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# Erratum to "A class of Fourier multipliers on $H^1(\mathbb{R}^2)$"

(Studia Math. 140 (2000), 289–298)

by

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As was observed by Prof. Waldemar Hebisch, Theorem 1 of [W] is false. A counterexample is as follows. Let $\phi, \psi \in \mathcal{D}(\mathbb{R})$ and $\operatorname{supp}\phi, \operatorname{supp}\psi \subset [-1/2, 1/2]$ and $\psi(\xi) = 1$ for $[-1/4, 1/4]$. Let $\psi_n(\xi) = \psi((\xi - 2^n)/2^n)$ and $\phi_n(\eta) = \phi(2^n\eta)$ and $\Phi_n(\xi, \eta) = \psi_n(\xi)\phi_n(\eta)$. For $f \in \mathcal{D}(\mathbb{R}^2)$ we put

$$Tf = \sum_{n=1}^{\infty} \Phi_n^{\vee} * f.$$

It is easy to check that $T$ satisfies the assumptions of Theorem 1 from [W]. Let $g(\xi, \eta) = g_1(\xi)g_2(\eta)$ where $g_1, g_2 \in \mathcal{D}(\mathbb{R})$, $\operatorname{supp} g_1, \operatorname{supp} g_2 \subset [-1, 1]$, and $g_2(\eta) = 1$ for $\eta \in [-1/2, 1/2]$. Let

$$\widehat{f} = \sum_{k=1}^{\infty} a_k g(\xi - 2^{n_k}, \eta)$$

where $(a_k) \in \ell^2 \setminus \ell^1$, and $(n_k)$ is a sequence such that $n_1 > 1$ and

$$\Big\|\sum_{k=1}^{N} a_k \phi_{n_k}^{\vee}\Big\|_1 \to \infty \quad \text{as } N \to \infty.$$

Then $f \in H^1(\mathbb{R}^2)$. Indeed, since the functions $g(\xi - 2^{n_k}, \eta)$ $(k = 1, 2, \ldots)$ are supported on disjoint dyadic frames,

$$\|f\|_{H_1} \simeq \int \Big(\sum |a_k g^{\vee}|^2\Big)^{1/2} = \Big(\sum a_k^2\Big)^{1/2} \Big(\int |g^{\vee}|^2\Big)^{1/2} < \infty.$$

On the other hand

$$\begin{aligned}(Tf)^{\wedge}(\xi, \eta) &= \sum_k a_k \psi_{n_k}(\xi)\phi_{n_k}(\eta) g_1(\xi - 2^{n_k}) g_2(\eta) \\ &= \sum_k a_k \phi_{n_k}(\eta) g_1(\xi - 2^{n_k}).\end{aligned}$$

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Hence

$$\|Tg\|_1 = \Big\| \sum_k a_k \phi_{n_k}^{\vee}(y) g_1^{\vee}(x) e^{2\pi i 2^{n_k} x} \Big\|_1 = \|g_1^{\vee}\|_1 \cdot \Big\| \sum a_k \phi_{n_k}^{\vee} \Big\|_1 = \infty.$$

Similarly one can prove that Corollaries 1 and 2 of [W] do not hold.

The author is deeply grateful to Waldemar Hebisch for pointing out the mistake.

## References

[W] M. Wojciechowski, *A class of Fourier multipliers on* $H^1(\mathbb{R}^2)$, Studia Math. 140 (2000), 289–298.

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