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Corrigendum to the paper "Semigroup actions on tori and stationary measures on projective spaces"

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by

YVES GUIVARC'H (Rennes) and ROMAN URBAN (Wrocław)

The argument given in the first sentence of Subsection 6.1 on page 63, which shows that $\Sigma = \mathbb{T}^d$, is not correct. This is because we cannot directly apply Corollary 5.22 to the set $p^{-1}(\Sigma)$ since this is a subset of $V = \mathbb{R}^d$ whereas Corollary 5.22 concerns subsets of $\widetilde{V} \setminus \{0\}$. However, the fact that $\Sigma = \mathbb{T}^d$ is true and can be justified as follows.

Let Σ be a closed Γ-invariant subset of $\mathbb{T}^d$ and suppose that 0 is a limit point of Σ. Consider $p^{-1}(\Sigma) \subset V = \mathbb{R}^d$, the inverse image of Σ under the canonical projection p. Let $\widetilde{p^{-1}(\Sigma)}$ be the projection of the set $p^{-1}(\Sigma) \subset V$ into the space $\widetilde{V} = V/\{\pm\mathrm{Id}\}$. Clearly, $\widetilde{p^{-1}(\Sigma)}$ is a closed Γ-invariant set in $\widetilde{V}$ and 0 is a limit point of $\widetilde{p^{-1}(\Sigma)}$. Applying Corollary 5.22 to the Γ-invariant set $\widetilde{p^{-1}(\Sigma)} \setminus \{0\} \subset \widetilde{V} \setminus \{0\}$ we get

$$\widetilde{p^{-1}(\Sigma)} \supset L_\Gamma \times \mathbb{R}^*_+ = \widetilde{L}_\Gamma/\{\pm\mathrm{Id}\},$$

and consequently

$$V \supset p^{-1}(\Sigma) \cup -p^{-1}(\Sigma) \supset \widetilde{L}_\Gamma.$$

By Lemma 5.1, L_Γ is not contained in a countable union of subspaces, in particular $\widetilde{L}_\Gamma$ contains at least one ray which is not contained in a rational subspace. Thus

$$p(p^{-1}(\Sigma) \cup -p^{-1}(\Sigma)) = \mathbb{T}^d,$$

and consequently

$$\Sigma \cup -\Sigma = \mathbb{T}^d.$$

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In particular, the normalized Haar measure of the set Σ is positive. Using exactly the same ergodic argument as on page 64, lines 16–13 from the bottom, we conclude that $\Sigma = \mathbb{T}^d$.

IRMAR
Université de Rennes 1
Campus de Beaulieu
35042 Rennes Cedex, France
E-mail: yves.guivarch@univ-rennes1.fr

Institute of Mathematics
Wrocław University
Plac Grunwaldzki 2/4
50-384 Wrocław, Poland
E-mail: urban@math.uni.wroc.pl

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