Erratum to “Can \( \mathcal{B}(\ell^p) \) ever be amenable?”
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by

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Some of the results of Section 3 of [2] are incorrect; in particular, we claimed that the implication (i) \( \Rightarrow \) (ii) of Lemma 3.3 was “routine”, whereas it appears to be false, or at least difficult to prove.

The main result of this section, Theorem 3.2, claims that a separable Banach algebra \( \mathfrak{A} \) is ultra-amenable (that is, all ultrapowers of \( \mathfrak{A} \) are amenable) if and only if \( \ell^\infty(\mathfrak{A}) \) is amenable. However, if we let \( \mathfrak{A} = \mathbb{C} \), then any ultrapower of \( \mathfrak{A} \) is also \( \mathbb{C} \), and hence trivially amenable. While \( \ell^\infty \) is amenable, this is not trivial, and in no sense do our arguments reduce to this special case.

Furthermore, the motivation for Section 3 was [1, Section 5], where the first named author studied similar ideas for ultra-amenability. The proof of [1, Proposition 5.4], (ii) \( \Rightarrow \) (i), also needs further justification, as currently the map \( \psi_0 \) is implicitly assumed to be at least bounded below. However, in this case, in light of [1, Proposition 4.7], it seems possible that this could be true, at least for certain well-behaved spaces \( \mathfrak{A} \).

Let us restate Theorem 3.2. In light of the example of \( \mathfrak{A} = \mathbb{C} \), it seems unlikely that Lemma 3.3, even suitably adjusted, could be true, and so fully correcting Theorem 3.2 seems out of reach.

**Theorem.** Let \( \mathfrak{A} \) be a Banach algebra, and consider the conditions:

(i) \( \ell^\infty(\mathfrak{A}) \) is amenable;
(ii) \( \ell^\infty(\mathbb{I}, \mathfrak{A}) \) is amenable for every index set \( \mathbb{I} \);
(iii) \( \mathfrak{A} \) is ultra-amenable.

Then (ii) \( \Rightarrow \) (i) and (ii) \( \Rightarrow \) (iii).

**Proof.** Clearly (ii) \( \Rightarrow \) (i), and as an ultrapower of \( \mathfrak{A} \) is a quotient of \( \ell^\infty(\mathbb{I}, \mathfrak{A}) \) for a suitable \( \mathbb{I} \), and as amenability passes to quotients, it follows that (ii) \( \Rightarrow \) (iii). \( \blacksquare \)

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Corollary 4.4 uses Theorem 3.2, but only implication (ii)⇒(iii), and hence remains true. The rest of [2] is unaffected. In particular, the tentative approach, outlined in Section 6, to showing that $\mathcal{B}(\ell^p)$ is not amenable, is not affected. We remark that the second named author has recently shown in [3] that, in particular, $\mathcal{B}(\ell^p)$ is not amenable for any $p \in [1, \infty)$; the arguments only rely upon Section 2 of [2] and are hence unaffected by this erratum.

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References

