STUDIA MATHEMATICA 170 (2) (2005)

Corrigenda to: "Optimal domains for the kernel operator associated with Sobolev's inequality"

(Studia Math. 158 (2003), 131–152)

by

GUILLERMO P. CURBERA (Sevilla) and WERNER J. RICKER (Eichstätt)

The notation and references used are from the original paper, which we reference here as [CR]. On p. 133 of [CR], the statement that "$[T,X] = L^1(\nu_X)$, without any restrictions on X", is incorrect. However, it is correct if X has absolutely continuous (briefly, a.c.) norm. The source of this inaccuracy is that X_{b} (i.e. the closure of the simple functions in X) fails to be a rearrangement invariant (briefly, r.i.) space in the sense of [2] because it may fail the Fatou property, that is, if $0 \le f_n \uparrow f$ a.e. with $f_n \in X$ and $\sup_n \|f_n\|_X < \infty$, then $f \in X$ and $\|f_n\|_X \to \|f\|_X$ (equivalently, the unit ball of X is closed with respect to convergence in measure). The requirement of this property in [2] (which we adopted in [CR]) is not assumed by other authors, [13], [16]. We now describe how this oversight affects the results of [CR].

If X_{b} does inherit the Fatou property from X, then the statement and proof of Proposition 3.3(c) in [CR] are correct. Under the assumptions of Section 3 of [CR] (namely, X is r.i. in the sense of [2] and $X \neq L^\infty([0,1])$), this condition on X_{b} is equivalent to X having a.c. norm; see [2, Theorem II.5.5]. So, we have the following correct version of

PROPOSITION 3.3(c). *If X is a r.i. space with a.c. norm, then $L^1(\nu_X)$ is weakly sequentially complete.*

Using this modified version of Proposition 3.3(c), "the same proofs" of Propositions 3.4(a) and 3.5 as given in [CR] remain valid and yield the following correct statements.

PROPOSITION 3.4(a). *Let $X \neq L^\infty([0,1])$ and $f\colon [0,1] \to \mathbb{R}$ be a measurable function.*

2000 *Mathematics Subject Classification*: Primary 47B38, 46E30; Secondary 47G10, 28B05.

The following two statements are equivalent:

(i) $f \in L^1(\nu_X)$.

(ii) *The function* $fF_X\colon [0,1] \to X$ *is Pettis* λ-*integrable.*

The following three statements are equivalent:

(iii) $\int_0^1 |f|\, d|x'\nu_X| < \infty$ *for every* $x' \in X'$.

(iv) *For every* $g \in X'$ *which is non-negative and decreasing*,

$$\int_0^1 |f(s)| s^{(1/n)-1} \int_0^s g(t)\, dt\, ds < \infty. \tag{12}$$

(v) $f \in [T, X]$.

If, *in addition*, X *has a.c. norm*, *then all five statements are equivalent.*

Concerning the other proposition mentioned above we have

PROPOSITION 3.5. *Let* X *be a r.i. space. Then* $L^1(\nu_X) \subseteq [T, X]$ *with equality whenever* X *has a.c. norm.*

The above corrections affect the rest of [CR] as follows.

• The first sentence of Remark 3.7 should now be: *The extended operator* $T = I_{\nu_X}$ *is never compact on either* $L^1(\nu_X)$ (*by Proposition* 3.6) *or on its optimal domain* $[T, X]$ (*by Proposition* 3.5).

• The last sentence in the proof of Proposition 5.1 should read: *Accordingly*, (12) *is satisfied and so* $f \in [T, X]$ *by Proposition* 3.4(a).

• The second and third sentences in the proof of Proposition 5.5 should now be: *Hence*, (12) *is satisfied and so* $f \in [T, X]$. *Conversely*, *if* $M_X \subset [T, X]$, *then* (12) *yields* $\int_0^1 |f(s)| Wg(s)\, ds < \infty$ *for each* $f \in M_X$ *and every* $0 \le g \in X'$ *decreasing*, *hence for all* $g \in X'$.

• The proof of Corollary 5.8 is incorrect (since it relies on the equality $[T, X] = L^1(\nu_X)$). However, its statement is correct and will be proved elsewhere. Note that Corollary 5.8 is not used anywhere in [CR].

Facultad de Matemáticas
Universidad de Sevilla
Aptdo. 1160
Sevilla 41080, Spain
E-mail: curbera@us.es

Math.-Geogr. Fakultät
Katholische Universität Eichstätt-Ingolstadt
D-85072 Eichstätt, Germany
E-mail: werner.ricker@ku-eichstaett.de

Received December 30, 2004
Revised version April 7, 2005 (5557)