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Isomorphisms of some reflexive algebras

by

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Abstract. Suppose $\mathcal{L}_1$ and $\mathcal{L}_2$ are subspace lattices on complex separable Banach spaces X and Y, respectively. We prove that under certain lattice-theoretic conditions every isomorphism from $\operatorname{alg}\mathcal{L}_1$ to $\operatorname{alg}\mathcal{L}_2$ is quasi-spatial; in particular, if a subspace lattice $\mathcal{L}$ of a complex separable Banach space X contains a sequence E_i such that $(E_i)_- \neq X$, $E_i \subseteq E_{i+1}$, and $\bigvee_{i=1}^{\infty} E_i = X$ then every automorphism of $\operatorname{alg}\mathcal{L}$ is quasi-spatial.

1. Introduction. Let X and Y be separable complex Banach spaces and let $B(X,Y)$ be the set of all bounded linear maps from X into Y. When $X = Y$, we use $B(X)$ instead of $B(X,Y)$. When X is a Hilbert space, we use H instead of X. For vector spaces $\mathcal{U}$ and $\mathcal{V}$, we write $L(\mathcal{U},\mathcal{V})$ for the set of all linear maps from $\mathcal{U}$ to $\mathcal{V}$. By a *subspace lattice* on X, we mean a collection $\mathcal{L}$ of closed subspaces of X with 0 and X in $\mathcal{L}$ such that for every family $\{M_r\}$ of elements of $\mathcal{L}$, both $\bigcap M_r$ and $\bigvee M_r$ belong to $\mathcal{L}$. If the operations of meet and join distribute over each other for any collections of subspaces in $\mathcal{L}$, then $\mathcal{L}$ is said to be *completely distributive*. If $L \in \mathcal{L}$, we denote by L_- the subspace $\bigvee\{M \in \mathcal{L} : L \not\subseteq M\}$ and denote by L_+ the subspace $\bigcap\{M \in \mathcal{L} : M \not\subseteq L\}$. For a subspace lattice $\mathcal{L}$ of X, we use $\operatorname{alg}\mathcal{L}$ to denote the algebra of all operators on X that leave members of $\mathcal{L}$ invariant.

For Hilbert spaces, a common practice is to disregard the distinction between a subspace and the orthogonal projection onto it. A Hilbert space subspace lattice $\mathcal{L}$ is called a *commutative subspace lattice* if it consists of mutually commuting projections. If $\mathcal{L}$ is a commutative subspace lattice then $\operatorname{alg}\mathcal{L}$ is called a *CSL algebra*.

If $\mathcal{L}$ is a subspace lattice on X, we define $\mathcal{J}_{\mathcal{L}} = \{L \in \mathcal{L} : L \neq 0$ and $L_- \neq X\}$. We say $\mathcal{J}_{\mathcal{L}}$ is *sequentially dense* in X if there exists a sequence $E_i \in \mathcal{J}_{\mathcal{L}}$ such that $E_i \subseteq E_{i+1}$ and $\bigvee_{i=1}^{\infty} E_i = X$. Quasi-spatiality of isomorphisms has been studied in [1, 2, 4, 5]. The main task of [4] is to show that if $\mathcal{L}$ is a commutative subspace lattice on a Hilbert space H such that $\mathcal{J}_{\mathcal{L}}$ is

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sequentially dense in H then every automorphism on $\operatorname{alg}\mathcal{L}$ is quasi-spatial. In this paper, we generalize the above result, with a relatively simpler proof, to non-commutative subspace lattices on Banach spaces; more specifically, we show that if $\mathcal{L}$ is any subspace lattice on a Banach space X such that $\mathcal{J}_{\mathcal{L}}$ is sequentially dense in X then every automorphism on $\operatorname{alg}\mathcal{L}$ is quasi-spatial. Our main result, Theorem 2.6, is stated in a slightly more general form; this also makes the presentation of the proof a little clearer.

2. The main result. For a subspace E of a Banach space X, we define $E^{\perp} = \{f^* \in X^* : f^*|_E = 0\}$. For any $x \in X$ and $f^* \in X^*$, we use $x \otimes f^*$ to denote the rank-one operator satisfying $x \otimes f^*(u) = f^*(u)x$ for all $u \in X$. It follows from [3] that $x \otimes f^* \in \operatorname{alg}\mathcal{L}$ if and only if there exists an $L \in \mathcal{J}_{\mathcal{L}}$ such that $x \in L$ and $f^* \in (L_-)^{\perp}$. In the following, we suppose $\mathcal{L}_1$ and $\mathcal{L}_2$ are subspace lattices on Banach spaces X and Y, respectively; and $\operatorname{alg}\mathcal{L}_1$ and $\operatorname{alg}\mathcal{L}_2$ are the corresponding subalgebras of $B(X)$ and $B(Y)$, respectively.

We will break the proof of the main result into a few lemmas.

LEMMA 2.1. *Suppose $\mathcal{J}_{\mathcal{L}_2}$ is sequentially dense in Y, ψ is an isomorphism from $\operatorname{alg}\mathcal{L}_2$ to $\operatorname{alg}\mathcal{L}_1$, and $E \in \mathcal{J}_{\mathcal{L}_1}$. Then for any $x \in E$, there exist $K \in \mathcal{J}_{\mathcal{L}_2}$, $y \in K$, $h^* \in (K_-)^{\perp}$, and $0 \neq g^* \in X^*$ such that $\psi(y \otimes h^*) = x \otimes g^*$.*

Proof. Take any $x \in E$ and $0 \neq l^* \in (E_-)^{\perp}$. Then $x \otimes l^* \in \operatorname{alg}\mathcal{L}_1$. Since ψ is surjective, there exists a $B \in \operatorname{alg}\mathcal{L}_2$ such that $\psi(B) = x \otimes l^*$. Since $\mathcal{J}_{\mathcal{L}_2}$ is sequentially dense in Y, there exist a $K \in \mathcal{J}_{\mathcal{L}_2}$ and $w \in K$ such that $y = Bw \neq 0$. Choose $0 \neq h^* \in (K_-)^{\perp}$ and set $A = \psi(w \otimes h^*)$ and $g^* = A^* l^*$. Then $\psi(y \otimes h^*) = \psi((Bw) \otimes h^*) = \psi(Bw \otimes h^*) = \psi(B)\psi(w \otimes l^*) = x \otimes l^* A = x \otimes g^*$. ∎

REMARK 2.2. Let K be as in Lemma 2.1. From the proof of Lemma 2.1, one can see that, for any $L \in \mathcal{J}_{\mathcal{L}_2}$ with $K \subseteq L$, there exist $y_1 \in L$, $h_1^* \in (L_-)^{\perp}$, and $0 \neq g_1^* \in X^*$ such that $\psi(y_1 \otimes h_1^*) = x \otimes g_1^*$.

LEMMA 2.3. *Suppose $E_i \in \mathcal{J}_{\mathcal{L}_1}$ with $E_i \subseteq E_{i+1}$, $\bigvee_{i=1}^{\infty} E_i = X$, and $K_i \in \mathcal{J}_{\mathcal{L}_2}$ with $K_i \subseteq K_{i+1}$ and $\bigvee_{i=1}^{\infty} K_i = Y$. If ϕ is an isomorphism from $\operatorname{alg}\mathcal{L}_1$ to $\operatorname{alg}\mathcal{L}_2$, then there exist $K_{n_i} \in \mathcal{J}_{\mathcal{L}_2}$ with $K_{n_i} \subseteq K_{n_{i+1}}$, $\bigvee_{i=1}^{\infty} K_{n_i} = Y$, and injective $T_i \in L(E_i, Y)$ with $\operatorname{ran}(T_i) \subseteq K_{n_i}$ such that $\phi(A)T_i x = T_i A x$ for every $x \in E_i$ and $A \in \operatorname{alg}\mathcal{L}_1$.*

Proof. For any $0 \neq f_i^* \in ((E_i)_-)^{\perp}$, there exist E_{m_i} and $x_i \in E_{m_i}$ such that $f_i^*(x_i) = 1$. By Lemma 2.1, there exist $y_i \in K_{n_i} \in \mathcal{J}_{\mathcal{L}_2}$, $h_i^* \in ((K_{n_i})_-)^{\perp}$, and $0 \neq g_i^* \in X^*$ such that $\phi^{-1}(y_i \otimes h_i^*) = x_i \otimes g_i^*$. Since $E_i \subseteq E_{i+1}$ and $\bigvee_{i=1}^{\infty} E_i = X$, there exist E_{p_i} and $u_i \in E_{p_i}$ such that $g_i^*(u_i) = 1$. Define $T_i \in L(E_i, Y)$ by

$$T_i x = \phi(x \otimes f_i^*) y_i, \qquad \forall x \in E_i \tag{2.1}$$

and define $S_i \in L(K_{n_i}, X)$ by

$$S_i y = \phi^{-1}(y \otimes h_i^*)u_i, \quad \forall y \in K_{n_i}. \tag{2.2}$$

It is clear from the definition of T_i that $\operatorname{ran}(T_i) \subseteq K_{n_i}$.

For any $x \in E_i$,

$$\begin{aligned} S_i T_i x &= \phi^{-1}(T_i x \otimes h_i^*)u_i = \phi^{-1}((\phi(x \otimes f_i^*)y_i) \otimes h_i^*)u_i \\ &= (x \otimes f_i^*)\phi^{-1}(y_i \otimes h_i^*)u_i = (x \otimes f_i^*)(x_i \otimes g_i^*)u_i \\ &= (x \otimes f_i^*)x_i = x. \end{aligned} \tag{2.3}$$

In particular, T_i and $S_i|_{V_i}$ are injective, where $V_i = \operatorname{ran}(T_i)$. Furthermore,

$$\phi(A)T_i x = \phi(A)\phi(x \otimes f_i^*)y_i = \phi(Ax \otimes f_i^*)y_i = T_i A x, \quad \forall x \in E_i,\ A \in \operatorname{alg}\mathcal{L}_1. \tag{2.4}$$

Similar to (2.1) and (2.2), we can construct T_{i+1} and S_{i+1}; by Remark 2.2 we can assume $K_{n_i} \subseteq K_{n_{i+1}}$. ∎

For any Banach space X, $f^* \in X^*$ and $E \subseteq X$, define

$$[E \otimes f^*]_X = \{x \otimes f^* : x \in E\}.$$

Lemma 2.4. *Suppose $E_i \in \mathcal{J}_{\mathcal{L}_1}$ with $E_i \subseteq E_{i+1}$, $\bigvee_{i=1}^\infty E_i = X$, and $K_i \in \mathcal{J}_{\mathcal{L}_2}$ with $K_i \subseteq K_{i+1}$, $\bigvee_{i=1}^\infty K_i = Y$. If ϕ is an isomorphism from $\operatorname{alg}\mathcal{L}_1$ to $\operatorname{alg}\mathcal{L}_2$, then for each $a_i^* \in ((E_i)_-)^\perp$, there is a $b_i^* \in Y^*$ such that $\phi([E_i \otimes a_i^*]_X) \subseteq [Y \otimes b_i^*]_Y$.*

Proof. Let T_i be as in Lemma 2.3. Then by (2.4) we have

$$\phi(A)T_i x = T_i A x, \quad \forall x \in E_i,\ A \in \operatorname{alg}\mathcal{L}_1. \tag{2.5}$$

It follows that $BT_i x = T_i \phi^{-1}(B)x$ for $x \in E_i$ and $B \in \operatorname{alg}\mathcal{L}_2$. This implies that whenever B is a rank-one operator, $\phi^{-1}(B)$ is also a rank-one operator, since $\bigvee_{i=1}^\infty E_i = X$ and T_i is injective. By the symmetry of X and Y, ϕ also maps rank-one operators to rank-one operators.

For each fixed m, fix $0 \neq x_1 \in E_m$ and $0 \neq a_m^* \in ((E_m)_-)^\perp$ and suppose $\phi(x_1 \otimes a_m^*) = y_1 \otimes b_m^*$ for some $y_1 \in Y$ and $b_m^* \in Y^*$. We will show

$$\phi([E_m \otimes a_m^*]_X) \subseteq [Y \otimes b_m^*]_Y.$$

Take any $x_2 \in E_m$ such that $\{x_1, x_2\}$ is linearly independent. Suppose $\phi(x_2 \otimes a_m^*) = y_2 \otimes c_m^*$ for some $y_2 \in Y$ and $c_m^* \in Y^*$. We only need to show $\{b_m^*, c_m^*\}$ is linearly dependent.

Applying (2.5) with $A = x_1 \otimes a_m^*$ and $A = x_2 \otimes a_m^*$, respectively, we obtain

$$b_m^*(T_i x)y_1 = a_m^*(x)T_i x_1, \quad \forall x \in E_i, \tag{2.6}$$

and

$$c_m^*(T_i x)y_2 = a_m^*(x)T_i x_2, \quad \forall x \in E_i, \tag{2.7}$$

Since $E_i \subseteq E_{i+1}$ and $\bigvee_{i=1}^{\infty} E_i = X$, there exist E_i and $x \in E_i$ such that $a_m^*(x) \neq 0$. Since T_i is injective and $\{x_1, x_2\}$ is linearly independent, $\{T_i x_1, T_i x_2\}$ is linearly independent; so $\{y_1, y_2\}$ is linearly independent, by (2.6) and (2.7).

Since ϕ maps rank-one operators to rank-one operators, $\phi((x_1+x_2)\otimes a_m^*)$ is a rank-one operator. Thus, $y_1 \otimes b_m^* + y_2 \otimes c_m^* = \phi((x_1 + x_2) \otimes a_m^*)$ is a rank-one operator. Since $\{y_1, y_2\}$ is linearly independent, $\{b_m^*, c_m^*\}$ is linearly dependent. ∎

For a subspace $\mathcal{S}$ of $L(\mathcal{U}, \mathcal{V})$, define $\operatorname{ref}_{\mathrm{a}}(\mathcal{S}) = \{T \in L(\mathcal{U}, \mathcal{V}) : Tx \in \mathcal{S}x,\ \forall x \in \mathcal{U}\}$. We say $\mathcal{S}$ is *algebraically reflexive* if $\operatorname{ref}_{\mathrm{a}}(\mathcal{S}) = \mathcal{S}$. It is well known and not hard to show that every one-dimensional subspace of $L(\mathcal{U}, \mathcal{V})$ is algebraically reflexive.

LEMMA 2.5. *Assuming the same hypotheses and notations as in Lemma* 2.3, *by rescaling T_i we can have $T_{i+1}|_{E_i} = T_i$ for $i = 1, 2, \ldots$.*

Proof. Fix any $a_i^* \in ((E_i)_-)^{\perp}$ and $v \in Y$, and define $D \in L(E_i, Y)$ by $Dx = \phi(x \otimes a_i^*)v$ for $x \in E_i$. If D is not the zero operator then D is injective; indeed, by Lemma 2.4, there exists $b_i \in Y^*$ such that $\phi(x \otimes a_i^*) = \lambda_x \otimes b_i^*$ for all $x \in E_i$. Since ϕ maps rank-one operators to rank-one operators, $\lambda_x \neq 0$ for all $0 \neq x \in E_i$. If D is not the zero operator then $b_i^*(v) \neq 0$, so D is injective; in particular, the operators T_i defined by (2.1) are injective (which we already knew). By the symmetry of X and Y, the operators S_i defined by (2.2) are also injective.

Suppose T_i, S_i, T_{i+1}, and S_{i+1} have been constructed as in Lemma 2.3. Then $S_{i+1}T_{i+1}x = x$ for all $x \in E_{i+1}$; in particular, $S_{i+1}T_{i+1}x = x$ for all $x \in E_i$. Let $V_i = \operatorname{ran}(T_i)$ and note that $V_i \subseteq K_{n_i} \subseteq K_{n_{i+1}}$. Consider $S_i|_{V_i}, S_{i+1}|_{V_i} \in L(V_i, X)$. Since the one-dimensional subspace generated by the transformation $S_i|_{V_i}$ is algebraically reflexive in $L(V_i, X)$ and

$$
\begin{aligned}
S_{i+1}T_i x &= \phi^{-1}(T_i x \otimes h_{i+1}^*)u_{i+1} = \phi^{-1}((\phi(x \otimes f_i^*)y_i) \otimes h_{i+1}^*)u_{i+1} \\
&= (x \otimes f_i^*)\phi^{-1}(y_i \otimes h_{i+1}^*)u_{i+1} = (x \otimes f_i^*)t_{i+1} \\
&= f_i^*(t_{i+1})x = f_i^*(t_{i+1})S_i T_i x, \qquad \forall x \in E_i,
\end{aligned}
$$

where $t_{i+1} = \phi^{-1}(y_i \otimes h_{i+1}^*)u_{i+1}$, it follows that $S_{i+1}|_{V_i} = c_i S_i|_{V_i}$ for some scalar c_i. Since S_{i+1} is injective, $c_i \neq 0$.

Replacing S_{i+1} by $(1/c_i)S_{i+1}$ and T_{i+1} by $c_i T_{i+1}$ and still calling them S_{i+1} and T_{i+1}, respectively, we have $S_{i+1}|_{V_i} = S_i|_{V_i}$, and for any $x \in E_i$, $S_{i+1}T_i x = S_i T_i x = x = S_{i+1}T_{i+1}x$. It follows that $T_{i+1}x = T_i x$ for all $x \in E_i$. ∎

We say ϕ is *quasi-spatial* if there exists an injective linear transformation $T \in L(D(T), Y)$, where $D(T)$ is the domain of T such that $D(T)$ is dense in X and invariant under $\operatorname{alg}\mathcal{L}_1$, the range of T is dense in Y, and

$$\phi(A)Tx = TAx, \qquad \forall x \in D(T),\ A \in \operatorname{alg}\mathcal{L}_1. \tag{2.8}$$

THEOREM 2.6. *Suppose $\mathcal{J}_{\mathcal{L}_1}$ is sequentially dense in X and $\mathcal{J}_{\mathcal{L}_2}$ is sequentially dense in Y. Then every isomorphism ϕ from* $\operatorname{alg}\mathcal{L}_1$ *to* $\operatorname{alg}\mathcal{L}_2$ *is quasi-spatial*; *in particular*, *ϕ preserves ranks of operators.*

Proof. By the assumptions, there exist $E_i \in \mathcal{J}_{\mathcal{L}_1}$ with $E_i \subseteq E_{i+1}$, $\bigvee_{i=1}^{\infty} E_i = X$, and $K_i \in \mathcal{J}_{\mathcal{L}_2}$ with $K_i \subseteq K_{i+1}$, $\bigvee_{i=1}^{\infty} K_i = Y$. Now we can construct T_i as in Lemma 2.3, with modifications as in Lemma 2.5. Let $E = \bigcup_{i=1}^{\infty} E_i$, the non-closed union of E_i, so E is dense in X. Clearly, E is invariant under $\operatorname{alg}\mathcal{L}_1$, and if $x \in E$ then $x \in E_i$ for some i. Define $Tx = T_i x$. By the agreement among T_i, it follows that T is a well-defined, injective, linear transformation on E; moreover, $\phi(A)Tx = TAx$ for all $x \in E$ and $A \in \operatorname{alg}\mathcal{L}_1$. Let $\operatorname{ran}(T)$ be the range of T and $K = \bigcup_{i=1}^{\infty} K_i$. Clearly K is dense in Y and $\operatorname{ran}(T) \subseteq K$; we will show $\operatorname{ran}(T) = K$. Take any $y \in K$. There exists K_{n_i} such that $y \in K_{n_i}$. By (2.2) of Lemma 2.3, $S_i y = \phi^{-1}(y \otimes h_i^*)u_i \in E_{p_i} \in E$. By (2.1) of Lemma 2.3,

$$\begin{aligned} TS_i y = T_{p_i} S_i y &= \phi(\phi^{-1}(y \otimes h_i^*)u_i \otimes f_{p_i}^*)y_{p_i} = (y \otimes h_i^*)\phi(u_i \otimes f_{p_i}^*)y_{p_i} \\ &= h_i^*(\phi(u_i \otimes f_{p_i}^*)y_{p_i})y = \mu_i y, \end{aligned}$$

where $\mu_i = h_i^*(\phi(u_i \otimes f_{p_i}^*)y_{p_i})$. Since T_{p_i} and S_i are injective, $\mu_i \neq 0$. Now $T({\mu_i}^{-1} S_i y) = y$, so $\operatorname{ran}(T) = K$.

Rank-preserving follows from (2.8) directly. ■

The following corollary is the main result of [4]. A special case of the corollary was proved earlier in [1] with an additional hypothesis of subspace lattices being completely distributive.

COROLLARY 2.7 ([4, Theorem 17]). *Suppose $\mathcal{L}_1$ and $\mathcal{L}_2$ are commutative subspace lattices on a Hilbert space H and $\mathcal{J}_{\mathcal{L}_1}$ is sequentially dense in H. Then every isomorphism from* $\operatorname{alg}\mathcal{L}_1$ *to* $\operatorname{alg}\mathcal{L}_2$ *is quasi-spatial.*

Proof. By [4, Theorem C], we can assume $\mathcal{L}_1 = \mathcal{L}_2$. Now the conclusion follows from Theorem 2.6. ■

Remark: The hypotheses in [4, Theorem 17] are stated differently from Corollary 2.7, but it is easy to check that they are equivalent.

THEOREM 2.8. *If $\mathcal{L}_1$ is a subspace lattice with $X_- \neq X$ and $\mathcal{L}_2$ is a subspace lattice with $Y_- \neq Y$, then every isomorphism from* $\operatorname{alg}\mathcal{L}_1$ *to* $\operatorname{alg}\mathcal{L}_2$ *is spatially implemented and every bounded isomorphism from* $\operatorname{alg}\mathcal{L}_1$ *to* $\operatorname{alg}\mathcal{L}_2$ *is spatially implemented by a bounded operator.*

Proof. Suppose ϕ is an isomorphism from $\operatorname{alg}\mathcal{L}_1$ to $\operatorname{alg}\mathcal{L}_2$. Take $E_i = X$ and $K_i = Y$, then the hypotheses of Theorem 2.6 are satisfied. Let T_i be defined by (2.1) and S_i be defined by (2.2) in Lemma 2.3. By (2.3),

$S_i \in L(Y, X)$ is surjective. By the first paragraph of the proof of Lemma 2.5, S_i is injective, so S_i has an inverse. Now the equality $S_i T_i x = x$ for all $x \in E_i$ $(= X)$ implies T_i is invertible with $T_i^{-1} = S_i$. Finally, (2.5) of Lemma 2.4 implies ϕ is spatially implemented. If ϕ is bounded, then so are T_i and S_i. ∎

COROLLARY 2.9. *If* $\mathcal{L}$ *is a subspace lattice on a Hilbert space* H *with* $0_+ \neq 0$, *then every automorphism of* $\operatorname{alg} \mathcal{L}$ *is spatial.*

Proof. Suppose $\mathcal{L}$ satisfies $0_+ \neq 0$ and ϕ is an automorphism of $\operatorname{alg} \mathcal{L}$. Let $\mathcal{L}^{\perp} = \{I - L : L \in \mathcal{L}\}$, where I is the identity operator on H. Then $\mathcal{L}^{\perp}$ satisfies $H_- \neq H$.

Define $\phi^*(A^*) = (\phi(A))^*$ for $A^* \in \operatorname{alg} \mathcal{L}^{\perp}$. Then ϕ^* is an automorphism of $\operatorname{alg} \mathcal{L}^{\perp}$. By Theorem 2.8, we have $\phi^*(A^*) = (\phi(A))^* = T A^* T^{-1}$ for some $T \in B(H)$. So ϕ is spatial. ∎

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