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Erratum to "Can $\mathcal{B}(\ell^p)$ ever be amenable?"

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by

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Some of the results of Section 3 of [2] are incorrect; in particular, we claimed that the implication (i)$\Rightarrow$(ii) of Lemma 3.3 was "routine", whereas it appears to be false, or at least difficult to prove.

The main result of this section, Theorem 3.2, claims that a separable Banach algebra $\mathfrak{A}$ is ultra-amenable (that is, all ultrapowers of $\mathfrak{A}$ are amenable) if and only if $\ell^\infty(\mathfrak{A})$ is amenable. However, if we let $\mathfrak{A} = \mathbb{C}$, then any ultrapower of $\mathfrak{A}$ is also $\mathbb{C}$, and hence trivially amenable. While ℓ^∞ is amenable, this is not trivial, and in no sense do our arguments reduce to this special case.

Furthermore, the motivation for Section 3 was [1, Section 5], where the first named author studied similar ideas for ultra-amenability. The proof of [1, Proposition 5.4], (ii)$\Rightarrow$(i), also needs further justification, as currently the map ψ_0 is implicitly assumed to be at least bounded below. However, in this case, in light of [1, Proposition 4.7], it seems possible that this could be true, at least for certain well-behaved spaces $\mathfrak{A}$.

Let us restate Theorem 3.2. In light of the example of $\mathfrak{A} = \mathbb{C}$, it seems unlikely that Lemma 3.3, even suitably adjusted, could be true, and so fully correcting Theorem 3.2 seems out of reach.

Theorem. *Let $\mathfrak{A}$ be a Banach algebra, and consider the conditions*:

(i) $\ell^\infty(\mathfrak{A})$ *is amenable*;
(ii) $\ell^\infty(\mathbb{I}, \mathfrak{A})$ *is amenable for every index set* $\mathbb{I}$;
(iii) $\mathfrak{A}$ *is ultra-amenable.*

Then (ii)$\Rightarrow$(i) *and* (ii)$\Rightarrow$(iii).

Proof. Clearly (ii)$\Rightarrow$(i), and as an ultrapower of $\mathfrak{A}$ is a quotient of $\ell^\infty(\mathbb{I}, \mathfrak{A})$ for a suitable $\mathbb{I}$, and as amenability passes to quotients, it follows that (ii)$\Rightarrow$(iii). ∎

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Corollary 4.4 uses Theorem 3.2, but only implication (ii)⇒(iii), and hence remains true. The rest of [2] is unaffected. In particular, the tentative approach, outlined in Section 6, to showing that $\mathcal{B}(\ell^p)$ is not amenable, is not affected. We remark that the second named author has recently shown in [3] that, in particular, $\mathcal{B}(\ell^p)$ is not amenable for any $p \in [1, \infty)$; the arguments only rely upon Section 2 of [2] and are hence unaffected by this erratum.

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References

[1] M. Daws, *Amenability of ultrapowers of Banach algebras*, Proc. Edinburgh Math. Soc. (2) 52 (2009), 307–338.

[2] M. Daws and V. Runde, *Can* $\mathcal{B}(\ell^p)$ *ever be amenable?*, Studia Math. 188 (2008), 151–174.

[3] V. Runde, $\mathcal{B}(\ell^p)$ *is never amenable*, preprint, arXiv:0907.3984v3 [math.FA].

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