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DERIVED EQUIVALENCE CLASSIFICATION OF WEAKLY SYMMETRIC ALGEBRAS OF DOMESTIC TYPE

BY

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Abstract. We complete the derived equivalence classification of all weakly symmetric algebras of domestic type over an algebraically closed field, by solving the problem of distinguishing standard and nonstandard algebras up to stable equivalence, and hence derived equivalence. As a consequence, a complete stable equivalence classification of weakly symmetric algebras of domestic type is obtained.

1. Introduction and the main results. Throughout the paper by an *algebra* we mean a finite-dimensional associative K-algebra with an identity over a fixed algebraically closed field K, which we will assume to be basic and indecomposable. For an algebra A, we denote by $\operatorname{mod} A$ the category of finite-dimensional right A-modules, and by D the standard duality $\operatorname{Hom}_K(-,K)$ on $\operatorname{mod} A$. An algebra A is called *selfinjective* if $A \cong \mathrm{D}(A)$ in $\operatorname{mod} A$, that is, the projective modules in $\operatorname{mod} A$ are injective. Further, A is called *symmetric* if A and $\mathrm{D}(A)$ are isomorphic as A-A-bimodules. Moreover, A is called *weakly symmetric* if, for any indecomposable projective module P in $\operatorname{mod} A$, the socle $\operatorname{soc}(P)$ and the top $\operatorname{top}(P)$ of P are isomorphic. Two selfinjective algebras A and Λ are *socle equivalent* if the factor algebras $A/\operatorname{soc}(A)$ and $\Lambda/\operatorname{soc}(\Lambda)$ are isomorphic.

For an algebra A, we denote by $\underline{\operatorname{mod}}\, A$ the stable category of $\operatorname{mod} A$ (with respect to projective modules), and by $\mathrm{D}^b(\operatorname{mod} A)$ the derived category of bounded complexes from $\operatorname{mod} A$. Two algebras A and Λ are said to be *stably equivalent* if their stable categories $\underline{\operatorname{mod}}\, A$ and $\underline{\operatorname{mod}}\, \Lambda$ are equivalent. Further, A and Λ are *derived equivalent* if $\mathrm{D}^b(\operatorname{mod} A)$ and $\mathrm{D}^b(\operatorname{mod} \Lambda)$ are equivalent as triangulated categories.

The *Cartan matrix* of an algebra A is $(\dim_K \operatorname{Hom}_A(P_i,P_j))_{1\le i,j\le n}$ for a complete family $P_1,\dots,P_n$ of pairwise nonisomorphic indecomposable projective modules in $\operatorname{mod} A$.

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Since Happel's work [21] interpreting tilting theory in terms of equivalences of derived categories, the machinery of derived equivalences has been of interest to representation-theorists. In [27] Rickard proved his celebrated criterion: two algebras A and Λ are derived equivalent if and only if Λ is the endomorphism algebra of a tilting complex over A. Since lots of interesting properties are preserved by derived equivalences, it is for many purposes important to classify algebras up to derived equivalence, instead of Morita equivalence (equivalence of categories of finite-dimensional modules). For instance, for selfinjective algebras the representation type is an invariant of the derived category (see [25, 28]). Further, derived equivalent selfinjective algebras are stably equivalent [28], and hence have isomorphic stable Auslander–Reiten quivers. It has also been proved in [29] that the class of symmetric algebras is closed under derived equivalences. Moreover, two derived equivalent algebras have the same number of pairwise nonisomorphic simple modules, isomorphic centers, and equivalent Cartan matrices. We also mention that derived equivalences of selfinjective algebras preserve the singularity types of orbit closures of finite-dimensional modules [36].

One of the central problems of modern representation theory is the determination of the derived equivalence classes of selfinjective algebras of tame representation type. Recall that by the remarkable Tame and Wild Theorem of Drozd (see [18, 16]), the class of finite-dimensional algebras over an algebraically closed field K may be divided into two disjoint classes. The first class, called the *tame algebras*, consists of those algebras for which the indecomposable modules occur in each dimension d in a finite number of discrete and a finite number of one-parameter families. The second class is formed by the *wild algebras* whose representation theory encompasses the representation theories of all finite-dimensional algebras over K (see [31, Chapter XIX] for details). Accordingly, we may realistically hope to classify the indecomposable finite-dimensional modules only for the tame algebras. Among the tame algebras we may distinguish the class of *representation-finite algebras* having only a finite number of isomorphic classes of indecomposable modules, for which the representation theory is rather well understood.

A prominent class of tame algebras is formed by the domestic algebras. Recall that an algebra A is called *domestic* if there exist a finite number of $K[x]$-A-bimodules $M_1, \ldots, M_n$ which are left free modules of finite rank over the polynomial algebra $K[x]$ in one variable over K and, for any dimension d, all but finitely many isomorphism classes of indecomposable right A-modules of dimension d are of the form $K[x]/(x-\lambda)^m \otimes M_i$ for some $\lambda \in K$, $m \geq 1$, and $i \in \{1, \ldots, n\}$. Moreover, if n is minimal, then A is called *n-parametric*. We refer to the survey article [33] for known results concerning the classification of tame selfinjective algebras.

In [28] Rickard classifies the derived equivalence classes of Brauer tree algebras (hence, representation-finite blocks of group algebras). The derived equivalence classes of all representation-finite selfinjective algebras have been classified by Asashiba [1]. In [23] Holm classifies the derived equivalence classes of algebras of the dihedral, semidihedral and quaternion type (hence, representation-infinite tame blocks of group algebras), which are tame and symmetric. These derived classifications have been essentially applied by Dugas [19] and Erdmann and Skowroński [20] to establish the periodicity of selfinjective algebras of finite representation type and of algebras of pure quaternion type, respectively.

In this paper, we are concerned with the problem of derived equivalence (respectively, stable equivalence) classification of all representation-infinite tame selfinjective algebras of domestic type. Recall that for domestic algebras, there is a common bound (independent of the fixed dimension) for the number of one-parameter families of indecomposable modules. By a result due to Crawley-Boevey [17], the class of domestic algebras coincides with the class of algebras having only finitely many isomorphism classes of generic modules (infinite-dimensional indecomposable modules of finite length over their endomorphism algebras). By general theory, the classification of isomorphism classes of selfinjective domestic algebras splits into two cases: the *standard algebras*, which admit simply connected Galois coverings, and the remaining *nonstandard algebras*. Standard representation-infinite selfinjective domestic algebras are isomorphic to the orbit algebras $\widehat{B}/G$ of the repetitive categories $\widehat{B}$ of tilted algebras B of Euclidean type with respect to actions of admissible infinite cyclic groups G of automorphisms of $\widehat{B}$ (see [32, 33]). Nonstandard representation-infinite selfinjective domestic algebras are very exceptional weakly symmetric algebras, and are socle and geometric deformations of the corresponding standard weakly symmetric domestic algebras (see [15, 33]). We also stress that in contrast to the representation-finite case, representation-infinite nonstandard selfinjective domestic algebras occur for all algebraically closed fields (of arbitrary characteristic).

The aim of the paper is to provide a complete derived equivalence (respectively, stable equivalence) classification of all representation-infinite domestic weakly symmetric algebras. The Morita equivalence classification of these algebras has been established by the authors in [12, 13, 15]. In particular, it is known that every representation-infinite weakly symmetric domestic algebra with singular Cartan matrix is symmetric and isomorphic to the trivial extension algebra $T(B) = B \ltimes \mathrm{D}(B)$ of a tilted algebra B of Euclidean type. Then it follows from the main results of [4, 5, 6] that the trivial extension algebras $T(C)$ of canonical algebras C of Euclidean types $\widetilde{\mathbb{A}}_{p,q}$ $(1 \le p \le q)$, $\widetilde{\mathbb{D}}_n$ $(n \ge 4)$, $\widetilde{\mathbb{E}}_6$, $\widetilde{\mathbb{E}}_7$, $\widetilde{\mathbb{E}}_8$ form a complete set of representatives of pairwise different derived equivalence (respectively, stable equivalence) classes of representation-infinite weakly symmetric domestic algebras with singular Cartan matrices. Hence, it remains to investigate the derived equivalence (respectively, stable equivalence) classes of the representation-infinite weakly symmetric domestic algebras with nonsingular Cartan matrices. These algebras are one-parametric selfinjective algebras (equivalently, admit exactly one generic module up to isomorphism), and consequently their stable Auslander–Reiten quiver consists of one component of Euclidean type and one $\mathbb{P}_1(K)$-family of stable tubes. We note that there are one-parametric selfinjective algebras which are not weakly symmetric (see [14] for the Morita equivalence classification of these algebras).

In Section 2 we define (by quivers and relations) the following families of one-parametric selfinjective algebras:

- $A(\lambda)$, $\lambda \in K \setminus \{0\}$;
- $A(p,q,\lambda)$, $1 \le p \le q$, $p+q \ge 3$, $\lambda \in K \setminus \{0\}$;
- $\Lambda(n)$, $n \ge 2$;
- $\Gamma(n)$, $n \ge 1$;
- $\Omega(n)$, $n \ge 1$.

The algebras $\Omega(n)$, $n \ge 1$, form a complete family of pairwise nonisomorphic nonstandard representation-infinite domestic algebras. Moreover, these algebras are symmetric only for K of characteristic 2. The following theorem is the main result of this paper.

THEOREM 1.1. *The algebras $A(\lambda)$, $A(p,q,\lambda)$, $\Lambda(n)$, $\Gamma(n)$ and $\Omega(n)$ form a complete set of representatives of derived equivalence* (*respectively, stable equivalence*) *classes of indecomposable representation-infinite weakly symmetric domestic algebras with nonsingular Cartan matrices.*

The above theorem together with the main result of [10] provides a complete derived equivalence (respectively, stable equivalence) classification of all one-parametric selfinjective algebras. The derived equivalence (respectively, stable equivalence) classification of standard (respectively, nonstandard) representation-infinite weakly symmetric domestic algebras has been established in our joint paper with Holm [9] (respectively, [11]). Hence, in order to prove the above theorem, we solve the subtle problem of distinguishing standard and nonstandard representation-infinite weakly symmetric domestic algebras up to derived equivalence (respectively, stable equivalence). In the symmetric case, the distinguishing of derived equivalence classes of these algebras was done in [24] by using Külshammer ideals of the centers of algebras, which for the symmetric algebras over algebraically closed fields of positive characteristic have been shown by Zimmermann [37] to be invariants of derived equivalences. We also mention that the weak symmetry of algebras is not known to be an invariant of derived equivalences (respectively, stable equivalences).

The following result is the crucial ingredient in our proof of Theorem 1.1.

THEOREM 1.2. *Let A be a nonstandard representation-infinite domestic selfinjective algebra and A' a selfinjective algebra socle equivalent but not isomorphic to A. Then A and A' are not stably equivalent, and hence are not derived equivalent.*

The following corollary follows from Theorem 1.1 and preservation of the domestic type of selfinjective algebras by derived and stable equivalences (due to results of [25] and [28]).

COROLLARY 1.3. *Let A be a nonstandard representation-infinite domestic selfinjective algebra and Λ a standard selfinjective algebra. Then A and Λ are not stably equivalent, and hence are not derived equivalent.*

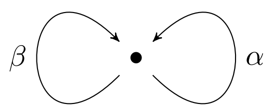
For basic background on the representation theory applied here we refer to the books [3, 7, 22, 30, 31, 35] and the survey article [33].

The main results of this paper were presented by the first named author during the conference Advances in Representation Theory of Algebras held in Montreal in June 2014.

2. Derived normal forms of one-parametric selfinjective algebras. In this section we present representatives of derived equivalences of one-parametric selfinjective algebras and describe their stable Auslander–Reiten quivers. Recall that the stable Auslander–Reiten quiver Γ_A^s of a selfinjective algebra A is obtained from its Auslander–Reiten quiver Γ_A by removing the projective vertices and the arrows attached to them. We observe that if two selfinjective algebras A and Λ are stably equivalent then the quivers Γ_A^s and Γ_Λ^s are isomorphic. Since all algebras considered in this paper are assumed to be basic, they can be presented by quivers and relations. Consider the families of algebras given by the following quivers and relations, occurring in Theorem 1.1.

The algebras $A(\lambda)$.

$A(\lambda)$:
$\lambda \in K \setminus \{0\}$

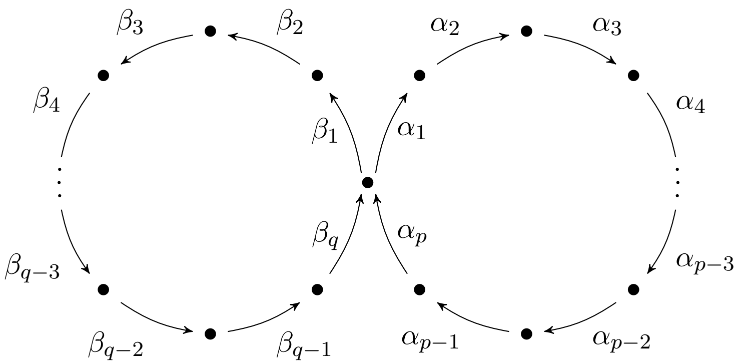


$$\alpha^2 = 0,\ \beta^2 = 0,\ \alpha\beta = \lambda\beta\alpha.$$

The algebra $A(\lambda)$ is standard weakly symmetric; its stable Auslander–Reiten quiver $\Gamma_{A(\lambda)}^s$ consists of a Euclidean component $\mathbb{Z}\widetilde{\mathbb{A}}_1$ and a $\mathbb{P}_1(k)$-family of stable tubes of rank 1. Moreover, $A(\lambda)$ is symmetric if and only if $\lambda = 1$ (see [12] for details).

The algebras $A(p,q,\lambda)$.

$A(p,q,\lambda)$:
$\lambda \in K \setminus \{0\}$
$1 \le p \le q$
$p+q \ge 3$

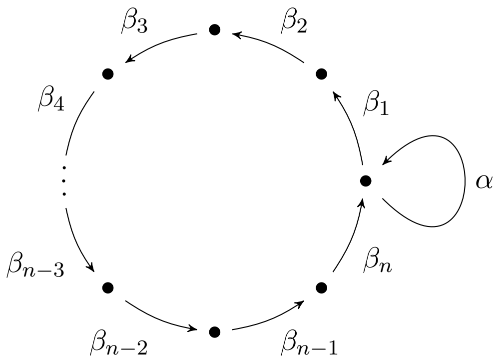


$$\alpha_1 \dots \alpha_p \beta_1 \dots \beta_q = \lambda \beta_1 \dots \beta_q \alpha_1 \dots \alpha_p,$$
$$\alpha_p \alpha_1 = 0,\ \beta_q \beta_1 = 0,$$
$$\alpha_i \dots \alpha_p \beta_1 \dots \beta_q \alpha_1 \dots \alpha_i = 0,\ 2 \le i \le p,$$
$$\beta_j \dots \beta_q \alpha_1 \dots \alpha_p \beta_1 \dots \beta_j = 0,\ 2 \le j \le q.$$

The algebra $A(p,q,\lambda)$ is standard weakly symmetric; the quiver $\Gamma^s_{A(p,q,\lambda)}$ consists of a Euclidean component of type $\mathbb{Z}\widetilde{\mathbb{A}}_{2n-1}$ ($n = p+q-1$, $2n-1 = 2(p+q)-3$) and a $\mathbb{P}_1(k)$-family of stable tubes of tubular type $(2p-1, 2q-1)$. Moreover, $A(p,q,\lambda)$ is symmetric if and only if $\lambda = 1$ (see [12] for details).

The algebras $\Lambda(n)$.

$\Lambda(n)$:
$n \ge 2$

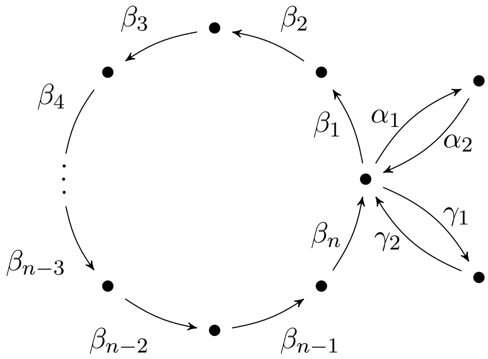


$$\alpha^2 = (\beta_1 \dots \beta_n)^2,\ \alpha\beta_1 = 0,\ \beta_n \alpha = 0,$$
$$\beta_j \dots \beta_n \beta_1 \dots \beta_n \beta_1 \dots \beta_j = 0,\ 2 \le j \le n.$$

The algebra $\Lambda(n)$ is standard symmetric; the quiver $\Gamma^s_{\Lambda(n)}$ consists of a Euclidean component of type $\mathbb{Z}\widetilde{\mathbb{A}}_{2n-1}$ and a $\mathbb{P}_1(k)$-family of stable tubes of tubular type (n,n) (see [12] for details).

The algebras $\Gamma(n)$.

$\Gamma(n)$:
$n \geq 1$

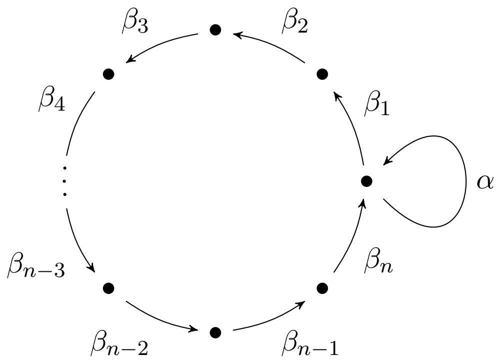


$$\alpha_1\alpha_2 = (\beta_1 \dots \beta_n)^2 = \gamma_1\gamma_2,$$
$$\alpha_2\beta_1 = 0,\ \gamma_2\beta_1 = 0,\ \beta_n\alpha_1 = 0,$$
$$\beta_n\gamma_1 = 0,\ \alpha_2\gamma_1 = 0,\ \gamma_2\alpha_1 = 0,$$
$$\beta_j \dots \beta_n\beta_1 \dots \beta_n\beta_1 \dots \beta_j = 0,\ 2 \leq j \leq n.$$

The algebra $\Gamma(n)$ is standard symmetric; the quiver $\Gamma^s_{\Gamma(n)}$ consists of a Euclidean component of type $\mathbb{Z}\widetilde{\mathbb{D}}_{2n+3}$ and a $\mathbb{P}_1(k)$-family of stable tubes of tubular type $(2, 2, 2n + 1)$ (see [13] for details).

The algebras $\Omega(n)$.

$\Omega(n)$:
$n \geq 1$



$$\alpha^2 = \alpha\beta_1 \dots \beta_n,\ \alpha\beta_1 \dots \beta_n + \beta_1 \dots \beta_n\alpha = 0,$$
$$\beta_n\beta_1 = 0,\ \beta_j \dots \beta_n\alpha\beta_1 \dots \beta_j = 0,\ 2 \leq j \leq n.$$

The algebra $\Omega(n)$ is nonstandard weakly symmetric; the quiver $\Gamma^s_{\Omega(n)}$ consists of a Euclidean component of type $\mathbb{Z}\widetilde{\mathbb{A}}_{2n-1}$ and a $\mathbb{P}_1(k)$-family of stable tubes of tubular type $2n - 1$ (see [15] for details).

We have the following theorems proved in [9] and [11], respectively.

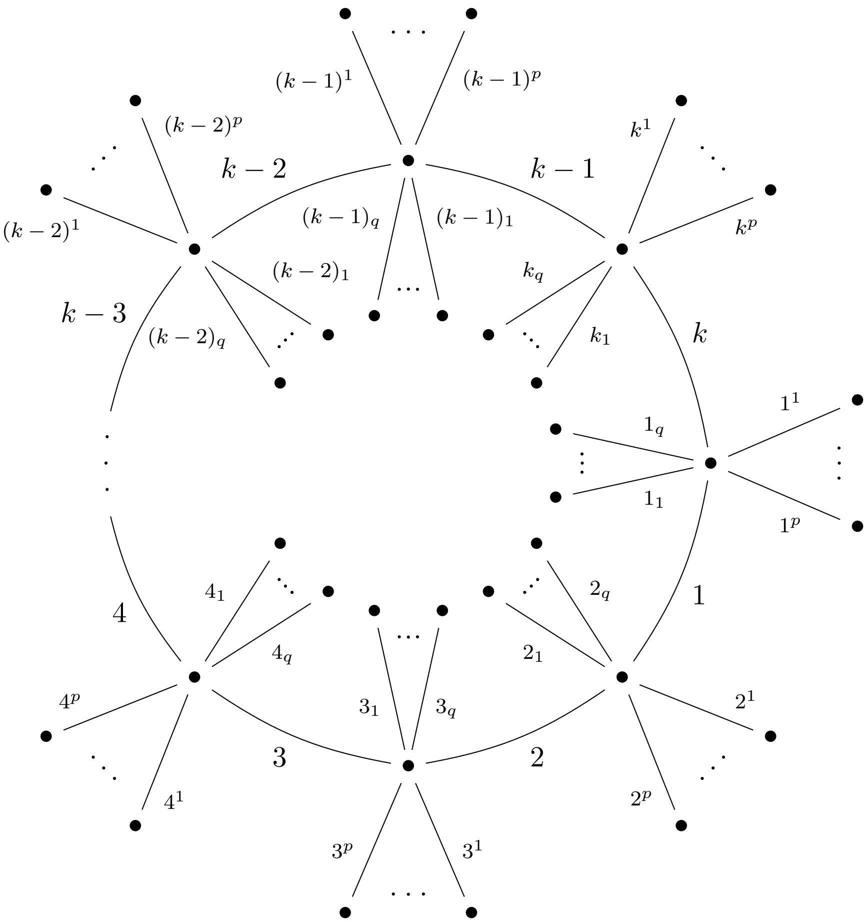
THEOREM 2.1. *Every standard weakly symmetric representation-infinite domestic algebra A with nonsingular Cartan matrix is derived equivalent* (*respectively, stably equivalent*) *to an algebra of the form $A(\lambda)$, $A(p,q,\lambda)$, $\Lambda(n)$, or $\Gamma(n)$.*

THEOREM 2.2. *Every nonstandard representation-infinite selfinjective algebra A is derived equivalent* (*respectively, stably equivalent*) *to an algebra of the form $\Omega(n)$. Moreover, $\Omega(m)$ and $\Omega(n)$ are derived equivalent* (*respectively, stably equivalent*) *if and only if $m = n$.*

We also need the following families of one-parametric but not weakly symmetric algebras.

The algebras $\Lambda(p,q,k,s,\theta)$. For the Brauer graph $\mathcal{T}_{p,q,k}$ of the form

$\mathcal{T}_{p,q,k}$:

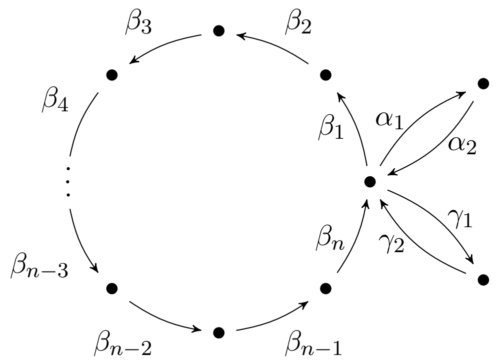


with $p, q \geq 0$ and $k \geq 2$, a positive integer s with $1 \leq s \leq k-1$, $\gcd(s+2,k) = 1$, $\gcd(s,k) = 1$ and $\theta \in K\backslash\{0\}$, we have a standard one-parametric selfinjective not weakly symmetric algebra $\Lambda(p,q,k,s,\theta)$ whose stable Auslander–

Reiten quiver $\Gamma^s_{\Lambda(p,q,k,s,\theta)}$ consists of a Euclidean component of type $\mathbb{Z}\widetilde{\mathbb{A}}_n$ ($n = 2(p+q+1)k-1$) and a $\mathbb{P}_1(k)$-family of stable tubes of tubular type $((2p+1)k,(2q+1)k)$ (see [10] and [14] for details).

The algebras $\Gamma^*(n)$.

$\Gamma^*(n)$:
$n \geq 1$



$$\alpha_1\alpha_2 = (\beta_1 \ldots \beta_n)^2 = \gamma_1\gamma_2,$$
$$\alpha_2\beta_1 = 0,\ \gamma_2\beta_1 = 0,\ \beta_n\alpha_1 = 0,$$
$$\beta_n\gamma_1 = 0,\ \alpha_2\alpha_1 = 0,\ \gamma_2\gamma_1 = 0,$$
$$\beta_j \ldots \beta_n\beta_1 \ldots \beta_n\beta_1 \ldots \beta_j = 0,\ 2 \leq j \leq n.$$

The algebra $\Gamma^*(n)$ is standard, one-parametric, selfinjective and not weakly symmetric, and its stable Auslander–Reiten quiver $\Gamma^s_{\Gamma^*(n)}$ consists of a Euclidean component of type $\mathbb{Z}\widetilde{\mathbb{D}}_{2n+3}$ and a $\mathbb{P}_1(k)$-family of stable tubes of tubular type $(2,2,2n+1)$ (see [9] and [10] for details).

The following theorem has been proved in [10].

THEOREM 2.3. *Every one-parametric selfinjective, not weakly symmetric, domestic algebra is derived equivalent* (*respectively, stably equivalent*) *to an algebra of the form* $\Lambda(p,q,k,s,\theta)$ *or* $\Gamma^*(n)$.

Moreover, we have the following direct consequence of the above theorems and description of the stable Auslander–Reiten quivers of the algebras $A(\lambda)$, $A(p,q,\lambda)$, $\Lambda(n)$, $\Gamma(n)$, $\Omega(n)$, $\Lambda(p,q,k,s,\theta)$, $\Gamma^*(n)$.

COROLLARY 2.4. *Let A and Λ be derived equivalent* (*respectively, stably equivalent*) *one-parametric selfinjective algebras, and assume that A is weakly symmetric. Then Λ is also weakly symmetric.*

We also obtain the following fact.

COROLLARY 2.5. *For any positive integer n, the algebra $\Omega(n)$ is not derived equivalent* (*respectively, stably equivalent*) *to an algebra of the form $A(p,q,\lambda)$ with $p \geq 2$, $\Lambda(n)$, $\Gamma(n)$, $\Lambda(p,q,k,s,\theta)$, $\Gamma^*(n)$.*

3. Proof of Theorem 1.2. Let A be a selfinjective algebra. We denote by Ω_A the *syzygy operator* on $\mathrm{mod}\,A$ which assigns to a module M in $\mathrm{mod}\,A$ the kernel $\Omega_A(M)$ of a minimal projective cover $P_A(M) \to M$ of M in $\mathrm{mod}\,A$. Then it induces a selfequivalence $\Omega_A : \underline{\mathrm{mod}}\,A \to \underline{\mathrm{mod}}\,A$ of the stable module category, and its inverse is the shift of a triangulated structure on $\underline{\mathrm{mod}}\,A$ (see [22]). We shall apply the following consequence of [7, Proposition X.1.12].

PROPOSITION 3.1. *Let $F : \underline{\mathrm{mod}}\,A \to \underline{\mathrm{mod}}\,\Lambda$ be a stable equivalence of selfinjective algebras A and Λ of Loewy length of at least* 3. *Then for any nonprojective indecomposable module M in* $\mathrm{mod}\,A$ *we have $F(\Omega_A(M)) \cong \Omega_\Lambda(F(M))$.*

We now describe the selfinjective algebras socle equivalent to the nonstandard, representation-infinite, domestic, weakly symmetric algebras $\Omega(n)$, $n \geq 1$. Consider the algebras

$$\Omega'(1,\lambda) = A(\lambda), \qquad \Omega'(n,\lambda) = A(1,n,\lambda) \quad \text{for } n \geq 2.$$

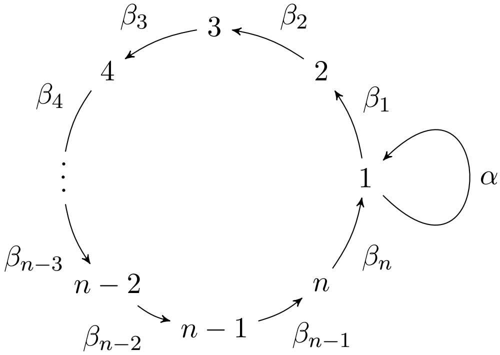
The following fact was proved in [15, Proposition 5.1].

LEMMA 3.2. *For any positive integer n and $\lambda \in K \setminus \{0\}$, the algebras $\Omega(n)$ and $\Omega'(n,\lambda)$ are socle equivalent but not isomorphic.*

The following proposition is a direct consequence of Theorems 2.1, 2.2 and the main theorem of [15].

PROPOSITION 3.3. *Let A be a nonstandard representation-infinite domestic selfinjective algebra and A' a selfinjective algebra socle equivalent but nonisomorphic to A. Then there is a positive integer n such that A is derived equivalent* (*respectively, stably equivalent*) *to $\Omega(n)$ and A' is derived equivalent* (*respectively, stably equivalent*) *to $\Omega'(n,\lambda)$ for some $\lambda \in K \setminus \{0\}$.*

The algebras $\Omega(n)$ and $\Omega'(n,\lambda)$, for $n \geq 1$, $\lambda \in K \setminus \{0\}$, are given by the same quiver $Q(n)$ of the form



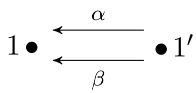
and slightly different relations:

$$\begin{aligned}
\Omega(n) : \alpha^2 &= \alpha\beta_1 \dots \beta_n,\ \alpha\beta_1 \dots \beta_n = -\beta_1 \dots \beta_n \alpha, \\
&\beta_n\beta_1 = 0,\ \beta_j \dots \beta_n \alpha \beta_1 \dots \beta_j = 0, \quad 2 \le j \le n, \\
\Omega'(n,\lambda) : \alpha^2 &= 0,\ \alpha\beta_1 \dots \beta_n = \lambda\beta_1 \dots \beta_n \alpha, \\
&\beta_n\beta_1 = 0,\ \beta_j \dots \beta_n \alpha \beta_1 \beta_2 \dots \beta_j = 0, \quad 2 \le j \le n.
\end{aligned}$$

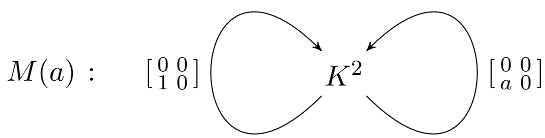
We shall identify $\operatorname{mod} \Omega(n)$ (respectively, $\operatorname{mod} \Omega'(n,\lambda)$) with the full subcategory of the category $\operatorname{rep}_K(Q(n))$ of finite-dimensional representations of the quiver $Q(n)$ over K satisfying the relations defining $\Omega(n)$ (respectively, $\Omega'(n,\lambda)$). We also mention that the algebras $\Omega'(n,\lambda)$, for $n \ge 1$ and $\lambda \in K \setminus \{0\}$, are special biserial in the sense of [34]. On the other hand, the algebras $\Omega(n)$, $n \ge 1$, are biserial but not special biserial, and hence do not admit a simply connected Galois covering (see [26]).

Proposition 3.4. *The algebra $\Omega(1)$ is not stably equivalent to $\Omega'(1,\lambda)$ for $\lambda \in K \setminus \{0\}$.*

Proof. We note first that $\Omega'(1,1) = A(1)$ is the commutative orbit algebra $\widehat{B}/(\varphi)$, where B is the path algebra of $K\Delta$ of the Kronecker quiver Δ of the form



and φ is a canonical automorphism of the repetitive category $\widehat{B}$ of B with $\varphi^2 = \nu_{\widehat{B}}$ (the Nakayama automorphism of $\widehat{B}$). Then the indecomposable modules in $\operatorname{mod} A(1)$ are the images of the indecomposable modules in $\operatorname{mod} \widehat{B}$ by the push-down functor $F_\lambda : \operatorname{mod} \widehat{B} \to \operatorname{mod} A(1)$ associated to the Galois covering functor $F : \widehat{B} \to \widehat{B}/(\varphi) = A(1)$, and the Auslander–Reiten quiver $\Gamma_{A(1)}$ is the orbit quiver $\Gamma_{\widehat{B}}/(\varphi)$ (see [2, 4, 32]). In particular, the stable Auslander–Reiten quiver $\Gamma^s_{A(1)}$ consists of one Euclidean component of type $\mathbb{Z}\widetilde{\mathbb{A}}_1$ and a $\mathbb{P}_1(K)$-family $\mathcal{T}(a)$, $a \in \mathbb{P}_1(K) = K \cup \{0\}$, of stable tubes of rank 1. Moreover, the module $M(a)$ lying on the mouth of $\mathcal{T}(a)$ is the push-down $F_\lambda(E(a))$ of the corresponding module lying on the mouth of the stable tube $\mathcal{T}^B_a$ of rank 1 in Γ_B (see [30, Section XI.4.3]), and is given by the following representation of the quiver $Q(1)$:



for $a \in K$, and

$$M(\infty) : \quad \begin{bmatrix}0&0\\0&0\end{bmatrix} \circlearrowright K^2 \circlearrowleft \begin{bmatrix}0&0\\1&0\end{bmatrix}$$

Fix $\lambda \in K \setminus \{0\}$. Since $\Omega'(1,\lambda) = A(\lambda)$ and $\Omega'(1,1) = A(1)$ are socle equivalent, we conclude that the stable Auslander–Reiten quivers $\Gamma^s_{A(\lambda)}$ and $\Gamma^s_{A(1)}$ are isomorphic. In particular, $\Gamma^s_{A(\lambda)}$ consists of one Euclidean component of type $\mathbb{Z}\widetilde{\mathbb{A}}_1$ and the $\mathbb{P}_1(K)$-family $\mathcal{T}(a)$, $a \in \mathbb{P}_1(K)$, of stable tubes of rank 1, described above. Therefore, the syzygy automorphism $\Omega_{A(\lambda)} : \underline{\operatorname{mod}}\, A(\lambda) \to \underline{\operatorname{mod}}\, A(\lambda)$ acts on the isomorphism classes of the modules $M(a)$, $a \in \mathbb{P}_1(K)$, lying on the mouth of the tubes $\mathcal{T}(a)$, $a \in \mathbb{P}_1(K)$. We describe this action explicitly. The unique indecomposable projective module $A(\lambda)$ in $\operatorname{mod} A(\lambda)$ is given by the following representation of $Q(1)$:

$$A(\lambda) : \quad \begin{bmatrix}0&0&0&0\\0&0&0&0\\1&0&0&0\\0&\lambda&0&0\end{bmatrix} \circlearrowright \bullet \circlearrowleft \begin{bmatrix}0&0&0&0\\1&0&0&0\\0&0&0&0\\0&0&1&0\end{bmatrix}$$

Let $a \in K$. Then a direct check shows that we have in $\operatorname{rep}_K(Q(1))$, and hence in $\operatorname{mod} A(\lambda)$, the exact sequence of representations

$$0 \to M\Big(-\frac{1}{\lambda}a\Big) \xrightarrow{f} A(\lambda) \xrightarrow{g} M(a) \to 0,$$

where $f : K^2 \to K^4$ is given by the matrix

$$\begin{bmatrix}0&0\\1&0\\-a&0\\0&\lambda\end{bmatrix}$$

and $g : K^4 \to K^2$ is given by the matrix

$$\begin{bmatrix}1&0&0&0\\0&a&1&0\end{bmatrix}.$$

Therefore, $\Omega_{A(\lambda)}(M(a)) \cong M\big(-\frac{1}{\lambda}a\big)$ for any $a \in K$. In particular, we obtain $\Omega_{A(\lambda)}(M(0)) \cong M(0)$. Observe that then also $\Omega_{A(\lambda)}(M(\infty)) \cong M(\infty)$.

Since the algebras $\Omega(1)$ and $A(1)$ are socle equivalent, we conclude that the quivers $\Gamma^s_{\Omega(1)}$ and $\Gamma^s_{A(1)}$ are isomorphic, and consequently $\Gamma^s_{\Omega(1)}$ consists of one Euclidean component of type $\mathbb{Z}\widetilde{\mathbb{A}}_1$ and the $\mathbb{P}_1(K)$-family $\mathcal{T}(a)$, $a \in \mathbb{P}_1(K)$, of stable tubes of rank 1, described above. Hence, the syzygy automorphism $\Omega_{\Omega(1)} : \underline{\operatorname{mod}}\, \Omega(1) \to \underline{\operatorname{mod}}\, \Omega(1)$ acts on the isomorphism classes of the modules $M(a)$, $a \in \mathbb{P}_1(K)$. The unique indecomposable projective module $\Omega(1)$ in $\operatorname{mod}\Omega(1)$ is given by the following representation of $Q(1)$:

$$\Omega(1):\quad \begin{bmatrix}0&0&0&0\\0&0&0&0\\1&0&0&0\\0&-1&0&0\end{bmatrix} \circlearrowright \bullet \circlearrowleft \begin{bmatrix}0&0&0&0\\1&0&0&0\\0&0&0&0\\0&-1&1&0\end{bmatrix}$$

Let $a \in K$. Then a direct check shows that we have in $\operatorname{rep}_K(Q(1))$, and hence in $\operatorname{mod}\Omega(1)$, the exact sequence of representations

$$0 \to M(a+1) \xrightarrow{u} \Omega(1) \xrightarrow{v} M(a) \to 0,$$

where $u : K^2 \to K^4$ is given by the matrix

$$\begin{bmatrix}0&0\\1&0\\-a&0\\0&-1\end{bmatrix}$$

and $v : K^4 \to K^2$ is given by the matrix

$$\begin{bmatrix}1&0&0&0\\0&a&1&0\end{bmatrix}.$$

Therefore, $\Omega_{\Omega(1)}(M(a)) \cong M(a+1)$ for any $a \in K$. Hence also $\Omega_{\Omega(1)}(M(\infty)) \cong M(\infty)$.

Assume now that $\Omega(1)$ is stably equivalent to $\Omega'(1,\lambda) = A(\lambda)$ for some $\lambda \in K \setminus \{0\}$, and $F : \underline{\operatorname{mod}}\,\Omega(1) \to \underline{\operatorname{mod}}\,A(\lambda)$ is an equivalence functor. Then there are two different elements $a, b \in \mathbb{P}_1(K)$ such that $F(M(a)) \cong M(0)$ and $F(M(b)) \cong M(\infty)$. Applying Proposition 3.1 we obtain the following isomorphisms in $\operatorname{mod} A(\lambda)$:

$$F(M(a)) \cong M(0) \cong \Omega_{A(\lambda)}(M(0)) \cong \Omega_{A(\lambda)}(F(M(a))) \cong F(\Omega_{\Omega(1)}(M(a))),$$
$$F(M(b)) \cong M(\infty) \cong \Omega_{A(\lambda)}(M(\infty)) \cong \Omega_{A(\lambda)}(F(M(b))) \cong F(\Omega_{\Omega(1)}(M(b))),$$

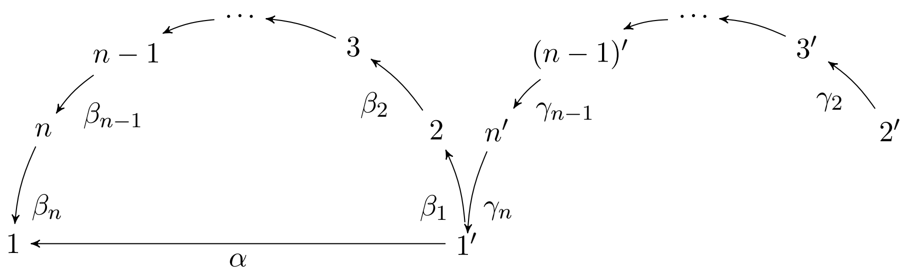
and hence the isomorphisms in $\operatorname{mod}\Omega(1)$:

$$\Omega_{\Omega(1)}(M(a)) \cong M(a) \quad \text{and} \quad \Omega_{\Omega(1)}(M(b)) \cong M(b).$$

This leads to a contradiction, because $\Omega_{\Omega(1)}$ fixes only one isomorphism class of modules $M(a)$, $a \in \mathbb{P}_1(K)$, in $\operatorname{mod}\Omega(1)$, namely the class of $M(\infty)$. Therefore, $\Omega(1)$ is not stably equivalent to $\Omega'(1,\lambda) = A(\lambda)$ for $\lambda \in K \setminus \{0\}$. ■

PROPOSITION 3.5. *Let $n \geq 2$ be a positive integer. The algebra $\Omega(n)$ is not stably equivalent to $\Omega'(n,\lambda)$ for $\lambda \in K \setminus \{0\}$.*

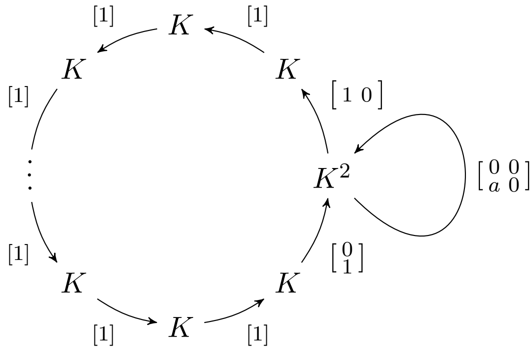
Proof. We first observe that $\Omega'(n,1)$ is a symmetric algebra isomorphic to the orbit algebra $\widehat{B(n)}/(\varphi_n)$, where $B(n)$ is the tilted algebra of Euclidean type $\widetilde{\mathbb{A}}_{2n-1}$ given by the quiver $\Delta(n)$ of the form



and the relation $\gamma_n\beta_1 = 0$, and φ_n is a canonical automorphism of the repetitive category $\widehat{B(n)}$ of $B(n)$ with $\varphi_n^2 = \nu_{\widehat{B(n)}}$. We also note that $B(n)$ is a tubular extension of the hereditary algebra $H(n)$ of Euclidean type $\widetilde{\mathbb{A}}_n$, which is the path algebra of the subquiver of $\Delta(n)$ given by the vertices $1', 2, 3, \ldots, n-1, n, 1$, using the nonsimple module lying on the mouth of the unique stable tube of $\Gamma_{H(n)}$ of rank n. Then the indecomposable modules in $\operatorname{mod} \Omega'(n,1)$ are the images of the indecomposable modules in $\operatorname{mod} \widehat{B(n)}$ by the push-down functor $F_\lambda : \operatorname{mod} \widehat{B(n)} \to \operatorname{mod} \Omega'(n,1)$ associated to the Galois covering functor $F : \widehat{B(n)} \to \widehat{B(n)}/(\varphi_n) = \Omega'(n,1)$, and the Auslander–Reiten quiver $\Gamma_{\Omega'(n,1)}$ of $\Omega'(n,1)$ is the orbit quiver $\Gamma_{\widehat{B(n)}}/(\varphi_n)$ (see [2, 4, 32]). In particular, the stable Auslander–Reiten quiver $\Gamma^s_{\Omega'(n,1)}$ consists of one Euclidean component of type $\mathbb{Z}\widetilde{\mathbb{A}}_{2n-1}$, a stable tube $\mathcal{T}(\infty)$ of rank $2n-1$, and a K-family $\mathcal{T}(a)$, $a \in K$, of stable tubes of rank 1.

Moreover, the module $M(a)$ lying on the mouth of $\mathcal{T}(a)$, $a \in K$, is the push-down $F_\lambda(E(a))$ of $E(a)$ lying on the mouth of the corresponding stable tube $\mathcal{T}_a^{H(n)}$ of $\Gamma_{H(n)}$ of rank 1 (see [30, Proposition XII.2.8]). For $a \in K$, the module $M(a)$ is given by the following representation of the quiver $Q(n)$:

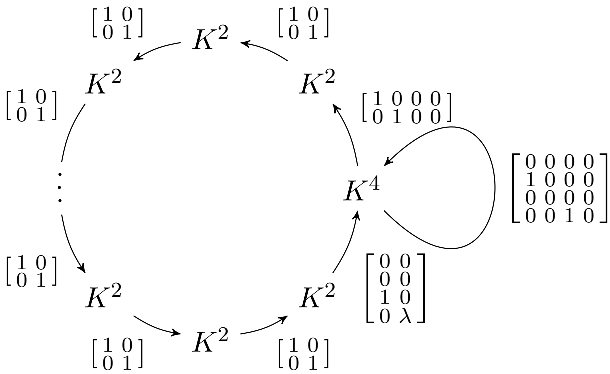
$M(a)$:



We also note that $M(a)$ has simple top and simple socle, both isomorphic to the simple module S_1 at the vertex 1.

Fix $\lambda \in K \setminus \{0\}$. Since $\Omega'(n,\lambda)$ and $\Omega'(n,1)$ are socle equivalent, we conclude that the quivers $\Gamma^s_{\Omega'(n,\lambda)}$ and $\Gamma^s_{\Omega'(n,1)}$ are isomorphic. In particular, $\Gamma^s_{\Omega'(n,\lambda)}$ consists of one Euclidean component of type $\mathbb{Z}\widetilde{\mathbb{A}}_{2n-1}$, a stable tube $\mathcal{T}(\infty)$ of rank $2n-1$, and the K-family $\mathcal{T}(a)$, $a \in K$, of stable tubes of rank 1, with $M(a)$ lying on the mouth of $\mathcal{T}(a)$. Therefore, the syzygy automorphism $\Omega_{\Omega'(n,\lambda)} : \underline{\mathrm{mod}}\, \Omega'(n,\lambda) \to \underline{\mathrm{mod}}\, \Omega'(n,\lambda)$ acts on the isomorphism classes of the modules $M(a)$, $a \in K$. The indecomposable projective module $P_1(n,\lambda)$ in $\mathrm{mod}\, \Omega'(n,\lambda)$ at the vertex 1 is given by the following representation of $Q(n)$:

$P_1(n,\lambda)$:



Let $a \in K$. Then a direct check shows that we have in $\mathrm{rep}_K(Q(n))$, and hence in $\mathrm{mod}\, \Omega'(n,\lambda)$, the exact sequence of representations

$$0 \longrightarrow M\left(-\frac{1}{\lambda}a\right) \xrightarrow{f} P_1(n,\lambda) \xrightarrow{g} M(a) \longrightarrow 0,$$

where $f = (f_i)$ and $g = (g_i)$ are defined as follows:

- $f_1 : K^2 \to K^4$ by the matrix

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ -a & 0 \\ 0 & \lambda \end{bmatrix};$$

- $f_i : K \to K^2$ by the matrix

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

for $i \in \{2, \ldots, n\}$;
- $g_1 : K^4 \to K^2$ by the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 1 & 0 \end{bmatrix};$$

- $g_i : K^2 \to K$ by the matrix

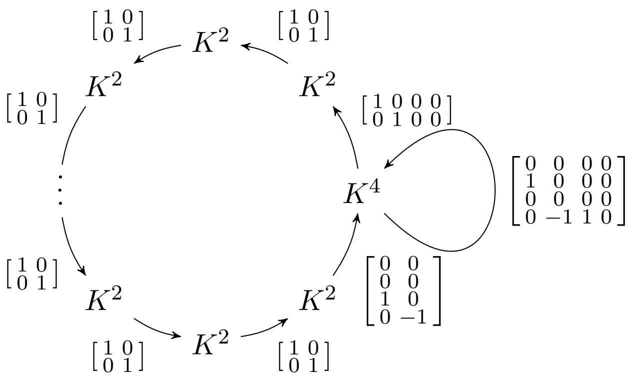
$$\begin{bmatrix} 1 & 0 \end{bmatrix}$$

for $i \in \{2, \ldots, n\}$.

Hence, $\Omega_{\Omega'(n,\lambda)}(M(a)) \cong M\big(-\frac{1}{\lambda}a\big)$ for any $a \in K$. In particular, we obtain $\Omega_{\Omega'(n,\lambda)}(M(0)) \cong M(0)$.

Since $\Omega(n)$ and $\Omega'(n,\lambda)$ are socle equivalent, the quivers $\Gamma^s_{\Omega(n)}$ and $\Gamma^s_{\Omega'(n,\lambda)}$ are isomorphic, and consequently $\Gamma^s_{\Omega(n)}$ consists of one Euclidean component of type $\mathbb{Z}\widetilde{\mathbb{A}}_{2n-1}$, a stable tube $\mathcal{T}(\infty)$ of rank $2n-1$, and the family $\mathcal{T}(a)$, $a \in K$, of stable tubes of rank 1, with the module $M(a)$ on the mouth of $\mathcal{T}(a)$. The indecomposable projective module $P_1(n)$ in $\operatorname{mod} \Omega(n)$ at the vertex 1 is given by the following representation of $Q(n)$:

$P_1(n)$:



Let $a \in K$. Then in $\operatorname{rep}_K(Q(n))$, and hence in $\operatorname{mod} \Omega(n)$, we have the exact sequence of representations

$$0 \to M(a+1) \xrightarrow{u} P_1(n) \xrightarrow{v} M(a) \to 0,$$

where $u = (u_i)$ and $v = (v_i)$ are defined as follows:

- $u_1 : K^2 \to K^4$ by the matrix

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ -a & 0 \\ 0 & -1 \end{bmatrix};$$

- $u_i : K \to K^2$ by the matrix

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

for $i \in \{2, \ldots, n\}$;

- $v_1 : K^4 \to K^2$ by the matrix
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 1 & 0 \end{bmatrix};$$
- $v_i : K^2 \longrightarrow K$ by the matrix
$$\begin{bmatrix} 1 & 0 \end{bmatrix}$$
for $i \in \{2, \dots, n\}$.

Therefore, $\Omega_{\Omega(n)}(M(a)) \cong M(a+1)$ for any $a \in K$.

Assume now that $\Omega(n)$ is stably equivalent to $\Omega'(n, \lambda)$ for some λ in $K \setminus \{0\}$, and $F : \underline{\mathrm{mod}}\, \Omega(n) \to \underline{\mathrm{mod}}\, \Omega'(n, \lambda)$ is an equivalence functor. Then there exists $a \in K$ such that $F(M(a)) \cong M(0)$ in $\mathrm{mod}\, \Omega'(n, \lambda)$. Applying Proposition 3.1, we obtain isomorphisms
$$\begin{aligned} F(M(a+1)) \cong F\big(\Omega_{\Omega(n)}(M(a))\big) &\cong \Omega_{\Omega'(n,\lambda)}\big(F(M(a))\big) \\ &\cong \Omega_{\Omega'(n,\lambda)}(M(0)) \cong M(0) \cong F(M(a)) \end{aligned}$$
and hence $M(a+1) \cong M(a)$, a contradiction. Therefore, $\Omega(n)$ is not stably equivalent to $\Omega'(n, \lambda)$ with $\lambda \in K \setminus \{0\}$. ∎

Observe now that Theorem 1.2 follows from Propositions 3.3–3.5.

4. Proof of Theorem 1.1. Let A and Λ be (basic, indecomposable) representation-infinite derived equivalent (respectively, stably equivalent) selfinjective algebras, and assume that A is weakly symmetric, domestic, with nonsingular Cartan matrix. Then it follows from Theorems 2.1 and 2.2 that A is derived equivalent, hence stably equivalent, to an algebra of one of the forms $A(\lambda)$, $A(p, q, \lambda)$, $\Lambda(n)$, $\Gamma(n)$, or $\Omega(n)$. In particular, A is a one-parametric selfinjective algebra. Since A and Λ are stably equivalent, it follows from [25] that Λ is also a one-parametric algebra. Moreover, Λ is also weakly symmetric, by Corollary 2.4. Finally, the Cartan matrix of Λ is nonsingular, because the weakly symmetric domestic algebras with singular Cartan matrices are isomorphic to the trivial extension algebras $T(B)$ of tilted algebras B of Euclidean type, which are 2-parametric algebras (see [2, 4]). In particular, applying Theorems 2.1 and 2.2 again, we conclude that Λ is of the form $A(\lambda)$, $A(p, q, \lambda)$, $\Lambda(n)$, $\Gamma(n)$, or $\Omega(n)$.

Observe now that A and Λ are both standard or both nonstandard. Indeed, if this is not the case, say A is nonstandard and Λ is standard, then $A = \Omega(n)$ and $\Lambda = \Omega'(n, \lambda)$ for some $n \geq 1$ and $\lambda \in K \setminus \{0\}$, and they are socle equivalent, by Lemma 3.2 and Proposition 3.3. This contradicts Theorem 1.2. Therefore, Theorem 1.1 follows from Theorems 2.1 and 2.2.

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