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On a simultaneous selection theorem

by

Takamitsu Yamauchi (Matsue)

Abstract. Valov proved a general version of Arvanitakis's simultaneous selection theorem which is a common generalization of both Michael's selection theorem and Dugundji's extension theorem. We show that Valov's theorem can be extended by applying an argument by means of Pettis integrals due to Repovš, Semenov and Shchepin.

1. Introduction. All spaces considered in this paper are assumed to be completely regular and Hausdorff. For a space X and a linear topological space E, let $C(X,E)$ denote the linear space of all continuous mappings from X to E. For a space Y, let 2^Y denote the power set of Y. For a subset A of a linear topological space E, let $\overline{\mathrm{conv}}(A)$ denote the closed convex hull of A. A mapping $\Phi : X \to 2^Y$ is said to be *lower semicontinuous* (*l.s.c.* for short) if for every open subset V of Y, the set $\{x \in X : \Phi(x) \cap V \neq \emptyset\}$ is open in X. As a common generalization of Michael's convex-valued selection theorem [9] and Dugundji's simultaneous extension theorem [5], Arvanitakis [1, Theorem 1.1] established the following simultaneous selection theorem.

Theorem 1.1 (Arvanitakis [1]). *Let X be a paracompact k-space, Y a completely metrizable space, E a locally convex complete linear topological space and $\Phi : X \to 2^Y \setminus \{\emptyset\}$ an l.s.c. mapping. Then there exists a linear mapping $S : C(Y,E) \to C(X,E)$ such that*

(1.1) $$S(f)(x) \in \overline{\mathrm{conv}}(f(\Phi(x))) \quad \text{for every } x \in X \text{ and } f \in C(Y,E).$$

Furthermore, S is continuous when both $C(Y,E)$ and $C(X,E)$ are equipped with the uniform topology or the topology of uniform convergence on compact sets.

Let $C_b(X,E)$ denote the set of all bounded continuous mappings from X to E. Valov [22, Theorem 1.2] proved that the assumption in Theorem 1.1 that X is a k-space can be dropped if $C(Y,E)$ and $C(X,E)$ are replaced

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with $C_b(Y,E)$ and $C_b(X,E)$, respectively, or if E is a Banach space. His proof is based on the existence of a barycenter map introduced by Banakh [2] and that of a perfect Milyutin mapping due to Repovš, Semenov and Shchepin [18].

In this note, we show that the assumption in Theorem 1.1 of X being a k-space can be dropped and that of E being complete can be relaxed (see Theorem 2.5). Our proof is based on an argument by means of Pettis integrals due to Repovš, Semenov and Shchepin [18].

Let $\mathbb{R}$ denote the space of real numbers with the usual topology. The covering dimension of a normal space X is denoted by $\dim X$. For undefined notions we refer to [6] or [19].

2. Proof of the main result. For a space X, let βX denote the Stone–Čech compactification of X and $P(\beta X)$ the set of all regular Borel probability measures on βX. For $\mu \in P(\beta X)$, let $\operatorname{supp}\mu$ denote the support of μ, that is, the intersection of all closed subsets F of βX such that $\mu(\beta X \setminus F) = 0$. Put $P_\beta(X) = \{\mu \in P(\beta X) : \operatorname{supp}\mu \subset X\}$. Let $C_b(X)$ denote the Banach space $C_b(X,\mathbb{R})$ with the sup-norm and $C_b(X)^*$ its dual. Then, for $\mu \in P_\beta(X)$, the mapping $L_\mu : C_b(X) \to \mathbb{R}$ defined by

$$L_\mu(f) = \int_{\beta X} \beta f \, d\mu = \int_{\operatorname{supp}\mu} f \, d\mu, \quad f \in C_b(X),$$

is a continuous linear functional on $C_b(X)$, where βf is the unique continuous extension of f to βX. Since each $\mu \in P_\beta(X)$ is a regular measure on βX, the correspondence $\mu \mapsto L_\mu$ is an injection on $P_\beta(X)$ into $C_b(X)^*$. Thus, we may assume that $P_\beta(X)$ is a subset of $C_b(X)^*$ and endow $P_\beta(X)$ with the relative topology induced by the weak* topology on $C_b(X)^*$. Then, a basic neighborhood of $\mu \in P_\beta(X)$ is of the form

$$\Big\{\nu \in P_\beta(X) : \Big| \int_{\operatorname{supp}\mu} f_i \, d\mu - \int_{\operatorname{supp}\nu} f_i \, d\nu \Big| < \varepsilon,\ i \in \{1,\dots,n\}\Big\},$$

where $f_1,\dots,f_n \in C_b(X)$ and $\varepsilon > 0$.

Let E be a locally convex linear topological space in which every closed convex hull of a compact subset is compact (for example, a quasi-complete space, see [7, §20, 6 (3)]). Then, for every $\mu \in P_\beta(X)$ and $f \in C(X,E)$, and every compact subset C of X, there exists the Pettis integral $\int_C f \, d\mu$ which is the unique element of E satisfying

$$\int_C f \, d\mu \in \mu(C)\,\overline{\operatorname{conv}}(f(C)), \tag{2.1}$$

$$\lambda\Big(\int_C f \, d\mu\Big) = \int_C (\lambda \circ f) \, d\mu \tag{2.2}$$

for each $\lambda \in E^*$ (see [19, Theorem 3.27]).

For spaces Z and X, a continuous surjection $p : Z \to X$ is said to be *zero-dimensional* if each fiber $p^{-1}(x)$, $x \in X$, is zero-dimensional. Note that if p is a zero-dimensional perfect mapping, then $\dim p^{-1}(x) = 0$ for each $x \in X$ (see [6, Theorem 6.2.7]). A continuous surjection $p : Z \to X$ is called a *Milyutin mapping* if there exists a continuous mapping $\nu : X \to P_\beta(Z)$ such that $\operatorname{supp} \nu(x) \subset p^{-1}(x)$ for each $x \in X$. Milyutin mappings were introduced by Pełczyński [15] under the name of "mappings with a regular averaging operator", and the name "Milyutin mapping" was introduced by Shchepin [20] (see also [16], [17], [21]). Following [18], we shall define a continuous linear mapping by means of zero-dimensional perfect Milyutin mappings (see also [16, Theorem 4.1], [17, A §3]).

Lemma 2.1. *Let Z and X be spaces, $p : Z \to X$ a zero-dimensional perfect Milyutin mapping, $\nu : X \to P_\beta(Z)$ a continuous mapping associated with p and E a locally convex linear topological space in which every closed convex hull of a compact subset is compact. For $h \in C(Z, E)$, define $T(h) : X \to E$ by*

$$T(h)(x) = \int_{p^{-1}(x)} h \, d\nu(x), \quad x \in X. \tag{2.3}$$

Then $T(h)$ is continuous.

To show Lemma 2.1, we shall apply the following fact.

Fact 2.2. *Let Z be a space and C its compact subspace satisfying $\dim C = 0$. Then, for every collection $\mathscr{V}$ of open subsets of Z which covers C, there exists a finite disjoint collection $\mathscr{G}$ of open subsets of Z which refines $\mathscr{V}$ and covers C.*

Proof. We give the proof for the sake of completeness (for a normal space and its closed subspace, see [3, §22]). Let $\mathscr{V}$ be a collection of open subsets of Z which covers C. Since C is compact and $\dim C = 0$, we have a finite collection $\mathscr{W}$ of open subsets of Z such that $C \subset \bigcup \mathscr{W}$, $\mathscr{W}$ refines $\mathscr{V}$ and the collection $\{W \cap C : W \in \mathscr{W}\}$ is disjoint. For each $z \in C$, we can take a unique $W(z) \in \mathscr{W}$ and an open subset $U(z)$ of Z so that $z \in U(z) \subset \overline{U(z)} \subset W(z)$ since Z is completely regular. Choose a finite set $\{z_1, \ldots, z_k\} \subset C$ such that $C \subset \bigcup_{j=1}^k U(z_j)$. For each $W \in \mathscr{W}$, put $U_W = \bigcup \{U(z_j) : W(z_j) = W, j \in \{1, \ldots, k\}\}$. Then we have $\overline{U_W} \subset W$. By putting $G_W = U_W \setminus \bigcup_{W' \neq W} \overline{U_{W'}}$ for each $W \in \mathscr{W}$, we have the required finite disjoint collection $\mathscr{G} = \{G_W : W \in \mathscr{W}\}$. ∎

Proof of Lemma 2.1. Our proof is based on the idea in [17, A §3.4]. Let $h \in C(Z, E)$, $x_0 \in X$ and W a neighborhood of the origin of E. Take a convex symmetric neighborhood V of the origin such that $4V \subset W$. Then $\mathscr{V} = \{h^{-1}(h(z) + V) : z \in p^{-1}(x_0)\}$ is a collection of open subsets of Z

which covers $p^{-1}(x_0)$. Since $p^{-1}(x_0)$ is compact and $\dim p^{-1}(x_0) = 0$, by Fact 2.2, there exists a finite disjoint collection $\mathscr{G} = \{G_1, \ldots, G_n\}$ of open subsets of Z which refines $\mathscr{V}$ and covers $p^{-1}(x_0)$. For each $i \in \{1, \ldots, n\}$, choose $a_i \in p^{-1}(x_0)$ such that $G_i \subset h^{-1}(h(a_i) + V)$. Then

$$h(G_i) \subset h(a_i) + V. \tag{2.4}$$

Put $K_i = p^{-1}(x_0) \cap G_i$. Then $K_i = p^{-1}(x_0) \setminus \bigcup_{j \neq i} G_j$, which is compact and $K_i \subset G_i$. Thus, since Z is completely regular and Hausdorff, there exists a continuous function $s_i : Z \to [0, 1]$ such that $K_i \subset \operatorname{Int} s_i^{-1}(1) \subset s_i^{-1}((0, 1]) \subset G_i$. Put $U_i = \operatorname{Int} s_i^{-1}(1)$. Then $p^{-1}(x_0) \subset \bigcup_{i=1}^n U_i$. Since p is a closed mapping, we may take a neighborhood N of x_0 such that $p^{-1}(N) \subset \bigcup_{i=1}^n U_i$. Since ν is continuous, there exists a neighborhood O of x_0 such that

$$\begin{aligned}(2.5)\quad &\Big(\int\limits_{p^{-1}(x)} s_i \, d\nu(x) - \int\limits_{p^{-1}(x_0)} s_i \, d\nu(x_0)\Big) h(a_i) \\ &\qquad = \Big(\int\limits_{\operatorname{supp} \nu(x)} s_i \, d\nu(x) - \int\limits_{\operatorname{supp} \nu(x_0)} s_i \, d\nu(x_0)\Big) h(a_i) \in \frac{1}{n} V\end{aligned}$$

for each $x \in O$ and $i \in \{1, \ldots, n\}$.

We claim that $T(h)(x) - T(h)(x_0) \in W$ for each $x \in N \cap O$. To show this, let $x \in N \cap O$. For each $i \in \{1, \ldots, n\}$, we have $\operatorname{supp} \nu(x) \cap (G_i \setminus U_i) = \emptyset$ since $\operatorname{supp} \nu(x) \subset p^{-1}(x) \subset \bigcup_{i=1}^n U_i$ and $G_i \cap \bigcup_{j \neq i} U_j = \emptyset$. Also, $p^{-1}(x) \setminus G_i \subset s_i^{-1}(0)$ and $U_i \subset s_i^{-1}(1)$. Thus,

$$\begin{aligned}(2.6)\quad \int\limits_{p^{-1}(x)} s_i \, d\nu(x) &= \int\limits_{p^{-1}(x) \setminus G_i} s_i \, d\nu(x) + \int\limits_{G_i \setminus U_i} s_i \, d\nu(x) + \int\limits_{U_i} s_i \, d\nu(x) \\ &= \int\limits_{U_i} s_i \, d\nu(x) = \nu(x)(U_i).\end{aligned}$$

Since $\sum_{i=1}^n \nu(x)(U_i) = \nu(x)(p^{-1}(x)) = 1$ and V is convex, by applying (2.4) and (2.6), we obtain

$$\begin{aligned} T(h)(x) &= \int\limits_{p^{-1}(x)} h \, d\nu(x) = \sum_{i=1}^n \int\limits_{U_i \cap p^{-1}(x)} h \, d\nu(x) \\ &\in \sum_{i=1}^n \nu(x)(U_i \cap p^{-1}(x)) \, \overline{\operatorname{conv}}(h(U_i \cap p^{-1}(x))) \\ &\subset \sum_{i=1}^n \nu(x)(U_i)(h(a_i) + \overline{V}) \subset \sum_{i=1}^n \nu(x)(U_i) h(a_i) + \overline{V} \\ &= \sum_{i=1}^n \Big(\int\limits_{p^{-1}(x)} s_i \, d\nu(x)\Big) h(a_i) + \overline{V}. \end{aligned}$$

Similarly,

$$T(h)(x_0) \in \sum_{i=1}^{n} \Big(\int\limits_{p^{-1}(x_0)} s_i \, d\nu(x_0) \Big) h(a_i) + \overline{V}.$$

Thus, by (2.5),

$$\begin{aligned} T(h)(x) - T(h)(x_0) &\in \sum_{i=1}^{n} \Big(\int\limits_{p^{-1}(x)} s_i \, d\nu(x) - \int\limits_{p^{-1}(x_0)} s_i \, d\nu(x_0) \Big) h(a_i) + 2\overline{V} \\ &\subset \sum_{i=1}^{n} \frac{1}{n} V + 2\overline{V} \subset 4V \subset W. \end{aligned}$$

Hence $T(h) : X \to E$ is continuous. ■

We have the following analogue of [22, Proposition 2.2].

Lemma 2.3. *Let Z, X, p, ν and E be as in Lemma 2.1. Let $T : C(Z, E) \to C(X, E)$ be the linear mapping defined by (2.3). Then:*

(i) *$T(h)(x) \in \overline{\mathrm{conv}}(h(p^{-1}(x)))$ for each $x \in X$ and $h \in C(Z, E)$,*
(ii) *$T(g \circ p) = g$ for each $g \in C(X, E)$, and*
(iii) *T is continuous when both $C(Z, E)$ and $C(X, E)$ are equipped with the uniform topology or the topology of uniform convergence on compact sets.*

Proof. (i) follows from (2.3) and (2.1) since $\nu(x)(p^{-1}(x)) = 1$.

For (ii), let $g \in C(X, E)$. Then for each $x \in X$ and $\lambda \in E^*$,

$$\begin{aligned} \lambda(T(g \circ p)(x))) &= \lambda\Big(\int\limits_{p^{-1}(x)} (g \circ p) \, d\nu(x) \Big) = \int\limits_{p^{-1}(x)} (\lambda \circ g \circ p) \, d\nu(x) \\ &= \lambda(g(x)) \int\limits_{p^{-1}(x)} d\nu(x) = \lambda(g(x))\nu(x)(p^{-1}(x)) = \lambda(g(x)). \end{aligned}$$

Since E^* separates points in E, we have $T(g \circ p)(x) = g(x)$, which shows (ii).

For (iii), we show T is continuous with respect to the topology of uniform convergence on compact sets. Let $h \in C(Z, E)$, K a compact subset of X and W a convex neighborhood of the origin of E. Since $p : Z \to X$ is a perfect mapping, $p^{-1}(K)$ is a compact subset of Z. Let $k \in C(Z, E)$ be such that $k(z) - h(z) \in W$ for each $z \in p^{-1}(K)$.

To complete the proof, it suffices to show that $T(k)(x) - T(h)(x) \in \overline{W}$ for each $x \in K$. For a contradiction, assume $T(k)(x') - T(h)(x') \notin \overline{W}$ for some $x' \in K$. By a separation theorem (see [19, Theorem 3.4]), there exists $\lambda \in E^*$ such that $\sup \lambda(\overline{W}) < \lambda(T(k)(x') - T(h)(x'))$. Put $c = \sup \lambda(\overline{W})$. Then for each $z \in p^{-1}(x') (\subset p^{-1}(K))$, we have $k(z) - h(z) \in W$, and hence

$\lambda(k(z)-h(z)) \le c$ for each $z \in p^{-1}(x')$. Thus,

$$c < \lambda(T(k)(x') - T(h)(x')) = \lambda\Big(\int\limits_{p^{-1}(x')} k\,d\nu(x') - \int\limits_{p^{-1}(x')} h\,d\nu(x')\Big)$$
$$= \int\limits_{p^{-1}(x')} (\lambda \circ (k-h))\,d\nu(x') \le \int\limits_{p^{-1}(x')} c\,d\nu(x') = c\,\nu(x)(p^{-1}(x')) = c.$$

This is a contradiction. Thus T is continuous with respect to the topology of uniform convergence on compact sets. Similarly, we can show that T is continuous with respect to the uniform topology. ∎

The following theorem is essentially proved by Michael [10, Theorem 1.2], [11, Theorem2] (see also [17, Theorem (2.4)*]).

Theorem 2.4 (Michael [10], [11]). *Let Z be a paracompact space with $\dim Z = 0$, Y a metrizable space and $\varphi : Z \to 2^Y$ an l.s.c. mapping with nonempty complete values for some compatible metric on Y. Then φ admits a continuous selection.*

Now we have the main theorem.

Theorem 2.5. *Let X be a paracompact space, Y a metrizable space, $\Phi : X \to 2^Y$ an l.s.c. mapping with nonempty complete values for some compatible metric on Y and E a locally convex linear topological space in which every closed convex hull of a compact subset is compact. Then there exists a linear mapping $S : C(Y,E) \to C(X,E)$ satisfying* (1.1). *Furthermore, S is continuous when both $C(Y,E)$ and $C(X,E)$ are equipped with the uniform topology or the topology of uniform convergence on compact sets.*

Proof. Our proof is the same as that of [22, Theorem 1.2]. We present it here for the sake of completeness. According to [18, Theorem 1.4] (see also [17, Theorem (3.9)]), there exists a paracompact space Z with $\dim Z = 0$ and a Milyutin mapping $p : Z \to X$. Then the mapping $\varphi : Z \to 2^Y$ defined by $\varphi(z) = \Phi(p(z))$ is an l.s.c. mapping with nonempty complete values for some compatible metric on Y. According to Theorem 2.4, φ admits a continuous selection $g : Z \to Y$. Let $T : C(Z,E) \to C(X,E)$ be the linear mapping defined by (2.3). Define $S : C(Y,E) \to C(X,E)$ by $S(f) = T(f \circ g)$ for each $f \in C(Y,E)$. Since g is a selection of φ, we have $g(p^{-1}(x)) \subset \Phi(x)$ for each $x \in X$. Thus, by (i) in Lemma 2.3, we have

$$S(f)(x) = T(f \circ g)(x) \in \overline{\mathrm{conv}}(f(g(p^{-1}(x)))) \subset \overline{\mathrm{conv}}(f(\Phi(x))).$$

The continuity of S follows from that of T and g. ∎

Remark 2.6. If Y in Theorem 2.5 is completely metrizable, then the assumption that the values of Φ are complete for some compatible metric on Y can be dropped. Indeed, let X and Φ be as in Theorem 2.5, Y a complete metric space and $\Phi : X \to 2^Y \setminus \{\emptyset\}$ an l.s.c. mapping. Then the mapping $\overline{\Phi} : X \to 2^Y \setminus \{\emptyset\}$ defined by $\overline{\Phi}(x) = \overline{\Phi(x)}$ for $x \in X$ is l.s.c. and complete-valued. Thus, by Theorem 2.5 replacing Φ with $\overline{\Phi}$, we obtain a linear mapping $S : C(Y, E) \to C(X, E)$ such that

$$S(f)(x) \in \overline{\mathrm{conv}}(f(\overline{\Phi(x)})) = \overline{\mathrm{conv}}(f(\Phi(x)))$$

for all $x \in X$ and $f \in C(Y, E)$.

Hence Theorem 2.5 is an extension of Theorem 1.1.

3. Applications. Applying Theorem 2.5, we have another proof of the following theorems due to Michael [13, Theorem 1.2], [12, Theorem 1.3].

THEOREM 3.1 (Michael [13]). *Let X be a paracompact space, E a locally convex linear topological space in which every closed convex hull of a compact subset is compact and Y a metrizable subset of E. Let $\Phi : X \to 2^Y$ be an l.s.c. mapping with nonempty complete values for some compatible metric on Y. Then there exists a continuous $f : X \to E$ such that $f(x) \in \overline{\mathrm{conv}}(\Phi(x))$ for every $x \in X$.*

Proof. By Theorem 2.5, there exists a linear mapping $S : C(Y, E) \to C(X, E)$ satisfying (1.1). Let $i : Y \to E$ be the inclusion mapping. Then the mapping $f = S(i) \in C(X, E)$ is as required. ∎

THEOREM 3.2 (Michael [12]). *Let X and Y be metric spaces and $p : X \to Y$ a surjective open mapping such that $p^{-1}(y)$ is complete for every $y \in Y$. Let E be a locally convex linear topological space in which every closed convex hull of a compact subset is compact, and let $C(X, E)$ and $C(Y, E)$ be endowed with the topology of uniform convergence on compact sets. Then there exists a continuous linear mapping $S : C(X, E) \to C(Y, E)$ such that*

$$S(f)(y) \in \overline{\mathrm{conv}}(f(p^{-1}(y)))$$

for every $f \in C(X, E)$ and $y \in Y$.

Proof. Since $p : X \to Y$ is an open mapping, $\Phi : Y \to 2^X$ defined by $\Phi(y) = p^{-1}(y)$ for $y \in Y$ is an l.s.c. mapping with nonempty complete values. Thus the conclusion follows from Theorem 2.5. ∎

REMARK 3.3. By [12, Example 1.4], the assumption in Theorem 3.2 that every $p^{-1}(y)$ is complete cannot be dropped. Thus, in Theorem 2.5, the assumption that Φ is compete-valued is necessary.

For a space Y, let $\mathscr{C}(Y)$ denote the set of all nonempty compact subsets of Y and put $\mathscr{C}'(Y) = \mathscr{C}(Y) \cup \{Y\}$. The weight of a space Y is denoted by $w(Y)$. For a metrizable space Y and an l.s.c. mapping $\Phi : X \to 2^Y \setminus \{\emptyset\}$, a triple (Z, g, φ) is called an *l.s.c. weak factorization* of Φ [4, 14] if Z is a metrizable space with $w(Z) \le w(Y)$, $g : X \to Z$ is a continuous mapping and $\varphi : Z \to \mathscr{C}(Y)$ is an l.s.c. mapping such that $\varphi(g(x)) \subset \Phi(x)$ for every $x \in X$. For an infinite cardinal τ, a space X is said to be *τ-collectionwise* *normal* if for every discrete collection $\mathscr{F}$ of closed subsets of X with $|\mathscr{F}| \le \tau$, there exists a disjoint collection $\{U_F : F \in \mathscr{F}\}$ of open subsets of X such that $F \subset U_F$ for each $F \in \mathscr{F}$. A space X is collectionwise normal if X is τ-collectionwise normal for every τ. It is well-known that a space X is normal if and only if X is ω-collectionwise normal, where ω is the smallest infinite cardinal.

COROLLARY 3.4. *Let X be a τ-collectionwise normal space, Y a completely metrizable space with $w(Y) \le \tau$, $\Phi : X \to \mathscr{C}'(Y)$ an l.s.c. mapping and E a locally convex linear topological space in which every closed convex hull of a compact subset is compact. Then there exists a linear mapping $S : C(Y,E) \to C(X,E)$ satisfying* (1.1). *Furthermore, S is continuous when both $C(Y,E)$ and $C(X,E)$ are equipped with the uniform topology or the topology of uniform convergence on compact sets.*

Proof. By [4] (see also [14, Lemma 3.6 and Theorem 5.1]), Φ admits a weak factorization (Z, g, φ). In view of Theorem 2.5 for $\varphi : Z \to \mathscr{C}(Y)$, there exists a linear mapping $T : C(Y,E) \to C(Z,E)$ such that

$$T(f)(z) \in \overline{\mathrm{conv}}(f(\varphi(z))) \quad \text{for every } z \in Z \text{ and } f \in C(Y,E).$$

Then the mapping $S : C(Y,E) \to C(X,E)$ defined by $S(f) = T(f) \circ g$ for $f \in C(Y,E)$ is as required. ∎

We have the following corollary analogous to [8, Theorem 1].

COROLLARY 3.5. *Let X be a τ-collectionwise normal space, A a completely metrizable subspace of X with $w(A) \le \tau$, and E a locally convex linear topological space in which every closed convex hull of a compact subset is compact. Then there exists a linear mapping $S : C(A,E) \to C(X,E)$ such that $S(f)$ is an extension of f and $S(f)(X) \subset \overline{\mathrm{conv}}(f(A))$ for every $f \in C(A,E)$. Furthermore, S is continuous when both $C(A,E)$ and $C(X,E)$ are equipped with the uniform topology or the topology of uniform convergence on compact sets.*

Proof. As in [22, §3], define $\Phi : X \to 2^A$ by $\Phi(x) = \{x\}$ if $x \in A$ and $\Phi(x) = A$, otherwise. Then Φ is l.s.c. and $\Phi(x) \in \mathscr{C}'(A)$ for each $x \in X$. Thus, by Corollary 3.4, we have a linear mapping $S : C(A,E) \to C(X,E)$ satisfying (1.1) with Y replaced by A. This S is as required. ∎

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Takamitsu Yamauchi
Department of Mathematics
Shimane University
Matsue, 690-8504, Japan
E-mail: t_yamauchi@riko.shimane-u.ac.jp

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