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An embedding theorem for Sobolev type functions with gradients in a Lorentz space

by

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Abstract. The purpose of this paper is to prove an embedding theorem for Sobolev type functions whose gradients are in a Lorentz space, in the framework of abstract metric-measure spaces. We then apply this theorem to prove absolute continuity and differentiability of such functions.

Introduction. In this article, we extend Morrey's embedding theorem (see for instance [GT, Thm. 7.17]) to Sobolev type functions whose generalized gradients are in a Lorentz space, when the underlying space is a metric-measure space. In fact, Morrey's theorem states that for any function f in the Sobolev space $W^{1,p}(\mathbb{R}^n)$, the following inequality is satisfied:

$$\operatorname*{ess\,sup}_{x,y\in B(a,R)} |f(x)-f(y)| \le CR^{1-n/p}\|\nabla f\|_{L^p}, \tag{1}$$

whenever $p > n$, for some constant C which depends on n and p. We state and prove an inequality, similar to (1), controlling the value of $\operatorname{ess\,sup}_{x,y\in B(a,R)} |f(x)-f(y)|$ by the Lorentz norm of the (generalized) gradient of f, for a real-valued function f whose domain is a metric-measure space (see Theorem 2.1). We use the definition of Sobolev spaces as in [H], in order to define the (generalized) gradient for functions over abstract spaces. Then we study continuity and differentiability of such functions. Our results sharpen those of [MS] for weighted Sobolev spaces and extend the main results of [KKM] to metric-measure spaces. See also [AC], [DS], [S], [Cc], [CL] and [Ca] for other work on this subject.

1. Preliminaries. First, we recall some basic definitions relating to metric-measure spaces, that is, metric spaces (X,d) with a Radon (outer)

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measure μ. We denote by $B(x,r)$ the open ball of radius $r>0$ with center at x in a metric space (X,d). For the basic concepts of metric-measure spaces, see for instance [He] and the references therein.

DEFINITION 1.1. Let (X,d,μ) be a metric-measure space and let Ω be a (measurable) subset of X. Then Ω is also a metric-measure space with the induced metric and measure. We say that Ω is *Bishop–Gromov regular* of dimension $n>0$ if

$$\frac{\mu(B(x,s)\cap\Omega)}{s^n}\le C_0\,\frac{\mu(B(y,r)\cap\Omega)}{r^n}$$

for all $0<r<2s$, $x,y\in\Omega$ and $d(x,y)\le s$, where $C_0\ge 1$ is a constant.

We say that Ω is *doubling* if there is a constant $C'>0$ such that

$$\mu(B(x,2r)\cap\Omega)\le C'\mu(B(x,r)\cap\Omega)$$

for all $x\in\Omega$ and $r>0$.

It is easy to see that if Ω is doubling, then it is Bishop–Gromov regular of dimension n for some $n>0$. Also, if Ω is doubling and μ is not identically zero on Ω, then $\mu(B(x,r)\cap\Omega)>0$ for all $x\in\Omega$ and $r>0$.

Now, we recall the definition of Lorentz spaces. For their basic properties, see for instance [SW], [BS] and [KK]; see also [CRS] and the references therein.

DEFINITION 1.2. Let (X,d,μ) be a metric-measure space and let Ω be a (measurable) subset of X. Let $u:X\to\mathbb{R}$ be a measurable function and let m be a positive number. We say that u belongs to the *Lorentz space* $\mathcal{L}^{m,1}(\Omega)$ if

$$\|u\|_{\mathcal{L}^{m,1}(\Omega)}:=\int_0^\infty \mu(\{z\in\Omega:|u(z)|>t\})^{1/m}\,dt<\infty.$$

Next, we state the following simple fact about connected doubling metric-measure spaces (cf. Remark 2.2).

PROPOSITION 1.3. *Let (X,d,μ) be a metric-measure space and assume that the ball $B(a,R)$ is a connected doubling subset of X. Suppose that μ is not identically zero on $B(a,R)$ and there exists $R^-\in\,]2R/3,R[$ such that $B(a,R^-)\ne B(a,R)$. Then*

$$0<\mu(B(x,r)\cap B(a,R))<\mu(B(x,r+t)\cap B(a,R))<\infty \tag{2}$$

for all $x\in B(a,R)$, $r\in\,]0,R/4]$ and $t>0$.

Proof. Since $B(a,R)$ is doubling and μ is not identically zero on $B(a,R)$, we know that $\mu(B(y,s)\cap B(a,R))>0$ for all $y\in B(a,R)$ and $s>0$. First, assume $R/4<d(x,a)<R$. Since the distance function is continuous and

$B(a,R)$ is connected, there exist $z \in X$ and $s > 0$ such that

$$B(z,s) \subset [B(x,r+t) \setminus B(x,r)] \cap B(a,R)$$

for any $r \in]0, R/4]$ and $0 < t < d(x,a) - R/4$. So, the condition (2) holds.

Also, when $d(x,a) \le R/4$, by a similar method (and using the condition $B(a,R^-) \ne B(a,R)$), we can obtain (2). ∎

Now, we recall the concept of geodesic spaces. Let (X,d) be a metric space. We say that X is a *geodesic space* if for any $x,y \in X$, there exists a *geodesic* $\gamma_{x,y}$ between x and y, i.e. there is an isometry $\gamma_{x,y} : [0, d(x,y)] \to X$ such that $\gamma(0) = x$ and $\gamma(d(x,y)) = y$.

PROPOSITION 1.4. *Let (X,d,μ) be a metric-measure space and let $B(a,R)$ be a ball in X. Suppose that X is geodesic and Bishop–Gromov regular of dimension n, i.e.*

$$\frac{\mu(B(x,s))}{s^n} \le C_0 \frac{\mu(B(y,r))}{r^n}$$

for all $0 < r < 2s$, $x,y \in X$ and $d(x,y) \le s$, where $C_0 \ge 1$ is a constant. Then $B(a,R)$ is Bishop–Gromov regular of dimension n, more precisely,

$$\frac{\mu(B(x,s) \cap B(a,R))}{s^n} \le C \frac{\mu(B(y,r) \cap B(a,R))}{r^n}$$

for all $0 < r < 2s$, $x,y \in B(a,R)$ and $d(x,y) \le s$, where C is a constant which depends on C_0 and n, but is independent of a and R.

Proof. Pick $x,y \in B(a,R)$ and $r,s \in \mathbb{R}$ with $0 < r < 2s$ and $d(x,y) \le s$. It is clear that if $B(y,r) \subset B(a,R)$, we immediately obtain the desired result. Also, without loss of generality, we may assume $r \le R/2$. Since X is geodesic, by considering a geodesic $\gamma_{a,y}$ between a and y, we see that there exists $z \in B(a,R)$ such that

$$B(z,r/2) \subset B(y,r) \cap B(a,R).$$

Moreover, since X is doubling, we know that

$$\mu(B(y,r)) \le C\, \mu(B(z,r/2)),$$

where C is a constant which depends on C_0 and n. This completes the proof of the proposition. ∎

2. Embedding theorem. In this section, we state and prove an embedding theorem for Sobolev type functions with gradients in the Lorentz space $\mathcal{L}^{n,1}(X)$. We use a Lipschitz type characterization of Sobolev spaces (see [H]) to define the (generalized) gradient for Sobolev type functions over metric-measure spaces.

THEOREM 2.1. *Let (X,d,μ) be a metric-measure space and assume that the ball $B(a,R)$ is Bishop–Gromov regular of dimension n. Suppose that*

$B(a,R)$ is connected, μ is not identically zero on $B(a,R)$ and there exists $R^- \in \,]2R/3, R[$ such that $B(a,R^-) \neq B(a,R)$. Suppose that $f : X \to \mathbb{R}$ is a measurable function satisfying

$$|f(x) - f(y)| \le d(x,y)[g(x) + g(y)]$$

for a.e. $x, y \in X$, where g is a non-negative function in the Lorentz space $\mathcal{L}^{n,1}(X)$. Then

$$|f(x) - f(y)| \le 1000\left(\frac{C_0 R^n}{\mu(B(a,R))}\right)^{1/n} \|g\|_{\mathcal{L}^{n,1}(B(a,R))} \tag{3}$$

for a.e. $x, y \in B(a,R)$, where the constant C_0 is as in Definition 1.1 (*with $\Omega = B(a,R)$*).

Proof. Set $\Omega := B(a,R)$. There is a set $E \subset X$ of measure zero such that

$$|f(z) - f(w)| \le d(z,w)[g(z) + g(w)]$$

for all $z, w \in X \setminus E$. For $i \in \mathbb{Z}$, define

$$F_i := \{z \in \Omega : g(z) \le 2^i\} \setminus E,$$

$$r_i := \min\left\{2R, \left(\mu(\Omega \setminus F_i)\,\frac{C_0 R^n}{\mu(\Omega)}\right)^{1/n}\right\}, \qquad s_i := \begin{cases} r_i & \text{if } r_i \le R/4, \\ 2R & \text{if } r_i > R/4. \end{cases}$$

First, we show that

$$\mu(\Omega \setminus F_i) \le \mu(B(x, r_i) \cap \Omega) \tag{4}$$

for all $i \in \mathbb{Z}$ and $x \in \Omega$. If $r_i = 2R$ or $r_i = 0$, then (4) is obvious; otherwise, $0 < r_i < 2R$, and since Ω is Bishop–Gromov regular, we have

$$\frac{\mu(\Omega)}{R^n} \le C_0\,\frac{\mu(B(x, r_i) \cap \Omega)}{r_i^n}$$

for all $i \in \mathbb{Z}$ and $x \in \Omega$. This implies (4).

Next, we show that for any $x_{k+1} \in F_{k+1}$, whenever $F_k \neq \emptyset$ there exists $x_k \in F_k$ such that $d(x_{k+1}, x_k) < s_k^+$ for any s_k^+ such that $\mu(B(x_{k+1}, s_k) \cap \Omega) < \mu(B(x_{k+1}, s_k^+) \cap \Omega)$; when $s_k = 2R$ define $s_k^+ := s_k = 2R$. This is obvious if $r_k > R/4$, so suppose that $r_k \le R/4$. By contradiction, suppose that $B(x_{k+1}, s_k^+) \cap F_k = \emptyset$. Then $B(x_{k+1}, s_k^+) \cap \Omega \subset \Omega \setminus F_k$, and therefore $\mu(B(x_{k+1}, s_k^+) \cap \Omega) \le \mu(\Omega \setminus F_k)$. Also, by (4), we obtain

$$\mu(B(x_{k+1}, s_k^+) \cap \Omega) \le \mu(\Omega \setminus F_k) \le \mu(B(x_{k+1}, s_k) \cap \Omega) < \mu(B(x_{k+1}, s_k^+) \cap \Omega),$$

a contradiction.

Now, we prove (3). Suppose that $x, y \in \Omega \setminus E$. There are $k_x, k_y \in \mathbb{Z}$ such that $g(x) \le 2^{k_x}$ and $g(y) \le 2^{k_y}$. Define $l := \inf\{i : F_i \neq \emptyset\} \in \mathbb{Z} \cup \{-\infty\}$.

Then there are two sequences $\{x_i\}$ and $\{y_j\}$ such that

$$\begin{cases} x_{k_x} := x, \\ x_i \in F_i, \\ d(x_i, x_{i-1}) < s^+_{i-1} \quad \text{for } k_x \geq i > l, \end{cases}$$

and

$$\begin{cases} y_{k_y} := y, \\ y_j \in F_j, \\ d(y_j, y_{j-1}) < s^+_{j-1} \quad \text{for } k_y \geq j > l. \end{cases}$$

Also, we have

$$|f(z) - f(w)| \leq d(z,w)[g(z) + g(w)] \leq d(z,w)[2^i + 2^i] \leq 2^{i+1}\, d(z,w)$$

for all $z, w \in F_i$. Therefore, we obtain

$$|f(x_i) - f(x_{i-1})| \leq 2^{i+1}\, s^+_{i-1}, \qquad |f(y_j) - f(y_{j-1})| \leq 2^{j+1}\, s^+_{j-1},$$

for all $k_x \geq i > l$ and $k_y \geq j > l$. Without loss of generality, we may assume that $k_x \geq k_y$. Then, for any integer $k_y > m > l$, we get

$$\begin{aligned} |f(x) - f(y)| &\leq \sum_{k_x \geq i > m} |f(x_i) - f(x_{i-1})| + |f(x_m) - f(y)| \\ &\leq \sum_{k_x \geq i > m} |f(x_i) - f(x_{i-1})| + |f(x_m) - f(y_m)| + |f(y_m) - f(y)| \\ &\leq \sum_{k_x \geq i > m} |f(x_i) - f(x_{i-1})| + |f(x_m) - f(y_m)| \\ &\quad + \sum_{k_y \geq j > m} |f(y_j) - f(y_{j-1})| \\ &\leq \sum_{k_x \geq i > m} 2^{i+1} s^+_{i-1} + 2^{m+2}\, R + \sum_{k_y \geq j > m} 2^{j+1} s^+_{j-1} \\ &\leq 2^{m+2}\, R + 2 \sum_{k_x \geq i > m} 2^{i+1} s^+_{i-1} \\ &\leq \left(\frac{C_0 R^n}{\mu(\Omega)} \right)^{1/n} (2^{m+2}\, \mu(\Omega)^{1/n}) + 2 \sum_{k_x \geq i > m} 2^{i+1} s^+_{i-1}. \end{aligned}$$

By Proposition 1.3, we can choose s^+_{i-1} to be any number greater than s_{i-1}, so

$$(5) \qquad |f(x) - f(y)| \leq \left(\frac{C_0 R^n}{\mu(\Omega)} \right)^{1/n} (2^{m+2} \mu(\Omega)^{1/n}) + 4 \sum_{\max\{k_x, k_y\} \geq i > m} 2^{i+1} s_{i-1}$$

for all $x, y \in \Omega \setminus E$ and $\min\{k_x, k_y\} > m > l$.

On the other hand, by definition, we have

$$\sum_{i\in\mathbb{Z}} 2^{i-1}\mu(\Omega\setminus F_i)^{1/n} \le \|g\|_{\mathcal{L}^{n,1}(\Omega)}.$$

Then, if $l=-\infty$, by letting $m\to-\infty$ in (5), we obtain

$$|f(x)-f(y)| \le 0 + 600\left(\frac{C_0R^n}{\mu(\Omega)}\right)^{1/n}\|g\|_{\mathcal{L}^{n,1}(\Omega)}$$

for all $x,y\in\Omega\setminus E$. Also, if $l\ne-\infty$, we know that

$$2^{l-1}\mu(\Omega)^{1/n} \le \int_0^{2^{l-1}} \mu(\{z\in\Omega : g(z)>t\})^{1/n}\,dt \le \|g\|_{\mathcal{L}^{n,1}(\Omega)};$$

then, by choosing $m=l+1$ and (5), we obtain

$$|f(x)-f(y)| \le 1000\left(\frac{C_0R^n}{\mu(\Omega)}\right)^{1/n}\|g\|_{\mathcal{L}^{n,1}(\Omega)}$$

for all $x,y\in\Omega\setminus E$. This completes the proof of the theorem. ■

REMARK 2.2. In Theorem 2.1, we can replace the connectedness assumption and the condition $B(a,R^-)\ne B(a,R)$ by the following (weaker) assumption:

- There exists a constant $K>1$ such that, for any $x\in B(a,R)$ and $r\in\,]0,R/4]$, we have $\mu(B(x,r)\cap B(a,R))<\mu(B(x,Kr)\cap B(a,R))$.

3. Applications. In this section, we apply Theorem 2.1 to study the continuity and differentiability of Sobolev type functions. First, we recall some definitions (see [M1] and [KKM]).

DEFINITION 3.1. Let (X,d,μ) be metric-measure space and let f be a real-valued function on X. We say that f is *essentially m-absolutely continuous*, for some $m>0$, if for any $\varepsilon>0$, there is $\delta>0$ such that for any family $\{B_i\}$ of pairwise disjoint balls in X, we have

$$\sum_i \operatorname*{ess\,sup}_{x,y\in B_i}|f(x)-f(y)|^m<\varepsilon \quad\text{whenever}\quad \sum_i\mu(B_i)<\delta.$$

Also, the *essential m-variation* of f is defined by

$$\mathcal{V}_m(f) := \sup_{\{B_i\}}\Big\{\sum_i \operatorname*{ess\,sup}_{x,y\in B_i}|f(x)-f(y)|^m\Big\},$$

where the supremum is taken over all pairwise disjoint families $\{B_i\}$ of balls in X.

THEOREM 3.2. *Let the notations and assumptions be as in Theorem* 2.1. *Moreover, assume that X is Bishop–Gromov regular of dimension n and geodesic. Then*

(i) *For a.e. $a \in X$, we have*

$$\limsup_{s\to 0} \operatorname*{ess\,sup}_{x,y\in B(a,s)} \frac{|f(x)-f(y)|}{s} < \infty.$$

(ii) *There exists a constant C which depends on C_0 and n* (*as in Definition* 1.1 *with $\Omega = X$*) *such that*

$$\mathcal{V}_n(f) \le C\biggl(\sup_{z,s} \frac{s^n}{\mu(B(z,s))}\biggr) \|g\|^n_{\mathcal{L}^{n,1}(X)}.$$

(iii) *In addition, if $\sup_{z,s} s^n/\mu(B(z,s)) < \infty$, then f is essentially n-absolutely continuous.*

Proof. First, we assume $n > 1$. We have (cf. [M2, Thm. 2.1])

$$\|g\|_{\mathcal{L}^{n,1}(B(a,R))} = \Bigl(\int_0^\infty \Phi'(t)^{1/(1-n)}\,dt\Bigr)^{(n-1)/n}\Bigl(\int_0^\infty \Phi'(t)\mu(\{z \in B(a,R) : g(z) > t\})\,dt\Bigr)^{1/n},$$

where $\Phi(t) := \int_0^t \mu(\{z \in B(a,R) : g(z) > s\})^{(1-n)/n}\,ds$. Then we obtain

$$\|g\|_{\mathcal{L}^{n,1}(B(a,R))} \le L\|g\|^{1/n}_{\mathcal{L}^{n,1}(B(a,R))} = L\Bigl(\int_{B(a,R)} \Phi\circ g\,d\mu\Bigr)^{1/n} < \infty, \tag{6}$$

where $L := \|g\|^{(n-1)/n}_{\mathcal{L}^{n,1}(X)} < \infty$. Also, it is easy to see that (6) is meaningful and valid when $n = 1$ (with $L = 1$ and $\Phi(t) \equiv t$).

Then, by Proposition 1.4, (3) and (6), we have

$$\begin{aligned}\operatorname*{ess\,sup}_{x,y\in B(a,s)} \frac{|f(x)-f(y)|}{s} &\le \frac{1000}{s}\biggl(\frac{Cs^n}{\mu(B(a,s))}\biggr)^{1/n} \|g\|_{\mathcal{L}^{n,1}(B(a,s))}\\ &\le LC\biggl(\frac{1}{\mu(B(a,s))}\int_{B(a,s)} \Phi\circ g\,d\mu\biggr)^{1/n},\end{aligned}$$

where C is a constant which depends on C_0 and n (associated with X). Now, Lebesgue's differentiation theorem (for doubling spaces, see for instance [He, Thm. 1.8]) implies (i). On the other hand, assume that $\{B(a_i,s_i)\}$ is any family of pairwise disjoint balls in X. Again, by Proposition 1.4, (2.1) and (3.1), we get

$$\begin{aligned}\sum_i \operatorname*{ess\,sup}_{x,y\in B(a_i,s_i)} |f(x)-f(y)|^n &\le L^n C^n \sum_i \frac{s_i^n}{\mu(B(a_i,s_i))} \int\limits_{B(a_i,s_i)} \Phi\circ g\,d\mu \\ &\le L^n C^n \Big(\sup_{z,s} \frac{s^n}{\mu(B(z,s))}\Big) \sum_i \int\limits_{B(a_i,s_i)} \Phi\circ g\,d\mu \\ &= L^n\,C^n \Big(\sup_{z,s} \frac{s^n}{\mu(B(z,s))}\Big) \int\limits_{\bigcup_i B(a_i,s_i)} \Phi\circ g\,d\mu.\end{aligned}$$

This completes the proof of theorem. ■

REMARK 3.3. If we replace "ess sup" by "sup" in Definition 3.1 and Theorem 3.2(iii), then the conclusion of Theorem 3.2(iii) remains valid after changing f on a set of measure zero.

Now, by the implications of the Poincaré inequality [HK, Thms. 3.2 and 3.4], properties of the Hardy–Littlewood maximal function [He, Thm. 2.2] (see also [KK, Thm. 1.2.1]), Marcinkiewicz's interpolation theorem for Lorentz spaces [BS, Ch. 4, Thm. 4.13, Rem. 4.15], characterization of the Lorentz space $\mathcal{L}^{n,1}(X)$ by Orlicz spaces [Cc] (see also [CL], [KKM] and [AC]) and properties of A_p-weights [KK, Thm. 5.3.1], we are able to recover and sharpen the results of [MS] for A_p-weighted Sobolev functions. Moreover, Theorem 3.2 extends the main results of [KKM] and [AC, Thm. 1.1] (see also [Cc] and [CL]) for functions whose domain is an abstract metric-measure space (see also [R]). Compare [S], [DS], [M1] and [Ca].

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References

[AC] A. Alberico and A. Cianchi, *Differentiability properties of Orlicz–Sobolev functions*, Ark. Mat. 43 (2005), 1–28.

[BS] C. Bennett and R. Sharpley, *Interpolation of Operators*, Pure Appl. Math. 129, Academic Press, Boston, 1988.

[Ca] A. P. Calderón, *On the differentiability of absolutely continuous functions*, Riv. Mat. Univ. Parma 2 (1951), 203–213.

[Cc] C. P. Calderón, *Lacunary differentiability of functions in* $\mathbb{R}^n$, J. Approx. Theory 40 (1984), 148–154.

[CL] C. P. Calderón and J. Lewis, *Maximal smoothing operators and some Orlicz classes*, Studia Math. 57 (1976), 285–296.

[CRS] M. J. Carro, J. A. Raposo and J. Soria, *Recent developments in the theory of Lorentz spaces and weighted inequalities*, Mem. Amer. Math. Soc. 187 (2007), no. 877.

[DS] R. A. DeVore and R. C. Sharpley, *On the differentiability of functions in* $\mathbb{R}^n$, Proc. Amer. Math. Soc. 91 (1984), 326–328.

[GT] D. Gilbarg and N. S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Classics Math., Springer, Berlin, 2001.

[H] P. Hajłasz, *Sobolev spaces on an arbitrary metric space*, Potential Anal. 5 (1996), 403–415.

[HK] P. Hajłasz and P. Koskela, *Sobolev met Poincaré*, Mem. Amer. Math. Soc. 145 2000, no. 688.

[He] J. Heinonen, *Lectures on Analysis on Metric Spaces*, Springer, New York, 2001.

[KKM] J. Kauhanen, P. Koskela and J. Malý, *On functions with derivatives in a Lorentz space*, Manuscripta Math. 100 (1999), 87–101.

[KK] V. Kokilashvili and M. Krbec, *Weighted Inequalities in Lorentz and Orlicz Spaces*, World Sci., River Edge, NJ, 1991.

[M1] J. Malý, *Absolutely continuous functions of several variables*, J. Math. Anal. Appl. 231 (1999), 492–508.

[M2] —, *Coarea properties of Sobolev functions*, in: Function Spaces, Differential Operators and Nonlinear Analysis (Teistungen, 2001), Birkhäuser, Basel, 2003, 371–381.

[MS] Y. Mizuta and T. Shimomura, *Continuity and differentiability for weighted Sobolev spaces*, Proc. Amer. Math. Soc. 130 (2002), 2985–2994.

[R] A. Ranjbar-Motlagh, *Generalized Stepanov type theorem with applications over metric-measure spaces*, Houston J. Math. 34 (2008), 623–635.

[S] E. M. Stein, *Editor's note: the differentiability of functions in* $\mathbb{R}^n$, Ann. of Math. (2) 113 (1981), 383–385.

[SW] E. M. Stein and G. Weiss, *Introduction to Fourier Analysis on Euclidean Spaces*, Princeton Math. Ser. 32, Princeton Univ. Press, Princeton, 1975.

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