STUDIA MATHEMATICA 162 (2) (2004)

Weak countable compactness implies quasi-weak drop property

by

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Abstract. Every weakly countably compact closed convex set in a locally convex space has the quasi-weak drop property.

1. Introduction. Let $(X, \|\ \|)$ be a Banach space and $B(X)$ the closed unit ball $\{x \in X : \|x\| \le 1\}$. By a *drop* induced by a point $x_0 \notin B(X)$, we mean the set $D(x_0, B(X)) := \operatorname{conv}(\{x_0\} \cup B(X))$. Daneš [3] proved that in any Banach space $(X, \|\ \|)$, for every closed set A at positive distance from $B(X)$, there exists an $x_0 \in A$ such that $D(x_0, B(X)) \cap A = \{x_0\}$. Modifying the assumption of the Daneš drop theorem, Rolewicz [25] began the study of the *drop property* for the closed unit ball. He defined the norm $\|\ \|$ to have the *drop property* if for every closed set A disjoint from $B(X)$ there exists an $x_0 \in A$ such that $D(x_0, B(X)) \cap A = \{x_0\}$, and he proved that if the norm $\|\ \|$ has the drop property then $(X, \|\ \|)$ is reflexive (see [25, Theorem 5]). Giles, Sims and Yorke [8] defined the norm $\|\ \|$ to have the *weak drop property* if for every weakly sequentially closed set A disjoint from $B(X)$, there exists an $x_0 \in A$ such that $D(x_0, B(X)) \cap A = \{x_0\}$, and they showed that this property is equivalent to $(X, \|\ \|)$ being reflexive. Kutzarova [11] and Giles and Kutzarova [7] extended the discussion of the drop and weak drop properties to closed bounded convex sets in Banach spaces. Cheng, Zhou and Zhang [2] and Zheng [29] considered the drop property of closed bounded convex sets in locally convex spaces. Concerning the drop and weak drop properties and their applications, a series of profound results have been obtained; see, for example, [1–3, 6-8, 11, 12, 14–20, 22–26, 29] and references therein. Recall that a closed bounded convex set B in X is said (see [7]) to have the *weak drop property* if for every weakly sequentially closed set A disjoint from B there exists an $x_0 \in A$ such that $D(x_0, B) \cap A = \{x_0\}$. For closed bounded

2000 *Mathematics Subject Classification*: 46A55, 46B20.

Key words and phrases: locally convex space; quasi-weak drop property; weak drop property; weakly compact set; weakly countably compact set.

convex sets in Banach spaces, the weak drop property is equivalent to weak compactness (see [7, Theorem 3] or [19, Proposition 4.4.7]).

In [23], we introduced a new drop property, called the quasi-weak drop property.

DEFINITION 1.1. Let B be a closed bounded convex set in a locally convex space X. If for any weakly closed set A disjoint from B, there exists an $x_0 \in A$ such that $D(x_0, B) \cap A = \{x_0\}$, then B is said to have the *quasi-weak drop property.*

It seems that the quasi-weak drop property should be weaker than the weak drop property since a weakly sequentially closed set need not be weakly closed even in a separable Banach space (see [23, Example 2.2]). By using streaming sequences introduced by Rolewicz (see [25] or [19, Chapter 4]) we proved [23, Theorem 2.2] the following somewhat surprising result: for closed bounded convex sets in Fréchet spaces (i.e. complete metrizable locally convex spaces), the quasi-weak drop property is equivalent to the weak drop property and both are equivalent to weak compactness (for subsets of Fréchet spaces, weak sequential compactness, weak countable compactness and weak compactness are equivalent to each other, see [10, p. 318]). Thus a Fréchet space is reflexive if and only if every closed bounded convex set in the space has the quasi-weak drop property [23, Corollary 2.1]. However, for closed bounded convex sets in locally convex spaces, the quasi-weak drop property is strictly weaker than the weak drop property even when the locally convex spaces are quasi-complete, (see [24, Example 3.1]). Concerning the relationship between weak compact properties and weak drop properties we proved the following results.

THEOREM 1.1 (see [24, Theorem 2.1]). *Let B be a weakly sequentially compact convex set in a locally convex space $(X, \mathcal{T})$. Then B has the weak drop property, that is, for any weakly sequentially closed set A disjoint from B, there exists an $x_0 \in A$ such that $D(x_0, B) \cap A = \{x_0\}$.*

THEOREM 1.2 (see [24, Theorem 3.1]). *Let B be a weakly compact convex set in a locally convex space $(X, \mathcal{T})$. Then B has the quasi-weak drop property, that is, for any weakly closed set A disjoint from B, there exists an $x_0 \in A$ such that $D(x_0, B) \cap A = \{x_0\}$.*

Moreover we showed [24, Theorem 3.2] that if $(X, \mathcal{T})$ is quasi-complete, then a closed bounded convex subset B of X has the quasi-weak drop property if and only if it is weakly compact. However, for closed bounded convex sets in sequentially complete locally convex spaces, quasi-weak drop property does not imply weak compactnesss (see [24, Example 3.2]).

In order to present our main result, we need to recall some concepts. Let S be a Hausdorff topological space and M be a subset of S. Then

M is *compact* if and only if each open cover of M has a finite subcover, equivalently if and only if each net on M has a cluster point (i.e. an adherent point) in M; M is *sequentially compact* if and only if each sequence of points in M has a subsequence which is convergent to a point in M; M is *countably compact* if and only if each countable open cover of M has a finite subcover, equivalently if and only if each sequence of points in M has a cluster point in M. It is immediate that compactness of M and sequential compactness of M both imply countable compactness of M; in general, no other implications are valid among these notions.

For a locally convex space $(X, \mathcal{T})$ we denote its topological dual by X^* and denote the weak topology on X by $\sigma(X, X^*)$. A subset M of $(X, \mathcal{T})$ is called *weakly compact* (respectively *weakly sequentially compact*, or *weakly countably compact*) if M is compact (respectively sequentially compact, or countably compact) in the weak topology $\sigma(X, X^*)$. Obviously every weakly compact set is weakly countably compact and every weakly sequentially compact set is also weakly countably compact. However, there exist weakly compact convex sets which are not weakly sequentially compact (see [5, p. 8] and [10, p. 311]); and there exist weakly sequentially compact (and so weakly countably compact) closed convex sets which are not weakly compact (see [5, pp. 8–9] and [10, p. 313]). Clearly, weak countable compactness is the weakest of the above three kinds of weak compactness.

In this paper we shall improve Theorem 1.2 as follows: every weakly countably compact, closed convex set in a locally convex space has the quasi-weak drop property.

2. Main result.

We need a number of lemmas.

LEMMA 2.1. *Let B be a weakly countably compact subset of a locally convex space $(X, \mathcal{T})$ and A be a weakly closed subset of $(X, \mathcal{T})$. Then $A - B$ is weakly sequentially closed.*

Proof. Let $(a_n - b_n)_{n \in \mathbb{N}}$ be a sequence in $A - B$ which is convergent to z in $(X, \sigma(X, X^*))$, where $a_n \in A$ and $b_n \in B$ for every n. We shall prove that $z \in A - B$. Take any absolutely convex 0-neighborhood W in $(X, \sigma(X, X^*))$. Then $z + \frac{1}{2}W$ is a z-neighborhood in $(X, \sigma(X, X^*))$. Hence there exists $n_0 \in \mathbb{N}$ such that $a_n - b_n \in z + \frac{1}{2}W$ for all $n \geq n_0$. Now $(b_n)_{n \in \mathbb{N}} \subset B$ and B is weakly countably compact, hence there exists $b \in B$ such that b is a weak cluster point of $(b_n)_{n \in \mathbb{N}}$. Clearly $b + \frac{1}{2}W$ is a b-neighborhood in $(X, \sigma(X, X^*))$, hence for any $n \in \mathbb{N}$, there exists $m \geq \max(n, n_0)$ such that $b_m \in b + \frac{1}{2}W$. Thus

$$a_m = b_m + (a_m - b_m) \in b + \tfrac{1}{2}W + z + \tfrac{1}{2}W = (b + z) + W.$$

This means that $b + z$ is a weak cluster point of $(a_n)_{n \in \mathbb{N}}$. By the hypothesis that A is weakly closed, we have $b+z \in A$. Thus $z=(b+z)-b \in A-B$. That is to say, $A-B$ is weakly sequentially closed. ■

LEMMA 2.2. *Let B be a closed bounded convex subset of a locally convex space $(X,\mathcal{T})$ and $a_0 \in X$. Then $D(a_0,B)$ is also a closed bounded convex set.*

Proof. It suffices to prove that $D(a_0,B)$ is closed. If $a_0 \in B$, then clearly $D(a_0,B)=B$ is closed. Next we assume that $a_0 \notin B$. Let $(\lambda_\delta a_0+(1-\lambda_\delta)b_\delta)_\delta$ be a net in $D(a_0,B)$ converging to z in X, where $0 \le \lambda_\delta \le 1$, $b_\delta \in B$. Then there exists a subnet $(\lambda_{\delta_i})_i$ such that $\lambda_{\delta_i} \xrightarrow{i} \lambda_0$. Clearly $0 \le \lambda_0 \le 1$. We shall argue in different ways, according to whether $\lambda_0=1$ or not.

(i) If $\lambda_0=1$, then $\lambda_{\delta_i} \xrightarrow{i} 1$. Since (b_{δ_i}) is a bounded net, we have $(1-\lambda_{\delta_i})\,b_{\delta_i} \xrightarrow{i} 0$. Combining this with $\lambda_{\delta_i}a_0+(1-\lambda_{\delta_i})b_{\delta_i} \xrightarrow{i} z$, we conclude that $\lambda_{\delta_i}a_0 \to z$. On the other hand, $\lambda_{\delta_i}a_0 \to a_0$. Therefore $z=a_0 \in D(a_0,B)$.

(ii) If $0 \le \lambda_0 < 1$, then $1-\lambda_0>0$. Since $\lambda_{\delta_i}a_0+(1-\lambda_{\delta_i})b_{\delta_i} \xrightarrow{i} z$ and $\lambda_{\delta_i}a_0 \xrightarrow{i} \lambda_0 a_0$, we have $(1-\lambda_{\delta_i})b_{\delta_i} \xrightarrow{i} z-\lambda_0 a_0$. And since $(\lambda_{\delta_i}-\lambda_0)b_{\delta_i} \xrightarrow{i} 0$, we have

$$(1-\lambda_0)b_{\delta_i}=(1-\lambda_{\delta_i})b_{\delta_i}+(\lambda_{\delta_i}-\lambda_0)b_{\delta_i} \xrightarrow{i} z-\lambda_0 a_0.$$

From this we know that

$$b_{\delta_i} \xrightarrow{i} \frac{z-\lambda_0 a_0}{1-\lambda_0} =: b_0.$$

Since every $b_{\delta_i} \in B$ and B is closed, we deduce that $b_0 \in B$. Thus

$$z=\lambda_0 a_0+(1-\lambda_0)b_0 \in D(a_0,B). \ ■$$

LEMMA 2.3. *Let B be a weakly countably compact, closed convex subset of a locally convex space $(X,\mathcal{T})$ and A be a weakly closed subset of $(X,\mathcal{T})$. Then $D(a_0,B)\cap A-B$ is weakly sequentially closed in $(X,\mathcal{T})$.*

Proof. Since B is weakly countably compact, it is bounded in $(X,\mathcal{T})$. By Lemma 2.2, $D(a_0,B)$ is a closed bounded convex subset of $(X,\mathcal{T})$ and hence $D(a_0,B)$ is weakly closed. Replacing "A" by "$D(a_0,B)\cap A$" in Lemma 2.1 we conclude that $D(a_0,B)\cap A-B$ is weakly sequentially closed. ■

LEMMA 2.4. *Let B be a weakly countably compact, closed convex subset of a locally convex space $(X,\mathcal{T})$ and $a_0 \notin B$. Then $D(a_0,B)$ is also a weakly countably compact, closed convex set.*

Proof. Again, by Lemma 2.2, $D(a_0,B)$ is a closed bounded convex set. Next we show that it is weakly countably compact. Put

$$K := \{(\lambda,\mu) \in \mathbb{R}^2 : \lambda \ge 0,\ \mu \ge 0,\ \lambda+\mu=1\}.$$

Then K is compact. With the topology induced by $\sigma(X, X^*)$, B is countably compact. By an elementary theorem of general topology, the topological product $K \times B$ is countably compact. If we map each $((\lambda, \mu), b) \in K \times B$ to the corresponding element $\lambda a_0 + \mu b \in (X, \sigma(X, X^*))$, we obtain a continuous map whose image $D(a_0, B)$ is countably compact in $(X, \sigma(X, X^*))$. ∎

LEMMA 2.5. *Let B be a weakly countably compact, closed convex subset of a locally convex space $(X, \mathcal{T})$ and A be a weakly closed subset of $(X, \mathcal{T})$ which is disjoint from B. Then for any $a_0 \in A$, and any ε, $0 < \varepsilon < 1$, there exists $a_1 \in D(a_0, B) \cap A$ such that*

$$D(a_1, B) \cap A \subset \{ta_1 + (1-t)b : b \in B,\ 1 - \varepsilon < t \leq 1\}.$$

Proof. By Lemma 2.4, $D(a_0, B)$ is a weakly countably compact, closed convex subset of $(X, \mathcal{T})$. By [5, p. 17], there exists a Banach disc $W \supset D(a_0, B)$. Denote $(\mathrm{sp}[W], p_W)$ by E_W, where $\mathrm{sp}[W]$ denotes the linear span of W and p_W denotes the Minkowski gauge of W. Then E_W is a Banach space. It is easy to see that

$$D(a_0, B) \cap A - B \subset W - W \subset \mathrm{sp}[W].$$

By Lemma 2.3, $D(a_0, B) \cap A - B$ is weakly sequentially closed. Note that the topology on $\mathrm{sp}[W]$ generated by p_W is finer than the one induced by the weak topology, hence $D(a_0, B) \cap A - B$ is closed in E_W. Since A is disjoint from B, $D(a_0, B) \cap A$ is disjoint from B and hence $0 \notin D(a_0, B) \cap A - B$. Thus we have

$$\alpha := \inf\{p_W(x - y) : x \in D(a_0, B) \cap A,\ y \in B\} > 0.$$

The result follows just as in the proof of [24, Lemma 2.1]. ∎

Now we can give our main result.

THEOREM 2.1. *Let $(X, \mathcal{T})$ be a locally convex space and B be a weakly countably compact, closed convex set in $(X, \mathcal{T})$. Then for any weakly closed set A disjoint from B and any $a_0 \in A$, there exists an $x_0 \in D(a_0, B) \cap A$ such that $D(x_0, B) \cap A = \{x_0\}$. That is, B has the quasi-weak drop property.*

Proof. Take a sequence $(\varepsilon_n)_{n \in \mathbb{N}}$ of positive real numbers such that

$$1 > \varepsilon_1 > \varepsilon_2 > \ldots > 0, \qquad \varepsilon_n \to 0\ (n \to \infty).$$

For $a_0 \in A$ and $\varepsilon_1 > 0$, by Lemma 2.5 there exists $a_1 \in D(a_0, B) \cap A$ such that

$$D(a_1, B) \cap A \subset \{ta_1 + (1-t)b : b \in B,\ 1 - \varepsilon_1 < t \leq 1\}.$$

For the above $a_1 \in A$ and $\varepsilon_2 > 0$, again by Lemma 2.5 there exists $a_2 \in D(a_1, B) \cap A$ such that

$$D(a_2, B) \cap A \subset \{ta_2 + (1-t)b : b \in B,\ 1 - \varepsilon_2 < t \leq 1\}.$$

Repeating this process, we obtain a sequence $(a_n)_{n\in\mathbb{N}} \subset A$ such that $a_{n+1} \in D(a_n, B) \cap A$ and

$$D(a_n, B) \cap A \subset \{ta_n + (1-t)b : b \in B,\ 1-\varepsilon_n < t \le 1\}.$$

By Lemma 2.4, $D(a_0, B)$ is a weakly countably compact, closed convex set. And since A is weakly closed, $D(a_0, B) \cap A$ is weakly countably compact. By Lemma 2.2, $D(a_n, B)$ is closed convex, hence it is weakly closed. Since

$$D(a_1, B) \supset D(a_2, B) \supset \ldots$$

is a sequence of weakly closed convex subsets of $(X, \mathcal{T})$ for which all the intersections $D(a_n, B) \cap (D(a_0, B) \cap A) = D(a_n, B) \cap A$ are non-empty, and $D(a_0, B) \cap A$ is weakly countably compact, we conclude that (see [10, p. 316])

$$\bigcap_{n=1}^{\infty} D(a_n, B) \cap (D(a_0, B) \cap A) = \bigcap_{n=1}^{\infty} D(a_n, B) \cap A \neq \emptyset.$$

Let $x_0 \in \bigcap_{n=1}^{\infty} D(a_n, B) \cap A$. Then $x_0 \in D(a_0, B) \cap A$. Next we show that

$$D(x_0, B) \cap A = \{x_0\}.$$

We may assume that $x_0 = \lambda_n a_n + (1-\lambda_n) b_n$, where $b_n \in B$, $1-\varepsilon_n < \lambda_n \le 1$. Hence

$$\begin{aligned} (1)\qquad a_n - x_0 &= a_n - \lambda_n a_n - (1-\lambda_n) b_n = (1-\lambda_n)(a_n - b_n) \\ &\in (1-\lambda_n)(D(a_0, B) - B). \end{aligned}$$

Since $D(a_0, B) - B$ is bounded in $(X, \mathcal{T})$ and $\lambda_n \to 1$, by (1) we know that the sequence $(a_n)_{n\in\mathbb{N}}$ is convergent to x_0 in $(X, \mathcal{T})$. Since $x_0 \in D(a_n, B)$, we have $D(x_0, B) \subset D(a_n, B)$, for every n. Take any $z \in D(x_0, B) \cap A$. Then $z \in D(a_n, B) \cap A$ for every n. We may assume that $z = \mu_n a_n + (1-\mu_n) b'_n$, where $b'_n \in B$ and $1-\varepsilon_n < \mu_n \le 1$. Thus

$$\begin{aligned} (2)\qquad z - a_n &= \mu_n a_n + (1-\mu_n) b'_n - a_n = (1-\mu_n)(b'_n - a_n) \\ &\in (1-\mu_n)(B - D(a_0, B)). \end{aligned}$$

Since $B - D(a_0, B)$ is bounded in $(X, \mathcal{T})$ and $\mu_n \to 1$, by (2) we know that the sequence $(a_n)_{n\in\mathbb{N}}$ is convergent to z in $(X, \mathcal{T})$. By the uniqueness of limits we have $z = x_0$. Therefore $D(x_0, B) \cap A = \{x_0\}$. ∎

REMARK 2.1. Theorem 2.1 is a proper improvement of [24, Theorem 3.1], since there exist weakly countably compact closed convex sets which are not weakly compact as we see in Section 1. Here we give another kind of counterexamples. Let us recall some basic facts concerning locally convex spaces (for example, see [4], [9], [13], [21] and [27]). A locally convex space X is said to be *barrelled* if every $\sigma(X^*, X)$-bounded subset of X^* is equicontinuous; to be *countably barrelled* (or $\aleph_0$-*barrelled*) if each $\sigma(X^*, X)$-bounded subset of X^*, which is the countable union of equicontinuous subsets of X^*, is itself equicontinuous; to be *ω-barrelled* (or *l^∞-barrelled*) if every $\sigma(X^*, X)$-bounded sequence in X^* is equicontinuous; to have *property* (C) if every $\sigma(X^*, X)$-bounded subset of X^* is relatively $\sigma(X^*, X)$-countably compact. Obviously we have

$$\text{barrelled} \Rightarrow \text{countably barrelled} \Rightarrow \omega\text{-barrelled} \Rightarrow \text{property (C)}.$$

None of the converses of the above implications is true. In particular, Levin and Saxon [13] exhibited an ω-barrelled Mackey space which is not barrelled. Valdivia [28] constructed an ω-barrelled Mackey space which is not countably barrelled. Recently Saxon and Sanchez Ruiz [27] presented a Mackey space with property (C) which is not ω-barrelled.

To sum up the above facts, we know that there exist Mackey spaces with property (C) which are not barrelled. Let $(X, \mathcal{T})$ be one. Then there exists a $\sigma(X^*, X)$-bounded subset B of X^* such that B is not $\mathcal{T}$-equicontinuous. Without loss of generality, we may assume that B is a $\sigma(X^*, X)$-closed absolutely convex set. Since $(X, \mathcal{T})$ is a Mackey space, the non-equicontinuous subset B is not $\sigma(X^*, X)$-compact. On the other hand, $(X, \mathcal{T})$ has property (C), hence B is $\sigma(X^*, X)$-countably compact. Let $E = X^*$ be endowed with any locally convex topology τ compatible with the dual pair $\langle X^*, X\rangle$. Then $(E, \tau)^* = X$. Clearly B is a weakly countably compact closed convex set in (E, τ), but it is not weakly compact.

The author would like to thank the referee for his valuable comments and suggestions.

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Received June 23, 2003
Revised version September 1, 2003 (5226)