Example 11 in [1]). Hence this is in fact an example of a semiprime left ue-ring which is not a left V-ring. In what follows we give the correct form of Theorem 18 and of the comment preceding it in [1]. We begin with the following definition.

**Definition.** A ring $R$ is called an *almost right V-ring* if every simple right $R$-module is either injective or projective.

Clearly, every right V-ring is an almost right V-ring and we will observe shortly that every right ue-ring is an almost right V-ring.

In reference [6] of [1], right V-rings $R$ with non-finitely generated right socle, say $S$, such that $R/S$ is a division ring are characterized. We note that these are semiprime right ue-rings which have the following properties too.
THEOREM (Theorem 18 in [1]). The following statements are equivalent for a semiprime ring $R$.

(1) $R$ is a right ue-ring.
(2) The intrinsic topology of $R$ is a non-discrete Hausdorff topology and a dense right ideal must be semisimple.
(3) $R$ is a regular, almost right V-ring and $R/\text{Soc}(R)$ is a division ring with $\text{Soc}(R) \neq 0$.
(4) For each right ideal $I$, either $R/I$ is non-singular or both $I$ and $R/I$ are semisimple $R$-modules and the Goldie dimension of $R$ is not finite.

Proof. (1)⇒(2). This is proved in [1].

(2)⇒(3). Everything is proved in [1], except the fact that $R$ is an almost right V-ring. To prove the latter fact, we note that $R$ is a right ue-ring and every maximal right ideal except the socle is a direct summand of $R$. Hence every simple $R$-module is either projective or isomorphic to $R/S$, where $S = \text{Soc}(R)$. Now we show that $R/S$ is injective and we are through. We must extend every homomorphism $f : I \to R/S$, where $I$ is a right ideal of $R$, to a homomorphism from $R$ into $R/S$. In view of Proposition 11 in [1], $I$ is either a direct summand of $R$ or semisimple. In case $I$ is a direct summand of $R$, $f$ can be naturally extended to a homomorphism from $R$ into $R/S$. Finally, if $I$ is semisimple, then $I \subseteq S$ and $I$ is generated by idempotents. Let $e \in I$ be an idempotent. Then $f(e) = f(e)e \in \frac{R}{S}S = 0$. Thus $f$ is the zero mapping and we are done.

(3)⇒(4). First, we note that the Goldie dimension of $R$ is not finite, for $R$ is a regular ring which is not semisimple. Now let $I \not\subseteq S = \text{Soc}(R)$ be a right ideal of $R$. Then in the proof of “(3)⇒(4)” in [1] it is already shown that $R/I$ is non-singular. Hence we may assume that $I \subseteq S$, and since $S$ is semisimple we have $S = I \oplus A$ for some right ideal $A$ of $R$. Put $\overline{R} = R/I$ and $\overline{A} = (A + I)/I = S/I$. Then either $\overline{A}$ is essential in $R/I$ in which case $R/I$ is non-singular (note that this is already proved in the last part of the proof of “(3)⇒(4)” in [1]), or there exists a simple submodule $\overline{B} = (B + I)/I$ of $\overline{R}$ such that $\overline{R} = \overline{A} \oplus \overline{B}$ (note that $\overline{A}$ is a maximal submodule of $\overline{R}$). As $\overline{A}$ is semisimple we infer that so is $\overline{R}$. Hence in this case both $I$ and $R/I$ are semisimple and we are done.

(4)⇒(1). If for every maximal right ideal $I$ of $R$, $R/I$ is non-singular, then no such $I$ is an essential right ideal. This means that $R$ is semisimple, which is absurd. Hence there must exist a maximal right ideal $I$ of $R$ such that $I$ is semisimple. Thus $I \subseteq S = \text{Soc}(R)$ implies that $I = S$ and we are done.

For the convenience of the reader we conclude this note with the following remark.
Remark. If we replace the word “right V-ring” with “almost right V-ring” in the abstract of [1] and in the phrase “and indeed any semiprime right ue-ring is a right V-ring” in the Introduction of [1], then the new form of Theorem 18 requires no alteration of any other statement in [1].

References


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