Corrigenda to:
“Optimal domains for the kernel operator associated with Sobolev’s inequality”
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by
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The notation and references used are from the original paper, which we reference here as [CR]. On p. 133 of [CR], the statement that “[T, X] = L^1(\nu_X), without any restrictions on X”, is incorrect. However, it is correct if X has absolutely continuous (briefly, a.c.) norm. The source of this inaccuracy is that X_b (i.e. the closure of the simple functions in X) fails to be a rearrangement invariant (briefly, r.i.) space in the sense of [2] because it may fail the Fatou property, that is, if 0 ≤ f_n ↑ f a.e. with f_n ∈ X and sup_n ||f_n||_X < ∞, then f ∈ X and ||f_n||_X → ||f||_X (equivalently, the unit ball of X is closed with respect to convergence in measure). The requirement of this property in [2] (which we adopted in [CR]) is not assumed by other authors, [13], [16]. We now describe how this oversight affects the results of [CR].

If X_b does inherit the Fatou property from X, then the statement and proof of Proposition 3.3(c) in [CR] are correct. Under the assumptions of Section 3 of [CR] (namely, X is r.i. in the sense of [2] and X \neq \mathbb{L}\infty([0,1])), this condition on X_b is equivalent to X having a.c. norm; see [2, Theorem II.5.5]. So, we have the following correct version of

**PROPOSITION 3.3(c).** If X is a r.i. space with a.c. norm, then L^1(\nu_X) is weakly sequentially complete.

Using this modified version of Proposition 3.3(c), “the same proofs” of Propositions 3.4(a) and 3.5 as given in [CR] remain valid and yield the following correct statements.

**PROPOSITION 3.4(a).** Let X \neq \mathbb{L}\infty([0,1]) and f: [0,1] → \mathbb{R} be a measurable function.
The following two statements are equivalent:

(i) \( f \in L^1(\nu_X) \).

(ii) The function \( fF_X : [0,1] \rightarrow X \) is Pettis \( \lambda \)-integrable.

The following three statements are equivalent:

(iii) \( \int_0^1 |f| \, d|x'|\nu_X| < \infty \) for every \( x' \in X' \).

(iv) For every \( g \in X' \) which is non-negative and decreasing,

\[
\int_0^1 |f(s)| \frac{1}{s^{(1/n)-1}} \int_0^s g(t) \, dt \, ds < \infty.
\]

(v) \( f \in [T,X] \).

If, in addition, \( X \) has a.c. norm, then all five statements are equivalent.

Concerning the other proposition mentioned above we have

**Proposition 3.5.** Let \( X \) be a r.i. space. Then \( L^1(\nu_X) \subseteq [T,X] \) with equality whenever \( X \) has a.c. norm.

The above corrections affect the rest of [CR] as follows.

- The first sentence of Remark 3.7 should now be: The extended operator \( T = I\nu_X \) is never compact on either \( L^1(\nu_X) \) (by Proposition 3.6) or on its optimal domain \([T,X]\) (by Proposition 3.5).

- The last sentence in the proof of Proposition 5.1 should read: Accordingly, (12) is satisfied and so \( f \in [T,X] \) by Proposition 3.4(a).

- The second and third sentences in the proof of Proposition 5.5 should now be: Hence, (12) is satisfied and so \( f \in [T,X] \). Conversely, if \( M_X \subseteq [T,X] \), then (12) yields \( \int_0^1 Wg(s) \, ds < \infty \) for each \( f \in M_X \) and every \( 0 \leq g \in X' \) decreasing, hence for all \( g \in X' \).

- The proof of Corollary 5.8 is incorrect (since it relies on the equality \([T,X] = L^1(\nu_X)\)). However, its statement is correct and will be proved elsewhere. Note that Corollary 5.8 is not used anywhere in [CR].