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Erratum to "On rings with a unique proper essential right ideal"

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by

O. A. S. Karamzadeh, M. Motamedi and
S. M. Shahrtash (Ahvaz)

Let $Q = \operatorname{End}(V)$, where V is an infinite-dimensional vector space over a field F. Put $R = S + F$, where S is the socle of Q. In [1, Example 11], we observed that R is a right ue-ring (i.e., it has a unique proper essential right ideal). Imitating the proof of this observation word for word, one can easily see that R is a left ue-ring too. But unfortunately this trivial fact was overlooked by us and we wrongly claimed otherwise. Moreover, we were led on by this to claim in [1, Theorem 18] that every semiprime right ue-ring is a right V-ring (note that the above ring R is well-known to be a right V-ring which is not a left V-ring, see Example 11 in [1]). Hence this is in fact an example of a semiprime left ue-ring which is not a left V-ring. In what follows we give the correct form of Theorem 18 and of the comment preceding it in [1]. We begin with the following definition.

Definition. A ring R is called an *almost right V-ring* if every simple right R-module is either injective or projective.

Clearly, every right V-ring is an almost right V-ring and we will observe shortly that every right ue-ring is an almost right V-ring.

In reference [6] of [1], right V-rings R with non-finitely generated right socle, say S, such that R/S is a division ring are characterized. We note that these are semiprime right ue-rings which have the following properties too.

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Theorem (Theorem 18 in [1]). *The following statements are equivalent for a semiprime ring R.*

(1) *R is a right ue-ring.*
(2) *The intrinsic topology of R is a non-discrete Hausdorff topology and a dense right ideal must be semisimple.*
(3) *R is a regular, almost right V-ring and $R/\mathrm{Soc}(R)$ is a division ring with $\mathrm{Soc}(R) \neq 0$.*
(4) *For each right ideal I, either R/I is non-singular or both I and R/I are semisimple R-modules and the Goldie dimension of R is not finite.*

Proof. (1)⇒(2). This is proved in [1].

(2)⇒(3). Everything is proved in [1], except the fact that R is an almost right V-ring. To prove the latter fact, we note that R is a right ue-ring and every maximal right ideal except the socle is a direct summand of R. Hence every simple R-module is either projective or isomorphic to R/S, where $S = \mathrm{Soc}(R)$. Now we show that R/S is injective and we are through. We must extend every homomorphism $f : I \to R/S$, where I is a right ideal of R, to a homomorphism from R into R/S. In view of Proposition 11 in [1], I is either a direct summand of R or semisimple. In case I is a direct summand of R, f can be naturally extended to a homomorphism from R into R/S. Finally, if I is semisimple, then $I \subseteq S$ and I is generated by idempotents. Let $e \in I$ be an idempotent. Then $f(e) = f(e)e \in \frac{R}{S}S = 0$. Thus f is the zero mapping and we are done.

(3)⇒(4). First, we note that the Goldie dimension of R is not finite, for R is a regular ring which is not semisimple. Now let $I \not\subseteq S = \mathrm{Soc}(R)$ be a right ideal of R. Then in the proof of "(3)⇒(4)" in [1] it is already shown that R/I is non-singular. Hence we may assume that $I \subseteq S$, and since S is semisimple we have $S = I \oplus A$ for some right ideal A of R. Put $\overline{R} = R/I$ and $\overline{A} = (A+I)/I = S/I$. Then either $\overline{A}$ is essential in R/I in which case R/I is non-singular (note that this is already proved in the last part of the proof of "(3)⇒(4)" in [1]), or there exists a simple submodule $\overline{B} = (B+I)/I$ of $\overline{R}$ such that $\overline{R} = \overline{A} \oplus \overline{B}$ (note that $\overline{A}$ is a maximal submodule of $\overline{R}$). As $\overline{A}$ is semisimple we infer that so is $\overline{R}$. Hence in this case both I and R/I are semisimple and we are done.

(4)⇒(1). If for every maximal right ideal I of R, R/I is non-singular, then no such I is an essential right ideal. This means that R is semisimple, which is absurd. Hence there must exist a maximal right ideal I of R such that I is semisimple. Thus $I \subseteq S = \mathrm{Soc}(R)$ implies that $I = S$ and we are done.

For the convenience of the reader we conclude this note with the following remark.

REMARK. If we replace the word "right V-ring" with "almost right V-ring" in the abstract of [1] and in the phrase "and indeed any semiprime right ue-ring is a right V-ring" in the Introduction of [1], then the new form of Theorem 18 requires no alteration of any other statement in [1].

References

[1] O. A. S. Karamzadeh, M. Motamedi and S. M. Shahrtash, *On rings with a unique proper essential right ideal*, Fund. Math. 183 (2004), 229–244.

Department of Mathematics
Chamran University
Ahvaz, Iran
E-mail: karamzadeh@ipm.ir

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