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Erratum to: "Painlevé null sets, dimension and compact embedding of weighted holomorphic spaces"

(Studia Math. 213 (2012), 169–187)

by

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In the proof of Proposition 3.3 in our paper [AT] there is a flaw which cannot be corrected. We made the wrong conclusion that the coincidence of the norm topology of $\|\cdot\|_{w_2}$ and the topology co on the unit ball $B_{w_1}(G)$ implies the existence of a compact set K in G and a positive constant A such that $\|f\|_{w_2} \le A\|f\|_K$ for all $f \in H_{w_1}(G)$. Obviously, we have to replace the latter by a weaker condition (see (ii) below) and restate Proposition 3.3 as follows.

PROPOSITION 0.1 (Proposition 3.3). *The following statements are equivalent:*

(i) $H_{w_1}(G) \subset H_{w_2}(G)$ *and the inclusion is compact.*
(ii) *Each bounded sequence* (f_k) *in* $H_{w_1}(G)$ *convergent to* 0 *in* $(H(G), co)$ *converges also to* 0 *in* $H_{w_2}(G)$.

Due to this flaw, our results in [AT] for the case of entire functions (Lemma 3.8, Theorem 3.9, Corollary 3.10 and Theorems 4.3 and 4.5 in their parts concerning $G = \mathbb{C}$) are unproven and may not be generally valid.

It is easy to see that Lemma 3.8, Theorem 3.9 and Corollary 3.10 are true for an arbitrary domain G (not only for $G = \mathbb{C}$) under the following additional assumption:

(BD) *For every function* f *in* $H_w(G)$ *there exists a bounded sequence* (f_k) *in* $H_{w0}(G)$ *that converges to* f *in* $(H(G), co)$.

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Lemma 0.2 (Lemma 3.8). *Suppose that $H_{w_1}(G)$ satisfies the condition* (BD). *If $H_{w_1}(G) \subset H_{w_2}(G)$ and the inclusion is compact, then $H_{w_1 0}(G)$ is dense in $H_{w_1}(G)$ with respect to the norm $\|\cdot\|_{w_2}$.*

Theorem 0.3 (Theorem 3.9). *Suppose that $H_{\widetilde{w}_1}(G)$ satisfies the condition* (BD). *For the inclusion $H_{w_1}(G) \subset H_{w_2}(G)$ to be compact it is necessary and sufficient that $\widetilde{w}_1(z)/w_2(z)$ vanishes at infinity on G.*

Lemma 0.2 is an immediate consequence of the condition (BD) and Proposition 0.1; then, applying Lemma 0.2, we can use the proof of Theorem 3.9 in [AT] to obtain Theorem 0.3.

Note that the condition (BD) holds if $B_w(G)$ coincides with $\overline{B_{w0}(G)}^{co}$, the closure of $B_{w0}(G)$ in the *co* topology. In particular, it holds for $H_w(G)$ whenever w is a positive radial weight on a balanced domain G and $H_{w0}(G)$ contains all polynomials (see [BS, Theorem 2.3] and [BBT, Theorem 1.13]).

Thus, Theorem 3.9 and Corollary 3.10 in [AT] are valid at least for radial weights w_1.

Similarly, Theorems 4.3 and 4.5 in [AT], in their parts concerning $G = \mathbb{C}$, are true at least for radial weights. The remaining parts of these two theorems, dealing with G whose complement has no one-point components, are valid for arbitrary weights.

Note also that in [AT] we used Proposition 3.3 in the proof of Lemma 4.4. We can remedy this by giving the following alternative proof.

Proof of Lemma 4.4 of [AT]. Since $H\overline{V}(G)$ and $\mathcal{V}H(G)$ have the same bounded subsets, $H\overline{V}(G)$ is semi-Montel if and only if each unit ball B_n of $H_{v_n}(G)$ is compact in $H\overline{V}(G)$ for each $n \in \mathbb{N}$. For B_n to be compact in $H\overline{V}(G)$ it is necessary and sufficient that the topology induced by the system $\{\|\cdot\|_{\overline{v}}\}_{\overline{v}\in\overline{V}}$ of norms and the *co* topology coincide on B_n. That is, each bounded sequence (f_k) in $H_{v_n}(G)$ convergent to 0 in the *co* topology also converges to 0 in $H\overline{V}(G)$, i.e. converges to 0 with respect to the norm $\|\cdot\|_{\overline{v}}$ for all $\overline{v} \in \overline{V}$. This is equivalent to the inclusion of $H_{v_n}(G)$ into $H_{\overline{v}}(G)$ being compact for all $n \in \mathbb{N}$ and $\overline{v} \in \overline{V}$. ∎

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