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Isomorphic classification of the tensor products $E_0(\exp \alpha i) \widehat{\otimes} E_\infty(\exp \beta j)$

by

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Abstract. It is proved, using so-called multirectangular invariants, that the condition $\alpha\beta = \tilde{\alpha}\tilde{\beta}$ is sufficient for the isomorphism of the spaces $E_0(\exp \alpha i) \widehat{\otimes} E_\infty(\exp \beta j)$ and $E_0(\exp \tilde{\alpha} i) \widehat{\otimes} E_\infty(\exp \tilde{\beta} j)$. This solves a problem posed in [14, 15, 1]. Notice that the necessity has been proved earlier in [14].

1. Introduction. Let $A = (a_{ip})_{i\in I,\, p\in\mathbb{N}}$ be a matrix of real numbers such that $0 \le a_{ip} \le a_{i,p+1}$, where I is a countable set. The *Köthe space* defined by the matrix A is the locally convex space $K(A) = K((a_{ip}))$ of all sequences $\xi = (\xi_i)_{i\in I}$ such that $|\xi|_p := \sum_{i\in I} a_{ip}|\xi_i| < \infty$ for all $p \in \mathbb{N}$, with the topology generated by the system of seminorms $\{|\xi|_p : p \in \mathbb{N}\}$. We denote the canonical basis by $e = \{e_i\}_{i\in I}$.

We say that $X = K(A)$ is *quasidiagonally isomorphic* to $\tilde{X} = K(\tilde{A})$ with $\tilde{A} = (\tilde{a}_{jp})_{j\in J,\, p\in\mathbb{N}}$ (and write $X \overset{\mathrm{qd}}{\simeq} \tilde{X}$) if there exists a bijection $\varphi : I \to J$ and a scalar sequence t_i such that the mapping $Te_i := t_i e_{\varphi(i)}$, $i \in I$, can be extended (by linearity and continuity) to an isomorphism $T : X \to \tilde{X}$.

A. Grothendieck considered in [5] an important particular class of Köthe spaces:

$$E_\lambda(a) := K((\exp(\lambda - 1/p)a_i)), \qquad a = (a_i),\ -\infty < \lambda \le \infty, \tag{1.1}$$

usually called *power series spaces* [4, 6] *of finite type* if $\lambda < \infty$ (without loss of generality we may consider only $\lambda = 0$), and *of infinite type* if $\lambda = \infty$. A complete isomorphic classification of the spaces (1.1) is due to B. Mityagin [7, 9]. Spaces of different type possess very different properties: $E_0(a)$ is not isomorphic to $E_\infty(b)$ if a or b is not bounded (see, e.g., [7]), moreover,

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every continuous linear operator $T : E_0(a) \to E_\infty(b)$ is bounded (compact if $b_i \to \infty$) [12].

In [13, 14] the second author introduced so-called *power Köthe spaces of the first type*:

$$E(\lambda, c) = K((\exp(-1/p + \lambda_i p)c_i)), \qquad c = (c_i),\ \lambda = (\lambda_i). \tag{1.2}$$

Including (up to isomorphism) all the spaces (1.1), this class also contains spaces of much more complicated, mixed "finite-infinite type" structure, in particular, *Cartesian and tensor products of power series spaces of different type* (for some results on isomorphic classification and linear topological structure of such spaces see, e.g., [13, 14, 3, 2]).

Our main goal is the following result solving a problem posed in [14, 15, 1].

THEOREM 1. *Let $\alpha, \beta, \tilde\alpha, \tilde\beta$ be positive numbers. Then the following statements are equivalent:*

(i) $X = E_0(\exp \alpha i) \widehat{\otimes} E_\infty(\exp \beta j)$ *is isomorphic to the space* $\tilde X = E_0(\exp \tilde\alpha i) \widehat{\otimes} E_\infty(\exp \tilde\beta j)$,
(ii) $X \overset{\mathrm{qd}}{\simeq} \tilde X$,
(iii) $\alpha\beta = \tilde\alpha\tilde\beta$.

This particular case is of a special interest because the sequences like $a_i = \exp \alpha i$ are exactly on the border between so-called *shift-stable sequences* (for which $\limsup a_{i+1}/a_i < \infty$) and *lacunary sequences* ($\lim a_{i+1}/a_i = \infty$). In fact, we need to prove only (iii)⇒(ii), since (ii)⇒(i) is obvious and (i)⇒(iii) has been proved in [14].

A crucial role in our proof is played by a *system of multirectangular characteristics* for the space (1.2) (see Section 2). It is worth mentioning that estimating multirectangular invariants through a single rectangle invariant (see Propositions 6 and 7) is similar, in a sense, to the transition from one interval to a union of intervals in Mityagin's investigation of the spaces (1.1) [8, 9].

2. Multirectangular invariants. Dealing with spaces (1.2) we always assume that

$$c_i > 1, \qquad \lambda_i \le 1.$$

Given $m \in \mathbb{N}$, an *m-rectangular characteristic* of the space $X = E(\lambda, c)$ is the function

$$\mu_m^X(\delta, \varepsilon; \tau, t) := \Big| \bigcup_{k=1}^{m} \{i : \delta_k < \lambda_i \le \varepsilon_k,\ \tau_k < c_i \le t_k\} \Big|, \tag{2.1}$$

where

$$\begin{aligned} &\delta = (\delta_k), \quad \varepsilon = (\varepsilon_k), \quad \tau = (\tau_k), \quad t = (t_k), \\ &0 \le \delta_k < \varepsilon_k \le 1, \quad 1 \le \tau_k < t_k < \infty \end{aligned} \tag{2.2}$$

and $|S|$ is the number of elements in S if S is finite, and $+\infty$ otherwise. This function counts those points (λ_i, c_i) that lie in the union of the m rectangles

$$P_k := (\delta_k, \varepsilon_k] \times (\tau_k, t_k], \quad k = 1, \dots, m. \tag{2.3}$$

Let $X = E(\lambda, c)$ and $\tilde{X} = E(\tilde{\lambda}, \tilde{c})$. We say that *the systems of m-rectangular characteristics (μ_m^X) and $(\mu_m^{\tilde{X}})$ are equivalent* (and write $(\mu_m^X) \approx (\mu_m^{\tilde{X}})$) if there exist a strictly increasing bijection $\varphi : [0,2] \to [0,1]$ and a positive constant Δ such that

$$\mu_m^X(\delta, \varepsilon; \tau, t) \le \mu_m^{\tilde{X}}(\varphi(\delta), \varphi^{-1}(\varepsilon); \tau/\Delta, \Delta t), \tag{2.4}$$

$$\mu_m^{\tilde{X}}(\delta, \varepsilon; \tau, t) \le \mu_m^X(\varphi(\delta), \varphi^{-1}(\varepsilon); \tau/\Delta, \Delta t) \tag{2.5}$$

for every $m \in \mathbb{N}$ and all parameters $\delta, \varepsilon, \tau, t$; here $\varphi(\delta) = (\varphi(\delta_k))$, $\varphi^{-1}(\varepsilon) = (\varphi^{-1}(\varepsilon_k))$, $\tau/\Delta = (\tau_k/\Delta)$, $\Delta t = (\Delta t_k)$.

We shall use the following characterization of the quasidiagonal isomorphism of power Köthe spaces of first type in terms of their systems of multirectangular characteristics ([3]).

PROPOSITION 2. *The spaces $X = E(\lambda, c)$ and $\tilde{X} = E(\tilde{\lambda}, \tilde{c})$ are quasidiagonally isomorphic if and only if $(\mu_m^X) \approx (\mu_m^{\tilde{X}})$.*

The following fact will be useful in further considerations.

PROPOSITION 3 ([13, 14]). *Let $a = (a_i)_{i \in \mathbb{N}}$, $b = (b_j)_{i \in \mathbb{N}}$, $c = (c_{ij})$, $c_{ij} = \max\{a_i, b_j\}$ and $\lambda = (\lambda_{ij})$, $\lambda_{ij} = b_j/c_{ij}$; let $\{e_i \otimes e_j\}$ and $\{e_{ij}\}$ be the canonical bases in $E_0(a) \mathbin{\widehat{\otimes}} E_\infty(b)$ and $E(\lambda, c)$. Then the mapping $e_i \otimes e_j \mapsto e_{ij}$, $(i,j) \in \mathbb{N}^2$, can be uniquely extended to an isomorphism $T : E_0(a) \mathbin{\widehat{\otimes}} E_\infty(b) \to E(\lambda, c)$.*

3. Proof of Theorem 1. In what follows, X and $\tilde{X}$ are the spaces from Theorem 1. By Proposition 3 we may assume that

$$X = E(\lambda, c), \quad \tilde{X} = E(\tilde{\lambda}, \tilde{c}),$$

where $c = (c_{ij})$, $\lambda = (\lambda_{ij})$ with

$$c_{ij} = \max\{\exp \alpha i, \exp \beta j\}, \quad \lambda_{ij} = \min\{1, \exp(\beta j - \alpha i)\},$$

and $\tilde{c} = (\tilde{c}_{ij})$, $\tilde{\lambda} = (\tilde{\lambda}_{ij})$, with

$$\tilde{c}_{ij} = \max\{\exp \tilde{\alpha} i, \exp \tilde{\beta} j\}, \quad \tilde{\lambda}_{ij} = \min\{1, \exp(\tilde{\beta} j - \tilde{\alpha} i)\}, \quad (i,j) \in \mathbb{N}^2.$$

First we obtain some estimates for a single rectangle characteristic with a special choice of parameters.

LEMMA 4. *If $\alpha\beta = \tilde\alpha\tilde\beta$ then there exists $C > 1$ such that*

$$\mu_1^X(e^{-1}, 1; \tau, t) \le \mu_1^{\tilde X}(\delta, 1; \tau, Ct) \tag{3.1}$$

for all $\delta \in [0, 1)$ and $1 \le \tau < t < \infty$.

Proof. In our case

$$\mu_1^X(e^{-1}, 1; \tau, t) = |L_1 \cup L_2|, \quad \mu_1^{\tilde X}(\delta, 1; \tau, Ct) \ge |\tilde L|,$$

where

$$L_1 = \left\{(i,j) : \frac{\alpha i - 1}{\beta} < j \le \frac{\alpha i}{\beta};\ \frac{\ln\tau}{\alpha} < i \le \frac{\ln t}{\alpha}\right\},$$
$$L_2 = \left\{(i,j) : i \le \frac{\beta j}{\alpha};\ \frac{\ln\tau}{\beta} < j \le \frac{\ln t}{\beta}\right\},$$
$$\tilde L = \left\{(i,j) : i \le \frac{\tilde\beta j}{\tilde\alpha};\ \frac{\ln\tau}{\tilde\beta} < j \le \frac{\ln t + \ln C}{\tilde\beta}\right\}.$$

Setting $M = ((\ln t)^2 - (\ln\tau)^2)/2\alpha\beta$ and taking into account that $\alpha\beta = \tilde\alpha\tilde\beta$ one can easily obtain the following estimates:

$$|L_1| \le \frac{(1+\beta)\ln t}{\alpha\beta}, \tag{3.2}$$

$$|L_2| \le \frac{(\ln t + \ln\tau + \beta)(\ln t - \ln\tau + \beta)}{2\alpha\beta} = M + \frac{2\beta\ln t + \beta^2}{2\alpha\beta}, \tag{3.3}$$

$$\begin{aligned} |\tilde L| &\ge \frac{(\ln t + \ln\tau + \ln C - \tilde\beta)(\ln t - \ln\tau + \ln C - \tilde\beta)}{2\tilde\alpha\tilde\beta} \\ &\quad - \frac{\ln t + \ln C}{\tilde\beta} \\ &\ge M + \frac{(\ln C)^2 + 2(\ln C - \tilde\beta - \tilde\alpha)\ln t - 2(\tilde\alpha + \tilde\beta)\ln C}{2\alpha\beta}. \end{aligned} \tag{3.4}$$

Now we choose a constant $C > 1$ so that $|L_1 \cup L_2| \le |\tilde L|$.

The desired estimates will be guaranteed if the sum of the right sides of (3.2) and (3.3) is smaller than the right side of (3.4):

$$\ln^2 C + 2(\ln C - (\tilde\alpha + \tilde\beta + 2\beta + 1))\ln t \ge 2(\tilde\alpha + \tilde\beta)\ln C + \beta^2.$$

It is easy to see that this inequality is true for all $t > \tau \ge 1$ if we choose C so that $\ln C \ge 2(\beta + \tilde\alpha + \tilde\beta) + 1 + \beta^2$. ∎

LEMMA 5. *If $\alpha\beta = \tilde\alpha\tilde\beta$ then there exist constants C and p such that*

$$\mu_1^X(\delta, \varepsilon; \tau, t) \le \mu_1^{\tilde X}(\delta^p, \varepsilon^{1/p}; \tau, t) \tag{3.5}$$

for

$$0 \le \delta < \varepsilon \le e^{-1}, \qquad 1 \le \tau, \qquad C\tau < t < +\infty. \tag{3.6}$$

Proof. Taking into account the expressions for Köthe matrices of the spaces X and $\tilde{X}$, we can obtain the following estimates:

$$\begin{aligned}\mu_1^X(\delta,\varepsilon;\tau,t) &= |\{(i,j) : \ln\delta < \beta j - \alpha i \le \ln\varepsilon;\ \ln\tau < \alpha i \le \ln t\}| \\ &\le \frac{(\ln\varepsilon - \ln\delta + \beta)(\ln t - \ln\tau + \alpha)}{\alpha\beta}, \\ \mu_1^{\tilde{X}}(\delta^p,\varepsilon^{1/p};\tau,t) &= \left|\left\{(i,j) : p\ln\delta < \tilde{\beta} j - \tilde{\alpha} i \le \frac{\ln\varepsilon}{p};\ \ln\tau < \tilde{\alpha} i \le \ln t\right\}\right| \\ &\ge \frac{\big(\frac{\ln\varepsilon}{p} - p\ln\delta - \tilde{\beta}\big)(\ln t - \ln\tau - \tilde{\alpha})}{\tilde{\alpha}\tilde{\beta}}.\end{aligned}$$

It follows from these estimates that the inequality (3.5) will hold for the parameters (3.6) if we take the constants so that

$$\ln C > 2\max\{\alpha, \tilde{\beta}\}, \qquad p > 2(\beta + \tilde{\alpha} + \tilde{\beta} + 1). \ \blacksquare$$

In the following two statements we obtain estimates (2.4) for arbitrary $m \in \mathbb{N}$, but for special unions of rectangles that are located along some horizontal or vertical strips.

PROPOSITION 6. *Let $\alpha\beta = \tilde{\alpha}\tilde{\beta}$ and let $p > 1$, $C > 1$ be the constants of Lemma 5. Then*

$$\mu_m^X(\delta,\varepsilon;\tau,t) \le \mu_m^{\tilde{X}}(\delta^p,\varepsilon^{1/p};\tau,t), \qquad m \in \mathbb{N}, \tag{3.7}$$

for all $\delta = (\delta_k)$, $\varepsilon = (\varepsilon_k) \in [0, e^{-1}]^m$ with $\delta_k < \varepsilon_k$, and $t = (t,\dots,t)$, $\tau = (\tau,\dots,\tau)$ with $1 \le \tau < t/C$; here $\delta^p := (\delta_k^p)_{k=1}^m$ and $\varepsilon^{1/p} := (\varepsilon_k^{1/p})_{k=1}^m$.

Proof. Representing the set $E := \bigcup_{k=1}^m (\delta_k^p, \varepsilon_k^{1/p}]$ as a disjoint union of intervals, $E = \bigcup_{j=1}^i (\tilde{\delta}_j^p, \tilde{\varepsilon}_j^{1/p}]$, and applying Lemma 3 to each rectangle $(\tilde{\delta}_j, \tilde{\varepsilon}_j] \times (\tau, t]$, we obtain

$$\begin{aligned}\mu_m^X(\delta,\varepsilon;\tau,t) &\le \sum_{j=1}^i \mu_1^X(\tilde{\delta}_j,\tilde{\varepsilon}_j;\tau,t) \\ &\le \sum_{j=1}^i \mu_1^{\tilde{X}}(\tilde{\delta}_j^p,\tilde{\varepsilon}_j^{1/p};\tau,t) = \mu_m^{\tilde{X}}(\delta^p,\varepsilon^{1/p};\tau,t). \ \blacksquare\end{aligned}$$

PROPOSITION 7. *Let $\alpha\beta = \tilde{\alpha}\tilde{\beta}$. Then there exists $C > 1$ such that*

$$\mu_m^X(e^{-1},1;\tau,t) \le \mu_m^{\tilde{X}}(\delta,1;\tau,Ct), \qquad m \in \mathbb{N},$$

for $0 < \delta < 1$ and all $\tau = (\tau_k)$, $t = (t_k) \in \mathbb{R}^m$ with $1 \le \tau_k < t_k < +\infty$; here $Ct = (Ct_k)_{k=1}^m$.

Proof. Let C be the constant of Lemma 4. Take any $m \in \mathbb{N}$, $\tau = (\tau_k)$, $t = (t_k)$ such that $1 \le \tau_k < t_k$, $k = 1,\dots,m$, and represent the set $E :=$

$\bigcup_{k=1}^{m}(\tau_k, Ct_k]$ as a union of disjoint intervals, $E = \bigcup_{j=1}^{i}(\tilde{\tau}_j, C\tilde{t}_j]$. Then, applying Lemma 4 to each rectangle $(e^{-1}, 1] \times (\tilde{\tau}_j, \tilde{t}_j]$, we obtain

$$\begin{aligned}\mu_m^X(e^{-1}, 1; \tau, t) &\le \sum_{j=1}^{i} \mu_1^X(e^{-1}, 1; \tilde{\tau}_j, \tilde{t}_j) \\ &\le \sum_{j=1}^{i} \mu_1^{\tilde{X}}(\delta, 1; \tilde{\tau}_j, C\tilde{t}_j) = \mu_m^{\tilde{X}}(\delta, 1; \tau, Ct). \ \blacksquare\end{aligned}$$

Now we are ready to prove Theorem 1. As noted above, we need only show (iii)⇒(ii).

By Proposition 2, it is sufficient to prove that the systems (μ_m^X) and $(\mu_m^{\tilde{X}})$ are equivalent, that is, there exists a constant Δ and a function φ such that for any m and for any collection (2.2) we have (2.4) and (2.5). Due to symmetry, it is sufficient to prove only (2.4).

Let $\alpha\beta = \tilde{\alpha}\tilde{\beta}$. Then we choose a constant C and p satisfying the conditions of Propositions 6 and 7. We are going to prove that (2.4) holds with $\Delta = C^2$ and any strictly increasing function $\varphi : [0, 2] \to [0, 1]$ such that $\varphi(x) = x^{1/p}$, $0 \le x \le e^{-1}$, namely

$$\Big|\Big\{i : (\lambda_i, c_i) \in \bigcup_{k=1}^{m} P_k\Big\}\Big| \le \Big|\Big\{i : (\tilde{\lambda}_i, \tilde{c}_i) \in \bigcup_{k=1}^{m} Q_k\Big\}\Big| \tag{3.8}$$

for any $m \in \mathbb{N}$, any system of rectangles (2.3) and

$$Q_k = (\varphi(\delta_k), \varphi^{-1}(\varepsilon)] \times (\tau_k/\Delta, \Delta t_k], \qquad k = 1, \dots, m.$$

To this end we introduce two auxiliary collections of rectangles in the following way. Taking from the set $\{e^{-1}, \delta_k,\ \varepsilon_k : k = 1, \dots, m\}$ only different numbers $\le e^{-1}$ in increasing order, we obtain a new set $\{\xi_k : k = 1, \dots, n\}$ with $\xi_n = e^{-1}$. Setting $\xi_{n+1} = 1$ and $\eta_s = C^{s-1}$, $s \in \mathbb{N}$, consider the rectangles

$$\begin{aligned} R_{r,s} &= (\xi_r, \xi_{r+1}] \times (\eta_s, \eta_{s+1}], \\ S_{r,s} &= \begin{cases} (\xi_r^p, \xi_{r+1}^{1/p}] \times (\eta_s, \eta_{s+1}] & \text{if } r < n, \\ (e^{-1/p}, 1] \times (\eta_s, C\eta_{s+1}] & \text{if } r = n, \end{cases} \end{aligned}$$

with $r = 1, \dots, n$ and $s \in \mathbb{N}$. Let

$$M = \Big\{(r, s) : R_{r,s} \cap \Big(\bigcup_{k=1}^{m} P_k\Big) \ne \emptyset\Big\}.$$

It is easily seen that

$$\bigcup_{k=1}^{m} P_k \subset \bigcup_{(r,s)\in M} R_{r,s}, \qquad \bigcup_{(r,s)\in M} S_{r,s} \subset \bigcup_{k=1}^{m} Q_k. \tag{3.9}$$

By Proposition 7, we have an estimate

$$\Big|\Big\{i : (\lambda_i, c_i) \in \bigcup_{s\colon (n,s)\in M} R_{n,s}\Big\}\Big| \le \Big|\Big\{i : (\tilde{\lambda}_i, \tilde{c}_i) \in \bigcup_{s\colon (n,s)\in M} S_{n,s}\Big\}\Big|. \tag{3.10}$$

On the other hand, by Proposition 6, we have

$$\Big|\Big\{i : (\lambda_i, c_i) \in \bigcup_{r<n\colon (r,s)\in M} R_{r,s}\Big\}\Big| \le \Big|\Big\{i : (\tilde{\lambda}_i, \tilde{c}_i) \in \bigcup_{r<n\colon (n,s)\in M} S_{r,s}\Big\}\Big| \tag{3.11}$$

for any s such that there exists $r < n$ with $(r, s) \in M$. By the construction of the rectangles, we observe that

$$\Big(\bigcup_{r<n\colon (r,s)\in M} S_{r,s}\Big) \cap \Big(\bigcup_{q\colon (n,q)\in M} S_{n,q}\Big) = \emptyset, \tag{3.12}$$

and for all $s_1 \neq s_2$ we have

$$\Big(\bigcup_{r<n} S_{r,s_1}\Big) \cap \Big(\bigcup_{r<n} S_{r,s_2}\Big) = \emptyset. \tag{3.13}$$

Combining (3.9)–(3.13), we get (3.8), which completes the proof.

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