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Compact operators on the weighted Bergman space $A^1(\psi)$

by

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Abstract. We show that a bounded linear operator S on the weighted Bergman space $A^1(\psi)$ is compact and the predual space $A_0(\varphi)$ of $A^1(\psi)$ is invariant under S^* if and only if $Sk_z \to 0$ as $z \to \partial D$, where k_z is the normalized reproducing kernel of $A^1(\psi)$. As an application, we give conditions for an operator in the Toeplitz algebra to be compact.

1. Introduction. Let φ be a positive continuous function on $[0,1)$. We say that φ is *normal* if there exist $0 < a < b$ and $r_0 < 1$ such that

$$\frac{\varphi(r)}{(1-r^2)^a} \searrow 0 \quad \text{and} \quad \frac{\varphi(r)}{(1-r^2)^b} \nearrow \infty \qquad (r_0 \le r \to 1^-). \tag{1}$$

The functions $\{\varphi, \psi\}$ will be called a *normal pair* if φ is normal and if, for some b satisfying (1), there exists $\alpha > b-1$ such that $\varphi(r)\psi(r) = (1-r^2)^\alpha$. Let dA denote the normalized Lebesgue measure on the open unit disk D in the complex plane, and dA_ψ the measure on D defined by $dA_\psi(z) = \psi(|z|)\, dA(z)$. The condition $\alpha > b-1$ ensures that the measure dA_ψ on D is finite.

Let $L^1(\psi)$ denote the Banach space of measurable functions f with norm $\|f\|_\psi = \int_D |f|\, dA_\psi < \infty$. Let $A^1(\psi)$ denote the closed subspace of $L^1(\psi)$ consisting of all analytic functions, which will be called the *weighted Bergman space*. In the case that $\varphi(r) = (1-r)^c$ for some constant $c > 0$, and that $\alpha = c$, $A^1(\psi)$ is the Bergman space $L^1_{\mathrm{a}}(D)$.

Let $L_\infty(\varphi)$ denote the Banach space consisting of all measurable functions such that $f(z)\varphi(|z|)$ is essentially bounded on D with norm $\|f\|_\varphi = \operatorname{ess\,sup}_{z\in D} |f(z)|\varphi(|z|)$. Let $A_\infty(\varphi)$ denote the closed subspace of $L_\infty(\varphi)$ consisting of all analytic functions, and

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$$A_0(\varphi) := \{f \in A_\infty(\varphi) : \lim_{r\to 1^-} \sup_{|z|=r} f(z)\varphi(|z|) = 0\},$$

a closed subspace of $A_\infty(\varphi)$.

Throughout this paper we shall use the following pairing between $A^1(\psi)$ and $A_\infty(\varphi)$:

$$\langle f, g\rangle = \int_D f(z)\overline{g(z)}(1-|z|^2)^\alpha \, dA(z). \tag{2}$$

For $z, w \in D$, let

$$K_z(w) = \frac{1+\alpha}{(1-\overline{z}w)^{2+\alpha}},$$

and define an operator Q on $L^1(\psi)$ by

$$(Qf)(z) = \langle f, K_z\rangle = \int_D f(w)\overline{K_z(w)}(1-|w|^2)^\alpha \, dA(w).$$

Then Q is a bounded projection from $L^1(\psi)$ onto $A^1(\psi)$ and $K_z(w)$ is the reproducing kernel of $A^1(\psi)$, that is, $f(z) = \langle f, K_z\rangle$ for any $f \in A^1(\psi)$. Since K_z is in $A^1(\psi)$ for any $z \in D$, the reproducing property $f(z) = \langle f, K_z\rangle$ also holds for $f \in A_\infty(\varphi)$ (see [6] for the details). The function $k_z(w) := K_z(w)/\|K_z\|_\psi$ will be called the *normalized reproducing kernel* of $A^1(\psi)$.

In the Bergman space $L^2_{\mathrm{a}}(D)$ setting, Axler and Zheng [1] proved that an operator S which is a finite sum of finite products of Toeplitz operators, is compact if and only if $\|Sk_z\| \to 0$ as $|z| \to 1^-$. This result also holds for the spaces $L^p_{\mathrm{a}}(D)$ $(1 < p < \infty)$ (see [7]), $A^2_v(\Omega)$ with Ω a regular bounded symmetric domain in C^n (see [2]), and $H^2(\Omega, dv)$ with Ω a smoothly bounded multiply connected domain in the complex plane (see [5]). Recently Miao and Zheng [4] proved that for a bounded operator S on $L^p_{\mathrm{a}}(D)$ $(1 < p < \infty)$ such that both $\sup_{z\in D} \|S_z 1\|_m$ and $\sup_{z\in D} \|S^*_z 1\|_m$ are finite, S is a compact operator on $L^p_{\mathrm{a}}(D)$ if and only if the Berezin transform of S tends to 0 at ∂D.

In this note, we will obtain a similar result for $A^1(\psi)$. More precisely, we show that a bounded linear operator S on $A^1(\psi)$ is compact and $A_0(\varphi)$ is an invariant subspace of S^* if and only if $\|Sk_z\|_\psi \to 0$ as $z \to \partial D$.

2. Preliminaries.

The following result can be found in [6].

LEMMA 1. *$A^1(\psi)^* = A_\infty(\varphi)$ and $A_0(\varphi)^* = A^1(\psi)$.*

We also need some results about the reproducing kernel of $A^1(\psi)$.

LEMMA 2. *There exist constants c and C such that*

$$c/\varphi(|z|) \le \|K_z\|_\psi \le C/\varphi(|z|).$$

Proof. The second inequality can be derived from Lemmas 5 and 8 in [6].

Now we prove the first inequality. If $|z| \le r_0$, the first inequality obviously holds. If $|z| > r_0$, using (1) and $\varphi(r)\psi(r) = (1-r^2)^\alpha$, we have

$$\frac{(1-|w|^2)^{\alpha-a}}{\psi(|w|)} \le \frac{(1-|z|^2)^{\alpha-a}}{\psi(|z|)} \quad \text{when } 1 > |w| > |z|,$$
$$\frac{(1-|w|^2)^{\alpha-b}}{\psi(|w|)} \le \frac{(1-|z|^2)^{\alpha-b}}{\psi(|z|)} \quad \text{when } r_0 \le |w| < |z|.$$

Hence

$$\begin{aligned}\|K_z\|_\psi &= \int_D \frac{(1+\alpha)\psi(|w|)}{|1-\bar z w|^{2+\alpha}}\,dA(w)\\ &\ge \frac{(1+\alpha)\psi(|z|)}{(1-|z|^2)^{\alpha-b}}\biggl(\int_{r_0\le|w|\le|z|}\frac{(1-|w|^2)^{\alpha-b}}{|1-\overline{z}w|^{2+\alpha}}\,dA(w)\\ &\qquad + \int_{|z|<|w|<1}\frac{(1-|w|^2)^{\alpha-a}}{|1-\overline{z}w|^{2+\alpha}(1-|z|^2)^{b-a}}\,dA(w)\biggr).\end{aligned}$$

Note that

$$\frac{1}{(1-|z|^2)^{b-a}} = \frac{1}{((1+|z|)(1-|z|))^{b-a}} \ge \frac{1}{(2|1-\overline{z}w|)^{b-a}},$$

and similarly

$$\frac{1}{(1-|w|^2)^{b-a}} \ge \frac{1}{(2|1-\overline{z}w|)^{b-a}}.$$

So there exists a positive constant c_1 such that

$$\begin{aligned}\|K_z\|_\psi &\ge \frac{c_1\psi(|z|)}{(1-|z|^2)^{\alpha-b}}\int_{r_0\le|w|}\frac{(1-|w|^2)^{\alpha-a}}{|1-\overline{z}w|^{2+\alpha+b-a}}\,dA(w)\\ &= \frac{c_1\psi(|z|)}{(1-|z|^2)^{\alpha-b}}\Bigl(\int_D - \int_{|w|\le r_0}\Bigr)\frac{(1-|w|^2)^{\alpha-a}}{|1-\overline{z}w|^{2+\alpha+b-a}}\,dA(w)\\ &=: \frac{c_1\psi(|z|)}{(1-|z|^2)^{\alpha-b}}\,(I_1(z)-I_2(z)).\end{aligned}$$

Now by Lemma 4.2.2 of [8], $I_1(z) \sim (1-|z|^2)^{-b}$ as $z \to \partial D$; and I_2 is bounded. Thus it is easy to see that there exists $c > 0$ so that

$$\|K_z\|_\psi \ge c\,\frac{\psi(|z|)}{(1-|z|^2)^\alpha} = c/\varphi(|z|).$$

The proof is now complete. ∎

LEMMA 3. *The normalized reproducing kernel k_z converges weakly* to* 0 *in $A^1(\psi)$ as $z \to \partial D$.*

Proof. For $g \in A_0(\varphi)$, by the reproducing property of K_z, we have

$$\langle k_z, g\rangle = \frac{\langle K_z, g\rangle}{\|K_z\|_\psi} = \frac{\overline{g(z)}}{\|K_z\|_\psi}.$$

Now it follows from the definition of $A_0(\varphi)$ and Lemma 2 that $\langle k_z, g\rangle \to 0$ as $z \to \partial D$. ∎

3. Compactness

THEOREM 1. *Suppose that S is a bounded linear operator on $A^1(\psi)$. Then S is compact and $A_0(\varphi)$ is an invariant subspace of S^* if and only if $\|Sk_z\|_\psi \to 0$ as $z \to \partial D$.*

Proof. Necessity. Suppose that S is a compact operator and $A_0(\varphi)$ is an invariant subspace of S^*. If $\|Sk_z\|_\psi \nrightarrow 0$ as $z \to \partial D$, then there exist a constant $\delta > 0$ and a sequence $\{z_n\}$ in D such that

$$z_n \to \partial D \quad \text{and} \quad \|Sk_{z_n}\|_\psi > \delta. \tag{3}$$

Since $\{k_{z_n}\}$ is a bounded sequence in $A^1(\psi)$ and S is compact, there exists a subsequence of $\{k_{z_n}\}$, also denoted by $\{k_{z_n}\}$, such that $\{Sk_{z_n}\}$ converges in $A^1(\psi)$. By Lemma 3, $z_n \to \partial D$ implies that $k_{z_n} \xrightarrow{w^*} 0$. Since $A_0(\varphi)$ is an invariant subspace of S^*, we have, for any $g \in A_0(\varphi)$,

$$\langle Sk_{z_n}, g\rangle = \langle k_{z_n}, S^*g\rangle \to 0.$$

Thus $Sk_{z_n} \xrightarrow{w^*} 0$. Since $\{Sk_{z_n}\}$ converges in $A^1(\psi)$, it must converge to its weak*-limit, that is, 0. This contradicts (3).

Sufficiency. Suppose that $\|Sk_z\|_\psi \to 0$ as $z \to \partial D$. By the reproducing property of K_z, one can see that

$$(S^*K_w)(z) = \langle S^*K_w, K_z\rangle = \overline{\langle SK_z, K_w\rangle} = \overline{(SK_z)(w)}.$$

So for $f \in A^1(\psi)$,

$$\begin{aligned}(Sf)(w) = \langle Sf, K_w\rangle = \langle f, S^*K_w\rangle &= \int_D f(z)\overline{(S^*K_w)(z)}(1-|z|^2)^\alpha\, dA(z)\\ &= \int_D f(z)(SK_z)(w)\varphi(|z|)\, dA_\psi(z).\end{aligned}$$

For $0 < t < 1$, define a compact supporting continuous function η_t on D by

$$\eta_t(z) = \begin{cases} 1, & |z| \le t,\\ \dfrac{1+t}{1-t} - \dfrac{2|z|}{1-t}, & t < |z| \le (1+t)/2,\\ 0, & (1+t)/2 < |z| < 1.\end{cases}$$

For any $0 < r, t < 1$, define the integral operators $S_{[r]}$ on $A^1(\psi)$ and $S_{[r,t]}$ from $A^1(\psi)$ to $L^1(\psi)$ as follows:

$$S_{[r]}f(w) = \int_D f(z)(SK_z)(w)\eta_r(z)\varphi(|z|)\,dA_\psi(z),$$
$$S_{[r,t]}f(w) = \int_D f(z)(SK_z)(w)\eta_r(z)\eta_t(w)\varphi(|z|)\,dA_\psi(z).$$

Using Lemma 2, we have

$$\begin{aligned}\|(S - S_{[r]})f\|_\psi &\le \int_D |f(z)|\,dA_\psi(z) \int_D |(SK_z)(w)|\varphi(|z|)(1-\eta_r(z))\,dA_\psi(w)\\ &\le C\|f\|_\psi \sup_{z\in D}(1-\eta_r(z))\|Sk_z\|_\psi.\end{aligned}$$

Since $\|Sk_z\|_\psi \to 0$ as $z \to \partial D$, we have $\sup_{z\in D}(1-\eta_r(z))\|Sk_z\|_\psi \to 0$ as $r \to 1^-$. Thus

$$\|S - S_{[r]}\| \to 0 \quad \text{as } r \to 1^-. \tag{4}$$

Seeing $S_{[r]}$ as an operator from $A^1(\psi)$ to $L^1(\psi)$, if we prove that it is compact, then it is also compact as an operator on $A^1(\psi)$. Similar to the above, we have

$$\|S_{[r]} - S_{[r,t]}\| \le C \sup_{z\in\frac{1+r}{2}D} \int_D |(Sk_z)(w)|(1-\eta_t(w))\,dA_\psi(w). \tag{5}$$

We will prove that

$$\sup_{z\in\frac{1+r}{2}D} \int_D |(Sk_z)(w)|(1-\eta_t(w))\,dA_\psi(w) \to 0$$

as $t \to 1^-$ for fixed $r < 1$.

Let $g_t(z) = \int_D |(Sk_z)(w)|(1-\eta_t(w))\,dA_\psi(w)$. Firstly we will show that $\{g_t : 0 < t < 1\}$ is equicontinuous and uniformly bounded on $\overline{\frac{1+r}{2}D}$.

Since the Taylor expansion of K_z is $K_z(w) = \sum_{m=0}^\infty \beta_m(w\overline{z})^m$, where $\beta_m = (\alpha+1)\cdots(\alpha+m+1)/m!$, for any $z_1, z_2 \in \overline{\frac{1+r}{2}D}$ we have

$$\begin{aligned}|K_{z_1}(w) - K_{z_2}(w)| &= \Big|\sum_{m=0}^\infty \beta_m(w\overline{z}_1)^m - \sum_{m=0}^\infty \beta_m(w\overline{z}_2)^m\Big|\\ &\le \sum_{m=1}^\infty \beta_m|w|^m|\overline{z}_1^m - \overline{z}_2^m| \le |z_1 - z_2|\sum_{m=1}^\infty \beta_m m r^{m-1}.\end{aligned}$$

The last series above is the derivative of the series $\sum_{m=0}^\infty \beta_m r^m$ for $\frac{1+\alpha}{(1-r)^{2+\alpha}}$ hence convergent in $|r| < 1$. So for any $\varepsilon > 0$, there exists a constant $\delta_1 > 0$ such that $|K_{z_1}(w) - K_{z_2}(w)| \le \varepsilon$ for any $z_1, z_2 \in \overline{\frac{1+r}{2}D}$ with $|z_1 - z_2| < \delta_1$, and so $\|K_z\|_\psi$ is uniformly continuous on $\overline{\frac{1+r}{2}D}$. Then for any $\varepsilon > 0$, there exist

$\delta_2 > 0$ such that $|k_{z_1}(w) - k_{z_2}(w)| < \varepsilon$ for $z_1, z_2 \in \overline{\frac{1+r}{2}D}$ with $|z_1 - z_2| < \delta_2$, whence

$$|g_t(z_1) - g_t(z_2)| \le \int_D |Sk_{z_1}(w) - Sk_{z_2}(w)|(1 - \eta_t(w))\, dA_\psi(w) \le \|S\| \int_D |k_{z_1}(w) - k_{z_2}(w)|\, dA_\psi(w) \le \varepsilon \|S\|\, \|1\|_\psi.$$

Since ε is arbitrary, $\{g_t : 0 < t < 1\}$ is equicontinuous. It is obvious that $\{g_t : 0 < t < 1\}$ is uniformly bounded.

For $z \in \overline{\frac{1+r}{2}D}$, Lebesgue's dominated convergence theorem implies that $g_t(z) \to 0$ as $t \to 1^-$. It follows from Ascoli's theorem that $\{g_t : 0 < t < 1\}$ is relatively compact in $C\big(\overline{\frac{1+r}{2}D}\big)$, the Banach space of continuous functions on $\overline{\frac{1+r}{2}D}$, so has a unique accumulation point, 0. Therefore $g_t \to 0$ as $t \to 1^-$. So (5) implies that

$$\|S_{[r]} - S_{[r,t]}\| \to 0 \quad \text{as } t \to 1^-. \tag{6}$$

Since the kernel of $S_{[r,t]}$ is a compact supporting continuous function on $D \times D$, it can be approximated uniformly by polynomials in $z, \overline{z}, w, \overline{w}$ on $D \times D$. Because a polynomial kernel induces a finite rank integral operator, $S_{[r,t]}$ is a compact operator (cf. [3]). Thus (6) implies that $S_{[r]}$ is compact, and (4) implies that S is compact.

Finally, we show that $A_0(\varphi)$ is invariant under S^*. Suppose that $g \in A_0(\varphi)$ and $S^*g \in A_\infty(\varphi) \setminus A_0(\varphi)$. By the definitions of $A_0(\varphi)$ and $A_\infty(\varphi)$, there exist some positive constant ε and a sequence $\{z_n\}$ in D such that $z_n \to \partial D$ as $n \to \infty$ and $|(S^*g)(z_n)|\varphi(|z_n|) > \varepsilon$. Thus using Lemma 2 and the reproducing property of K_z, we have

$$|\langle Sk_{z_n}, g\rangle| = |\langle k_{z_n}, S^*g\rangle| = \frac{|(S^*g)(z_n)|}{\|K_{z_n}\|_\psi} \ge \frac{1}{C}\, |(S^*g)(z_n)|\varphi(|z_n|) > \frac{\varepsilon}{C}.$$

This contradicts the hypothesis $\|Sk_z\|_\psi \to 0$ as $z \to \partial D$, and completes the proof of Theorem 1. ∎

In the last argument of the proof of Theorem 1, $g \in A_\infty(\varphi)$ is only needed. So the following result holds.

Corollary. *If S is a compact operator on $A^1(\psi)$ and $S^*A_0(\varphi) \subset A_0(\varphi)$, then $S^*A_\infty(\varphi) \subset A_0(\varphi)$.*

Remark. If S is a compact operator on $L^2_{\mathrm{a}}(D)$, then $\|Sk_z\| \to 0$ as $z \to \partial D$. Examples in [1] and [4] show that the converse does not hold. However, in $A^1(\psi)$, $\|Sk_z\| \to 0$ as $z \to \partial D$ is a sufficient condition for the compactness of S. Moreover, the following example shows that $A_0(\varphi)$ being an invariant subspace of S^* in Theorem 1 is necessary.

EXAMPLE. Suppose that $f \in A_\infty(\varphi) \setminus A_0(\varphi)$. By the definitions of $A_\infty(\varphi)$ and $A_0(\varphi)$, there exists a sequence $\{z_n\}$ in D such that $z_n \to \partial D$ and $|f(z_n)|\varphi(|z_n|) \nrightarrow 0$.

Suppose that $0 \neq g \in A^1(\psi)$. Then there exists $h \in A_\infty(\varphi)$ such that $\langle g, h\rangle \neq 0$. Let $S = g \otimes f$. Then S is a compact operator on $A^1(\psi)$. However,

$$\begin{aligned}|\langle Sk_z, h\rangle| &= |\langle k_z, (g\otimes f)^* h\rangle| = |\langle k_z, (f\otimes g^{**})h\rangle| \\ &= |\langle k_z, f\rangle\langle g, h\rangle| = |\langle g, h\rangle| \frac{|f(z_n)|}{\|K_{z_n}\|_\psi} \\ &\geq C|\langle g, h\rangle|\, |f(z_n)|\varphi(|z_n|) \nrightarrow 0,\end{aligned}$$

where the inequality comes from Lemma 2, and g^{**} denotes the image of g in the double dual space of $A^1(\psi)$. So we have $\|Sk_{z_n}\|_\psi \nrightarrow 0$.

For $u \in L^\infty(D)$, define the Toeplitz operator T_u by $T_u(f) = Q(uf)$, where $f \in A^1(\psi)$. Let $\mathcal{T}(L^\infty)$ denote the closed subalgebra of $B(A^1(\psi))$ generated by $\{T_u : u \in L^\infty(D)\}$.

THEOREM 2. *Suppose that $S \in \mathcal{T}(L^\infty)$. Then S is compact if and only if $\|Sk_z\|_\psi \to 0$ as $z \to \partial D$.*

Proof. By Theorem 1, it is sufficient to prove that $A_0(\varphi)$ is invariant under S^* when $S \in \mathcal{T}(L^\infty)$. By the definition of $\mathcal{T}(L^\infty)$, it is sufficient to prove that $A_0(\varphi)$ is invariant under T_u^* for any $u \in L^\infty(D)$. Let $g \in A_0(\varphi)$. Then

$$(T_u^* g)(z) = \overline{\langle K_z, T_u^* g\rangle} = \overline{\langle T_u K_z, g\rangle} = \overline{\langle uK_z, g\rangle}.$$

Hence

$$\begin{aligned}|(T_u^* g)(z)\varphi(|z|)| &\leq \int_D |u(w)K_z(w)g(w)|(1-|w|^2)^\alpha \varphi(|z|)\, dA(w) \\ &\leq \|u\|_\infty \int_D \varphi(|z|)|K_z(w)g(w)|\varphi(|w|)\, dA_\psi(w) \\ &= \|u\|_\infty \Big(\int_{rD} \varphi(|z|)|K_z(w)g(w)|\varphi(|w|)\, dA_\psi(w) \\ &\quad + \int_{D\setminus rD} \varphi(|z|)|K_z(w)g(w)|\varphi(|w|)\, dA_\psi(w)\Big),\end{aligned} \tag{7}$$

where $0 < r < 1$. Since $g \in A_0(\varphi)$, for any $\varepsilon > 0$, there exists r such that $|g(w)|\varphi(|w|) \leq \varepsilon$ for $w \in D\setminus rD$. Then it follows from Lemma 2 that the second integral on the right hand side of (7) is $\leq \varepsilon \int_{D\setminus rD} \varphi(|z|)|K_z(w)|\, dA_\psi(w)$ $\leq C\varepsilon$ for fixed r. Since $g \in A_0(\varphi)$, there exists a constant M such that $\sup_{w\in D} |g(w)|\varphi(|w|) \leq M$. Since $\varphi(|z|)|K_z(w)|$ converges uniformly to 0 on rD as $z \to \partial D$, the first integral on the right hand side of (7) converges to 0 as $z \to \partial D$. So we have

$$\limsup_{z\to\partial D} |(T_u^* g)(z)|\varphi(|z|) \le \|u\|_\infty C\varepsilon.$$

Since ε is arbitrary, $|(T_u^* g)(z)|\varphi(|z|) \to 0$ as $z \to \partial D$. Thus $T^* g \in A_0(\varphi)$. ∎

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