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Completely bounded lacunary sets for compact non-abelian groups

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Abstract. In this paper, we introduce and study the notion of completely bounded Λ_p sets ($\Lambda_p^{\rm cb}$ for short) for compact, non-abelian groups G. We characterize $\Lambda_p^{\rm cb}$ sets in terms of completely bounded $L^p(G)$ multipliers. We prove that when G is an infinite product of special unitary groups of arbitrarily large dimension, there are sets consisting of representations of unbounded degree that are Λ_p sets for all $p < \infty$, but are not $\Lambda_p^{\rm cb}$ for any $p \geq 4$. This is done by showing that the space of completely bounded $L^p(G)$ multipliers is a proper subset of the space of $L^p(G)$ multipliers.

1. Introduction. Sidon sets and Λ_p sets on compact abelian groups G have been thoroughly studied for many years. Every Sidon set is a Λ_p set for all $p < \infty$, but the converse is not true if G is an infinite group. Both classes of sets can be characterized in terms of L^p multipliers on G. In [11], Harcharras introduced the notion of completely bounded (non-commutative) Λ_p sets (called $\Lambda_p^{\rm cb}$ sets) for compact abelian groups. These are defined in terms of the canonical operator space structure on $L^p(G)$ obtained using Pisier's operator space complex interpolation. All Sidon sets are $\Lambda_p^{\rm cb}$ and all $\Lambda_p^{\rm cb}$ sets are Λ_p. Both inclusions are proper. The relationship between $\Lambda_p^{\rm cb}$ sets and completely bounded multipliers on $L^p(G)$ was studied by Harcharras and Pisier who showed, for example, that not all L^p multipliers are completely bounded. See [2], [6], [11], [12], [21] for proofs of these various facts.

Sidon and Λ_p sets have also been studied in the context of non-abelian, compact groups; [10] and [14] provide good overviews. In this paper, we introduce the analogous concept of completely bounded Λ_p sets for $2 < p < \infty$, for such groups. These notions are more complicated than for abelian groups as the dual object of a non-abelian group does not have a group structure. As in the abelian case, we show that $\Lambda_p^{\rm cb}$ sets can be characterized in terms

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of completely bounded $L^p(G)$ multipliers. Sidon sets are seen to be Λ_p^{cb} for all p and Λ_p^{cb} sets are always Λ_p.

In contrast to the case of abelian groups, not all infinite, compact, non-abelian groups admit infinite Sidon or even Λ_p sets. An important example of a group which does is an infinite product of special unitary groups. For these groups, we provide examples of sets of representations of unbounded degree that are Λ_p for all $p < \infty$, but are not Λ_p^{cb} for any $p \geq 4$. We do this by constructing an L^p multiplier which is not completely bounded. It would be interesting to know if there are any Λ_p^{cb} sets consisting of representations of unbounded degree that are not Sidon.

2. Preliminaries

2.1. Lacunary sets on compact groups. Let G be a compact group equipped with normalized Haar measure dg and denote by $\widehat{G}$ its dual object, the set of pairwise inequivalent, unitary, irreducible representations of G. For $\sigma \in \widehat{G}$, we let d_σ denote the dimension of the underlying Hilbert space $\mathcal{H}_\sigma$, known as the *degree* of σ. When G is abelian, $\widehat{G}$ is a discrete group consisting of the continuous characters on G.

Given $f \in L^1(G)$ and $\sigma \in \widehat{G}$, the Fourier transform of f at σ is defined as

$$\widehat{f}(\sigma) = \int_G f(x)\sigma(x^{-1})\,dx,$$

$\widehat{f}(\sigma)$ being a matrix of size $d_\sigma \times d_\sigma$. We call f a *trigonometric polynomial* if $\widehat{f}(\sigma) \neq 0$ for only finitely many σ; then we have

$$f(x) = \sum_{\sigma\in\widehat{G}} d_\sigma \operatorname{tr}(\widehat{f}(\sigma)\sigma(x))$$

where tr denotes the usual matrix trace. (Of course, in the abelian case, for each x, $\widehat{f}(\sigma)\sigma(x)$ is a complex number.)

Let $E \subseteq \widehat{G}$. A trigonometric polynomial f is called an *E-polynomial* if $\hat{f}(\sigma) = 0$ for all $\sigma \notin E$.

Definition 2.1.

(1) A set $E \subseteq \widehat{G}$ is called a *Sidon set* if there is a constant C such that

$$\sum_{\sigma\in E} d_\sigma \operatorname{tr}\left[\left(\widehat{f}(\gamma)\widehat{f}(\gamma)^*\right)^{1/2}\right] \leq C\|f\|_\infty$$

for all E-polynomials f.

(2) Let $2 < p < \infty$. A set $E \subseteq \widehat{G}$ is called a *Λ_p set* if there is a constant C_p such that $\|f\|_p \leq C_p\|f\|_2$ for all E-polynomials f.

It is known that all Sidon sets are Λ_p for all $p < \infty$ and the compactness of G ensures that any Λ_p set is also a Λ_q set for any $q < p$. Every infinite abelian group admits an infinite Sidon set, as well as sets that are Λ_p for all $p < \infty$, but not Sidon. Proofs of these facts can be found in the standard references [14] and [16]. It is also known that for each infinite abelian group G and each $p > 2$ there are Λ_p sets that are not Λ_q for any $q > p$. A constructive proof was given by Rudin [24] for G the circle group and even integers p. Using probabilistic arguments, Bourgain [4] proved the general case.

In contrast to the abelian case, there are non-abelian groups G which admit no infinite Sidon or even Λ_p sets. This is true, for instance, if G is a compact, connected, semisimple Lie group such as $SU(2)$ [15]. For the structure of groups which do admit infinite Sidon or Λ_p sets and for examples of both Sidon sets and Λ_p sets that are not Sidon see [5], [8] and [13].

Sidon and Λ_p sets play an important role in the study of multipliers. Given $E \subseteq \widehat{G}$ we denote

$$l^{\infty}(E) = \Big\{\phi = (\phi(\sigma))_{\sigma\in E} \in \prod B(\mathcal{H}_\sigma) : \sup_{\sigma} \|\phi(\sigma)\|_{\mathcal{B}(\mathcal{H}_\sigma)} < \infty\Big\}$$

where $\|\cdot\|_{\mathcal{B}(\mathcal{H}_\sigma)}$ denotes the operator norm.

Definition 2.2. Suppose $\phi \in l^{\infty}(\widehat{G})$. An operator $T_\phi : L^p(G) \to L^p(G)$ defined by

$$\widehat{T_\phi f}(\sigma) = \phi(\sigma)\widehat{f}(\sigma) \quad \forall f \in L^p$$

is said to be a (left) L^p *multiplier* if it is bounded.

We denote the space of (left) L^p multipliers by $M_p(G)$. One can analogously define the space of right L^p multipliers, $M_p^r(G)$. It is well known that $M_1(G) \simeq M(G)$, the space of finite regular Borel measures on G, $M_2(G) \simeq l^{\infty}(\widehat{G})$ and $M_p(G) \simeq M_{p'}^r(G)$ where $1/p + 1/p' = 1$ (isometrically isomorphic in all cases).

A duality argument can be used to show that if G is abelian, then $E \subseteq \widehat{G}$ is Sidon if and only if whenever $\phi \in l^{\infty}(E)$ there exists $\mu \in M(G)$ such that $\widehat{\mu}(\gamma) = \phi(\gamma)$ for all $\gamma \in E$. Thus Sidon sets are interpolation sets for $M(G)$. An application of Khintchine's inequality shows that Λ_p sets are interpolation sets for $M_p(G)$ when $p > 2$ (see [11] for a proof).

In the case of a non-abelian group G, Figà-Talamanca and Rider proved the following analogous result.

Theorem 2.3 ([8]). *Let $E \subseteq \widehat{G}$ and $2 < p < \infty$.*

- (i) *E is Λ_p if and only if whenever $\phi \in l^{\infty}(E)$, then there exists $\beta \in l^{\infty}(\widehat{G})$ such that $\beta(\sigma) = \phi(\sigma)$ for all $\sigma \in E$ and $T_\beta \in M_p(G)$.*
- (ii) *E is Sidon if and only if whenever $\phi \in l^{\infty}(E)$ there exists $\mu \in M(G)$ such that $\widehat{\mu}(\sigma) = \phi(\sigma)$ for all $\sigma \in E$.*

2.2. Operator space structure for $L^p(G)$. For convenience, we will briefly discuss the basic theory of operator spaces. Let $\mathcal{B}(\mathcal{H})$ be the space of all bounded linear operators on $\mathcal{H}$. A closed subspace E of $\mathcal{B}(\mathcal{H})$ is called a *concrete operator space*. Given a concrete operator space $E \subset \mathcal{B}(\mathcal{H})$, let $\mathbb{M}_n(E)$ denote the set of all $n \times n$ matrices with entries in E. The space $\mathbb{M}_n(E)$ is naturally embedded into $\mathcal{B}(\mathcal{H}^n)$ and with the norm inherited from $\mathcal{B}(\mathcal{H}^n)$ is a Banach space.

Ruan [23] defined abstract operator spaces as follows. Let E be a Banach space with a sequence of norms $\|\cdot\|_n$ on $\mathbb{M}_n(E)$ satisfying

(1) $\left\|\begin{array}{c|c} x & 0 \\ \hline 0 & y \end{array}\right\|_{m+n} = \max(\|x\|_m, \|y\|_n)$ and

(2) $\|\alpha x \beta\|_n \le \|\alpha\|\|x\|_n\|\beta\|$ for all $\alpha, \beta \in \mathbb{M}_n(\mathbb{C})$ and $x \in \mathbb{M}_n(E)$.

If $(\mathbb{M}_n(E), \|\cdot\|_n)$ is a Banach space for each n, then E is called an *operator space*. The morphisms in the category of operator spaces are completely bounded maps. Ruan [23] proved that every abstract operator space is a concrete operator space.

We refer to [7] and [22] for more detailed information on operator spaces.

Definition 2.4. Let E_1 and E_2 be operator spaces. A linear map $T: E_1 \to E_2$ is said to be *completely bounded* if the maps $T \otimes I_{\mathbb{M}_n} : \mathbb{M}_n(E_1) \to \mathbb{M}_n(E_2)$ satisfy

$$\|T\|_{\mathrm{cb}(E_1,E_2)} := \sup_{n\ge 1} \|T \otimes I_{\mathbb{M}_n}\|_{\mathcal{B}(\mathbb{M}_n(E_1),\mathbb{M}_n(E_2))} < \infty.$$

We will denote by $CB(E_1, E_2)$ the Banach space of all completely bounded maps from E_1 to E_2 with the norm $\|\cdot\|_{\mathrm{cb}(E_1,E_2)}$ defined above. The dual of the operator space E, denoted E^*, can be defined by taking $\mathbb{M}_n(E^*) = CB(E, \mathbb{M}_n(\mathbb{C}))$.

For any compact group G, $L^\infty(G)$ has a canonical operator space structure being a C^* algebra. Let $L^1(G)$ inherit the operator space structure from the dual space $L^\infty(G)^*$. By [3], $L^1(G)^*$ is completely isomorphic to $L^\infty(G)$. The canonical operator space structure on $L^p(G)$ is the interpolated operator space structure $(L^1, L^\infty)_{1/p}$ as developed by Pisier [20].

For $1 \le p < \infty$, let S_p be the space of compact operators on l^2 with norm

$$\|T\|_{S_p} := (\mathrm{tr}\,|T|^p)^{1/p}$$

where $|T| = (T^*T)^{1/2}$. Denote by $L^p(G, S_p)$ (or $L^p(S_p)$ for short if G is clear) the Banach space of S_p-valued measurable functions f such that

$$\|f\|_{L^p(G,S_p)} := \Big(\int_G \|f(x)\|_{S_p}^p\,dx\Big)^{1/p} < \infty,$$

and by $L_E^p(G, S_p)$ the set of $f \in L^p(G, S_p)$ with $\widehat{f} = 0$ off E.

Pisier's result stated below provides a condition for a bounded map on L^p to be completely bounded.

Proposition 2.5 ([21]). *Let $1 \le p < \infty$. A linear map $T : L^p(G) \to L^p(G)$ is completely bounded if and only if the mapping $T \otimes I_{S_p}$ is bounded on $L^p(G, S_p)$. Moreover,*

$$\|T\|_{\mathrm{cb}(p)} := \|T\|_{\mathrm{cb}(L^p,L^p)} = \|T \otimes I_{S_p}\|_{L^p(S_p)\to L^p(S_p)}.$$

We write $M_p^{\mathrm{cb}}(G)$ for the completely bounded Fourier multipliers on L^p,

$$M_p^{\mathrm{cb}}(G) := \{T \in M_p(G) : \|T\|_{\mathrm{cb}(p)} < \infty\}.$$

When G is a compact group it is known that $M_p^{\mathrm{cb}}(G) = M_p(G)$ if $p = 1, 2$ ([21]). As $M_p^{\mathrm{cb}}(G) \subseteq M_2^{\mathrm{cb}}(G)$, an interpolation argument implies that $M_q^{\mathrm{cb}}(G) \subseteq M_p^{\mathrm{cb}}(G)$ when $q \ge p \ge 2$ ([20]). It was shown in [1], [6] and [21] that when G is abelian, then $M_p^{\mathrm{cb}}(G) \subsetneq M_p(G)$ for $1 < p \ne 2 < \infty$.

2.3. Completely bounded Λ_p-sets. The concept of a completely bounded Λ_p set, denoted Λ_p^{cb}, was introduced in [11] for compact abelian groups.

Definition 2.6. Let $2 < p < \infty$ and G be a compact abelian group. A subset $E \subseteq \widehat{G}$ is called a Λ_p^{cb} *set* if there exists a constant C, depending only on p and E, such that

$$\|f\|_{L^p(G,S_p)} \le C\Big(\Big\|\Big(\sum_{\gamma\in E} \widehat{f}(\gamma)^*\widehat{f}(\gamma)\Big)^{1/2}\Big\|_{S_p} + \Big\|\Big(\sum_{\gamma\in E} \widehat{f}(\gamma)\widehat{f}(\gamma)^*\Big)^{1/2}\Big\|_{S_p}\Big) \tag{2.1}$$

for all S_p-valued E-polynomials f defined on G.

Remark 2.7. (1) By considering $f = g \otimes x$, where g is an E-polynomial on G and $x \in S_p$ with $\|x\|_{S_p} = 1$, it is straightforward to see that $\Lambda_p^{\mathrm{cb}} \subseteq \Lambda_p$.

(2) An application of the operator version of Jensen's inequality shows that the right hand side of (2.1) is dominated by $\|f\|_{L^p(G,S_p)}$.

(3) Unlike the situation in the classical setting, the fact that $S_2 \subsetneq S_p$ for $p > 2$ implies we never have $L_E^p(G, S_p) \approx L_E^2(G, S_2)$. However, if E is Λ_p^{cb}, then $L_E^2(G, S_2) \subseteq L_E^p(G, S_p)$.

Completely bounded Λ_p sets in $\mathbb{Z}$ were extensively studied by Harcharras [11]. Motivated by Rudin's work [24] on Λ_p sets in $\mathbb{Z}$, Harcharras gave a sufficient combinatorial criterion for the construction of $\Lambda_{2s}^{\mathrm{cb}}$ sets for integers s, and she used this to show that there are $\Lambda_{2s}^{\mathrm{cb}}$ sets that are not Λ_q for any $q > 2s$. She also showed that Sidon sets in $\mathbb{Z}$ are Λ_p^{cb} for all $p < \infty$; there are Λ_p sets that are not Λ_p^{cb}; and with Banks [2], that there are non-Sidon Λ_p^{cb} sets in $\mathbb{Z}$. Subsequently, it was shown in [12] that every infinite compact abelian group admits a non-Sidon, Λ_p^{cb} set.

The goal of this paper is to study analogous notions on compact non-abelian groups. To motivate the definition, we first discuss the Fourier transform of S_p-valued functions on G.

Let $f \in L^1(G, S_p)$. The vector-valued Fourier transform of f at $\sigma \in \widehat{G}$ is defined as

$$\widehat{f}(\sigma) = \int_G f(x) \otimes \sigma(x^{-1})\, dx,$$

where the integral is understood as an element of $M_{d_\sigma}(S_p)$, the $d_\sigma \times d_\sigma$ matrices with entries in S_p. It is convenient to view $\widehat{f}(\sigma)$ as a $d_\sigma \times d_\sigma$ matrix with entries from S_p. For general properties of this Fourier transform we refer the reader to [9].

Given an S_p-valued matrix $A = (A_{ij})_{i,j=1}^n$ with $A_{ij} \in S_p$, we define $\operatorname{Tr} A = \sum_{i=1}^n A_{ii}$. With this notation, the S_p-valued polynomial f has Fourier series

$$f(x) = \sum_{\sigma \in \widehat{G}} d_\sigma \operatorname{Tr}(\widehat{f}(\sigma)\sigma(x)).$$

As before, we call f an E-polynomial if $\widehat{f}(\sigma) = 0$ whenever $\sigma \notin E$.

We are now ready to extend the definition of Λ_p^{cb} to the setting of a non-abelian compact group. Note that the adjoint of $A = (A_{ij}) \in M_{d_\sigma}(S_p)$ can be identified with $B = (B_{ij})$ where $B_{ij} = (A_{ji})^*$ and $|A|^2 = A^*A$.

Definition 2.8. Let $E \subseteq \widehat{G}$ and $2 < p < \infty$. We say that E is a completely bounded Λ_p set (Λ_p^{cb} set) if there exists a constant C such that

$$(2.2) \qquad \|f\|_{L^p(G,S_p)} \le C\Big(\Big\|\Big(\sum_{\sigma\in E} d_\sigma \operatorname{Tr}|\widehat{f}(\sigma)|^2\Big)^{1/2}\Big\|_{S_p} + \Big\|\Big(\sum_{\sigma\in E} d_\sigma \operatorname{Tr}|(\widehat{f}(\sigma))^*|^2\Big)^{1/2}\Big\|_{S_p}\Big)$$

whenever $f = \sum_{\sigma\in E} d_\sigma \operatorname{Tr}(\widehat{f}(\sigma)\sigma)$ is an S_p-valued trigonometric E-polynomial.

Remark 2.9. (1) The definition reduces to that in Definition 2.6 when G is abelian.

(2) In Prop. 3.2 we show that the opposite inequality always holds.

3. A multiplier characterization of Λ_p^{cb} sets. The goal of this section is to obtain an L^p multiplier space characterization of Λ_p^{cb} in the spirit of Theorem 2.3(i). In order to prove Theorem 2.3(i), Figà-Talamanca and Rider [8] (see also [18, Remark 2.7]) obtained a non-abelian variation on Khintchine's inequality. To be precise, they showed that if A_σ is a $d_\sigma \times d_\sigma$ matrix and $\sum_{\sigma\in\widehat{G}} d_\sigma \operatorname{tr}(A_\sigma A_\sigma^*) < \infty$, then given any $p < \infty$ there exist unitary transformations $\{U_\sigma\}$ such that $\sum d_\sigma \operatorname{tr}(U_\sigma A_\sigma \sigma(x))$ is the Fourier series of an $L^p(G)$ function. For this, they considered certain lacunary subsets of the set of irreducible unitary representations of the compact group

$$\mathcal{G} = \prod_{\sigma\in\widehat{G}} U(d_\sigma),$$

where $U(d)$ denotes the group of $d\times d$ unitary matrices. Motivated by their strategy, we first obtain the following estimate, which was the genesis of the definition of Λ_p^{cb}.

Given $V\in\mathcal{G}$, we write $V=(V_\sigma)_{\sigma\in\widehat{G}}$ where $V_\sigma\in U(d_\sigma)$ and denote by dV the Haar probability measure on $\mathcal{G}$.

THEOREM 3.1. *Let G be any compact group and $\mathcal{G}=\prod_{\sigma\in\widehat{G}} U(d_\sigma)$. For each $p>2$ there is a constant $C=C(p)$ such that given any finite collection $\{A^\sigma\}_{\sigma\in\widehat{G}}$, with $A^\sigma\in M_{d_\sigma}(S_p)$, we have*

$$\int_{\mathcal{G}} \Big\| \sum_{\sigma\in\widehat{G}} d_\sigma \operatorname{Tr} A^\sigma V_\sigma\Big\|_{S_p}^p dV \le C\operatorname{tr}\Big[\Big(\sum_\sigma d_\sigma \sum_{j,l} |A^\sigma_{jl}|^2\Big)^{p/2} + \Big(\sum_\sigma d_\sigma \sum_{j,l} |(A^\sigma_{jl})^*|^2\Big)^{p/2}\Big]. \tag{3.1}$$

Proof. Let $\{x^\sigma_{jk} : 1\le j,k\le d_\sigma,\ \sigma\in\widehat{G}\}$ be a collection of independent, complex-valued, Gaussian random variables with mean zero and variance 1, defined on a probability space (Ω_1, P_1). For each $\omega\in\Omega_1$, let $X_\sigma(\omega)$ be the random operator on the Hilbert space $\mathcal{H}_{d_\sigma}$ represented by the matrix

$$\Big\{\frac{1}{\sqrt{d_\sigma}} x^\sigma_{jk}(\omega) : 1\le j,k\le d_\sigma\Big\}$$

with respect to the standard basis. These are independent random operators.

Let $\pi_\sigma : \mathcal{G}\to U(d_\sigma)$ be the projection maps, so that $\pi_\sigma(V)=V_\sigma$. These are independent random variables that are uniformly distributed on $U(d_\sigma)$.

Now view the $\{X_\sigma\}$ and $\{\pi_\sigma\}$ as independent random variables defined in the obvious way on the probability space $(\Omega\ ,P)$, where $\Omega=\Omega_1\times\mathcal{G}$ and P is the product measure.

We have

$$\int_\Omega \Big\|\sum_{\sigma\in\widehat{G}} d_\sigma \operatorname{Tr} A^\sigma \pi_\sigma(\omega)\Big\|_{S_p}^p dP(\omega) = \int_{\mathcal{G}} \Big\|\sum_{\sigma\in\widehat{G}} d_\sigma \operatorname{Tr} A^\sigma V_\sigma\Big\|_{S_p}^p dV,$$

hence upon applying [18, Cor. 2.4, p. 84], we see that for each $2\le p<\infty$

there is a constant $c = c(p)$ such that

$$\int_{\mathcal{G}} \Big\| \sum_{\sigma\in\widehat{G}} d_\sigma \operatorname{Tr} A^\sigma V_\sigma \Big\|_{S_p}^p dV = \int_{\Omega} \Big\| \sum_{\sigma\in\widehat{G}} d_\sigma \operatorname{Tr} A^\sigma \pi_\sigma \Big\|_{S_p}^p dP \le c \int_{\Omega} \Big\| \sum_{\sigma\in\widehat{G}} d_\sigma \operatorname{Tr} A^\sigma X_\sigma \Big\|_{S_p}^p dP.$$

Expanding out gives

$$\operatorname{Tr} A^\sigma X_\sigma = \sum_{j,k} \frac{1}{\sqrt{d_\sigma}} A_{jk}^\sigma x_{jk}^\sigma.$$

Applying Lust-Piquard's non-commutative Khintchine inequality [17] (see also [21, p. 105]), for another constant $C = C(p)$ we deduce that

$$\int_{\mathcal{G}} \Big\| \sum_{\sigma\in\widehat{G}} d_\sigma \operatorname{Tr} A^\sigma V_\sigma \Big\|_{S_p}^p dV \le c \int_{\Omega} \Big\| \sum_{\sigma\in\widehat{G}} \sqrt{d_\sigma} \sum_{j,k} A_{jk}^\sigma x_{jk}^\sigma \Big\|_{S_p}^p dP \le C \operatorname{tr}\Big[\Big(\sum_\sigma d_\sigma \sum_{j,l} |A_{jl}^\sigma|^2\Big)^{p/2} + \Big(\sum_\sigma d_\sigma \sum_{j,l} |(A_{jl}^\sigma)^*|^2\Big)^{p/2}\Big]. \ \blacksquare$$

We also need the following proposition, a vector-valued Jensen inequality. This is known in more generality, but for the sake of completeness we include the proof for the version we use.

PROPOSITION 3.2. *Let G be a compact group and $A^\sigma \in M_{d_\sigma}(S_p)$ for each $\sigma \in \widehat{G}$. If $p > 2$, then*

$$\Big\| \sum_{\sigma\in\widehat{G}} d_\sigma \operatorname{Tr}(A^\sigma \sigma) \Big\|_{L^p(G,S_p)}^p \ge \frac{1}{2} \operatorname{tr}\Big[\Big(\sum_{\sigma\in\widehat{G}} d_\sigma \operatorname{Tr} |A^\sigma|^2\Big)^{p/2} + \Big(\sum_{\sigma\in\widehat{G}} d_\sigma \operatorname{Tr} |(A^\sigma)^*|^2\Big)^{p/2}\Big].$$

Proof. Since G is compact, the vector-valued Jensen's inequality (c.. [19]) implies

$$\begin{aligned} I &:= \Big\| \sum_{\sigma\in\widehat{G}} d_\sigma \operatorname{Tr}(A^\sigma \sigma) \Big\|_{L^p(G,S_p)}^p \\ &= \int \operatorname{tr}\Big[\Big(\sum_{\sigma\in\widehat{G}} d_\sigma (\operatorname{Tr}(A^\sigma\sigma))^* \sum_{\sigma\in\widehat{G}} d_\sigma \operatorname{Tr}(A^\sigma\sigma)\Big)^{p/2}\Big] dg \\ &\ge \operatorname{tr}\Big[\Big(\sum_{\sigma,\psi} d_\sigma d_\psi \int (\operatorname{Tr} A^\sigma \sigma(g))^* (\operatorname{Tr} A^\psi \psi(g))\, dg\Big)^{p/2}\Big]. \end{aligned}$$

Upon expanding the Tr function and using orthogonality of the coordinate functions, it follows that

$$I \geq \operatorname{tr}\Big(\sum_{\sigma} d_\sigma^2 \int_G \sum_{k,l} |A_{kl}^\sigma|^2 |\sigma_{lk}(g)|^2 \, dg\Big)^{p/2} = \operatorname{tr}\Big(\sum_{\sigma} d_\sigma \operatorname{Tr}|A^\sigma|^2\Big)^{p/2}.$$

We can similarly deduce that $I \geq \operatorname{tr}(\sum_\sigma d_\sigma \operatorname{Tr}|(A^\sigma)^*|^2)^{p/2}$ using commutativity, and this completes the proof. ∎

Here is our multiplier characterization of Λ_p^{cb} sets, the non-commutative analogue of Theorem 2.3(i).

THEOREM 3.3. *Let $p > 2$. The subset $E \subseteq \widehat{G}$ is Λ_p^{cb} if and only if whenever $\phi \in l^\infty(E)$, then there exists $\beta \in l^\infty(\widehat{G})$ such that $\beta|_E = \phi$ and $T_\beta \in M_p^{\mathrm{cb}}(G)$.*

Proof. Suppose E is Λ_p^{cb}. Assume first that $\phi(\sigma) = U_\sigma$ is a unitary matrix for each $\sigma \in E$. Set $\beta(\sigma) = \phi(\sigma)$ for all $\sigma \in E$ and $\beta(\sigma) = 0$ otherwise. Let

$$F(g) = \sum_{\sigma \in \widehat{G}} d_\sigma \operatorname{Tr}(A_\sigma \sigma(g)) \in L^p(G, S_p).$$

As E is Λ_p^{cb} and $T_\beta \otimes I_{S_p}(F) = \sum_{\sigma \in E} d_\sigma \operatorname{Tr}(U_\sigma A_\sigma \sigma)$ is an E-function, there is a constant C (independent of F) such that

$$\|T_\beta \otimes I_{S_p}(F)\|_{L^p(S_p)}^p \leq C \operatorname{tr}\Big[\Big(\sum_{\sigma \in E} d_\sigma \operatorname{Tr}|U_\sigma A_\sigma|^2\Big)^{p/2} + \Big(\sum_{\sigma \in E} d_\sigma \operatorname{Tr}|(U_\sigma A_\sigma)^*|^2\Big)^{p/2}\Big].$$

Because U_σ is unitary, $\operatorname{Tr}|U_\sigma A_\sigma|^2 = \operatorname{Tr}|A_\sigma|^2$ and $\operatorname{Tr}|(U_\sigma A_\sigma)^*|^2 = \operatorname{Tr}|A_\sigma^*|^2$. From Prop. 3.2 we deduce that

$$\|T_\beta \otimes I_{S_p}(F)\|_{L^p(S_p)}^p \leq 2^s C \|F\|_{L^p(S_p)}^p,$$

proving that $T_\beta \otimes I_{S_p}$ is a bounded operator from $L^p(S_p)$ to $L^p(S_p)$. Thus $T_\beta \in M_p^{\mathrm{cb}}(G)$.

Since any ϕ in the unit ball of $l^\infty(E)$ can be written as the average of four functions, $\phi_j \in l^\infty(E)$, where $\phi_j(\sigma)$ is unitary for every $\sigma \in E$, the same conclusion follows by the triangle inequality for all ϕ.

Conversely, assume that given any $\phi \in l^\infty(E)$ there exists $\beta \in l^\infty(\widehat{G})$ such that $\beta|_E = \phi$ and $T_\beta \in M_p^{\mathrm{cb}}(G)$. Let $V = \{T_\phi \in M_p^{\mathrm{cb}}(G) : \phi|_E = 0\} \subseteq M_p^{\mathrm{cb}}(G)$. It is easy to see that V is a closed subspace of $M_p^{\mathrm{cb}}(G)$.

Now consider the map $Q : l^\infty(E) \to M_p^{\mathrm{cb}}(G)/V$ that sends ϕ to the equivalence class of $T_\beta \in M_p^{\mathrm{cb}}(G)$ with $\beta|_E = \phi$. An application of the closed graph theorem shows that this map is bounded. Hence there is a constant C_0 such that given $\phi \in l^\infty(E)$, there is a choice of $\beta \in l^\infty(E)$ such

that $\beta|_E = \phi$ and
$$\|T_\beta\|_{\mathrm{cb}(p)} \le C_0\|\phi\|_\infty.$$

Let $f = \sum_{\sigma\in E} d_\sigma \operatorname{Tr}(A_\sigma\sigma) \in L^p_E(G, S_p)$ be an E-polynomial. Set $B_\sigma(g) = A_\sigma\sigma(g)$ and define F on $G \times \prod_{\sigma\in\widehat{G}} U(d_\sigma)$ by
$$F(g,U) := \sum_{\sigma\in E} d_\sigma \operatorname{Tr}(U_\sigma A_\sigma\sigma(g)) = \Big(\sum_{\sigma\in E} d_\sigma \operatorname{Tr}(B^*_\sigma U^*_\sigma)\Big)^*.$$
Let $\mathcal{G} = \prod_{\sigma\in\widehat{G}} U(d_\sigma)$ and define F_g on $\mathcal{G}$ by $F_g(U) = F(g,U)$. By Theorem 3.1 there is a constant C such that for any (fixed) $g \in G$,
$$\begin{aligned}\|F_g(U)\|^p_{L^p(\mathcal{G},S_p)} &\le C \operatorname{tr}\Big[\Big(\sum_{\sigma\in E} d_\sigma \operatorname{Tr}|B_\sigma|^2\Big)^{p/2} + \Big(\sum_{\sigma\in E} d_\sigma \operatorname{Tr}|B^*_\sigma|^2\Big)^{p/2}\Big]\\ &= C \operatorname{tr}\Big[\Big(\sum_{\sigma\in E} d_\sigma \operatorname{Tr}|A_\sigma|^2\Big)^{p/2} + \Big(\sum_{\sigma\in E} d_\sigma \operatorname{Tr}|A^*_\sigma|^2\Big)^{p/2}\Big] =: \mathrm{RHS},\end{aligned}$$
and the latter is finite by Prop. 3.2. Integrating both the sides over G gives
$$\int_G\int_{\mathcal{G}} \|F_g(U)\|^p_{S_p}\, dU\, dg = \int_G \|F(g,U)\|^p_{L^p(\mathcal{G},S_p)}\, dg \le \mathrm{RHS}.$$
Hence there exists some $U \in \mathcal{G}$ such that
$$\int_G \|F(g,U)\|^p_{S_p}\, dg \le \mathrm{RHS}.$$
But $F(g,U) = T_U \otimes I_{S_p}(f)(g)$ (understanding $U = (U_\sigma) \in l^\infty(\widehat{G})$ in the natural sense), thus
$$\|T_U \otimes I_{S_p}(f)\|^p_{L^p(G,S_p)} = \int_G \|F(g,U)\|^p_{S_p}\, dg \le \mathrm{RHS} < \infty.$$

Let $\phi \in l^\infty(E)$ be defined by $\phi(\sigma) = U^*_\sigma$ for all $\sigma \in E$. As $\|\phi\| \le 1$, there exists $\beta \in l^\infty(\widehat{G})$ such that $\beta|_E = \phi$, $T_\beta \in M^{\mathrm{cb}}_p(G)$ and
$$\|T_\beta\|_{\mathrm{cb}(p)} = \|T_\beta \otimes I_{S_p}\|_{L^p(S_p)\to L^p(S_p)} \le C_0$$
where C_0 is the constant found above. Since $\beta(\sigma)U_\sigma = I_{d_\sigma}$ for all $\sigma \in E$, one can easily see that $f = T_\beta \otimes I_{S_p} \circ T_U \otimes I_{S_p}(f)$. Thus
$$\begin{aligned}\|f\|^p_{L^p(G,S_p)} &= \|T_\beta \otimes I_{S_p} \circ T_U \otimes I_{S_p}(f)\|^p_{L^p(S_p)} \le C_0\|T_U \otimes I_{S_p}(f)\|^p_{L^p(S_p)}\\ &\le CC_0 \operatorname{tr}\Big[\Big(\sum_{\sigma\in E} d_\sigma \operatorname{Tr}|A_\sigma|^2\Big)^s + \Big(\sum_{\sigma\in E} d_\sigma \operatorname{Tr}|A^*_\sigma|^2\Big)^s\Big].\end{aligned}$$
Since f was an arbitrary E-polynomial, this proves E is Λ^{cb}_p. ∎

We can quickly deduce from this that the Λ^{cb}_p sets are nested, as expected.

Corollary 3.4. *If $2 < p < q < \infty$ and E is Λ^{cb}_q, then E is Λ^{cb}_p.*

Proof. Assume E is Λ_q^{cb} and let $\phi \in l^\infty(E)$. Obtain $\beta \in l^\infty$ such that $\beta|_E = \phi$ and $T_\beta \in M_q^{\mathrm{cb}}(G)$. But $M_q^{\mathrm{cb}}(G) \subseteq M_p^{\mathrm{cb}}(G)$, thus the other direction of the theorem implies E is Λ_p^{cb}. ∎

Theorem 3.1 implies that if $G = \prod_j U(n_j)$ and $E = \{\pi_j\}$ where π_j is the unitary representation on G defined by $\pi_j(U) = U_j$, then E is a Λ_p^{cb} set. In fact, E is known to be a Sidon set [8]. More generally, one can deduce from the previous theorem that all Sidon sets are Λ_p^{cb}.

COROLLARY 3.5. *Let G be a compact group. Then any Sidon set is Λ_p^{cb} for all $p > 2$.*

Proof. If E is Sidon, then the multiplier characterization of Sidon (Thm. 2.3) implies that for every $\phi \in l^\infty(E)$ there is a finite, regular, Borel measure μ such that $\widehat{\mu}|_E = \phi$. It is known that all such measures act as completely bounded operators on L^1 and L^∞, and hence on all L^p by Pisier's complex interpolation theorem [20]. By Theorem 3.3, E is Λ_p^{cb} for all $p > 2$. ∎

4. Multipliers that are not completely bounded; Λ_p sets that are not Λ_4^{cb}. It is well known that there are infinite, compact, non-abelian groups that do not admit infinite Sidon or even Λ_p sets. The product group $G = \prod_j SU(n_j)$ is the prototypical example of a group that does. Indeed, let $\pi_j : G \to SU(n_j)$ be the projection onto the jth factor. The set $\{\pi_j, \overline{\pi_j} : j = 1, 2, \ldots\}$ is known as the *FTR set* (for Figà-Talamanca and Rider). As explained in [5], it is the prototypical example of a Sidon set in the non-abelian setting. When $n_j \to \infty$, the set $E = \{\pi_{2j} \times \pi_{2j+1} : j = 1, 2, \ldots\}$ is known to be Λ_p for all $p < \infty$, but not Sidon [13]. In this section we will show that E is not Λ_p^{cb} for any $p \geq 4$. Our method is inspired by Pisier's construction of a Λ_p, non-Λ_p^{cb} set in the abelian setting [21].

For notational simplicity we will write $\pi_{2J} = \chi^J$ and $\pi_{2J+1} = \psi^J$. There is no loss in assuming $4 < n_{2J} \leq n_{2J+1}$. If we represent $\chi^J \times \psi^J$ as a matrix with respect to the standard basis, then the diagonal entries are $(\chi^J)_{jj}(\psi^J)_{kk}$ where $j = 1, \ldots, n_{2J}$, $k = 1, \ldots, n_{2J+1}$ and $(\chi^J)_{jj}$, $(\psi^J)_{kk}$ are the diagonal entries of the standard matrix representations of χ^J and ψ^J, respectively. We will refer to $(\chi^J)_{jj}(\psi^J)_{kk}$ as the (j, k) *diagonal entry.*

For $j, k = 1, \ldots, n_{2J}$, let $u_{jk}^J = n_{2J}^{-1/2} \exp(2\pi i jk/n_{2J})$, $b_{jk}^J = n_{2J}^{-1/4} u_{jk}^J$ and $a_{jk}^J = n_{2J}^{1/2} \overline{u_{jk}^J}$. For $j = 1, \ldots, n_{2J}$, $k = n_{2J} + 1, \ldots, n_{2J+1}$, let $a_{jk}^J = 1$. Note that $(u_{jk}^J)_{j,k=1}^{n_{2J}}$ is an $n_{2J} \times n_{2J}$ unitary matrix. We define a multiplier T_ϕ by setting $\phi(\chi^J \times \psi^J)$ to be the diagonal matrix whose (j, k) diagonal entry is a_{jk}^J. Define $\phi(\sigma) = 0$ for $\sigma \notin E$. Since $|a_{jk}| = 1$, each $\phi(\chi^J \times \psi^J)$ is a unitary matrix. Because E is Λ_p for all $p < \infty$, we have $T_\phi \in M_p(G)$.

Consider the functions $F_J : G \to S_4$ given by

$$F_J(g) = \sum_{j,k=1}^{n_{2J}} b_{jk} E_{jk} \chi_{jj}^J \psi_{kk}^J(g),$$

where $\{E_{jk}\}$ is the canonical basis for S_4. The functions F_J are E-functions and the multiplier T_ϕ acts on F_J by

$$T_\phi(F_J)(g) = \sum_{j,k=1}^{n_{2J}} a_{jk} b_{jk} E_{jk} \chi_{jj}^J(g) \psi_{kk}^J(g).$$

We will show that

$$\frac{\|T_\phi(F_J)\|_{L^4(S_4)}}{\|F_J\|_{L^4(S_4)}} \to \infty \quad \text{as } J \to \infty. \tag{4.1}$$

If $\beta|_E = \phi$, then $T_\beta(F_J) = T_\phi(F_J)$, hence $T_\beta \notin M_4^{\mathrm{cb}}(G)$. It follows from Theorem 3.3 that E is not Λ_4^{cb}.

We will use the following calculations.

Lemma 4.1.

(i) $\int_{SU(N)} |V_{kk}|^4 \, dV = 2/(N(N+1))$ *for all* $k = 1, \ldots, N$.
(ii) $\int_{SU(N)} |V_{kk}|^2 |V_{mm}|^2 \, dV = 1/(N^2-1)$ *for* $k \neq m$.

Proof. Part (i) is [14, Lemma 29.10]. (It is proven there for $U(N)$, but the same arguments hold for $SU(N)$).

For (ii), fix $p \neq 1, 2$ and write $n_p = 1$ and $n_j = j$ for $j \neq p$. Set $a_2 = 1$ and $a_j = 0$ for $j \neq 2$. Then $\prod_{j=1}^N |V_{n_j j}|^{2a_j} = |V_{22}|^2$, so the same reasoning as for [14, Lemma 29.8] implies that

$$\begin{aligned}\int |V_{11}|^2 |V_{22}|^2 &= (1 + a_p) \int |V_{11}|^2 \prod |V_{n_j j}|^{2a_j} \\ &= (1 + a_1) \int |V_{n_p p}|^2 \prod |V_{n_j j}|^{2a_j} = \int |V_{1p}|^2 |V_{22}|^2.\end{aligned}$$

Summing over $p \neq 2$ and using the fact that $\sum_{p=1}^N |V_{1p}|^2 = 1$ gives

$$\begin{aligned}\int |V_{11}|^2 |V_{22}|^2 &= \frac{1}{N-1} \sum_{p \neq 2} \int |V_{1p}|^2 |V_{22}|^2 \\ &= \frac{1}{N-1} \Big(\sum_p \int |V_{1p}|^2 |V_{22}|^2 - |V_{12}|^2 |V_{22}|^2 \Big) \\ &= \frac{1}{N-1} \Big(\int |V_{22}|^2 - \int |V_{12}|^2 |V_{22}|^2 \Big) \\ &= \frac{1}{N-1} \left(\frac{1}{N} - \frac{(N-1)!}{(N+2-1)!} \right) = \frac{1}{N^2-1},\end{aligned}$$

where the last but one equality comes from [14, 29.9 and 29.10]. ∎

We will temporarily fix J. For notational convenience we will omit the subscripts or superscripts J and write $N = n_{2J}$, $M = n_{2J+1}$. To prove (4.1), we begin by noting that since $E_{kj}E_{mn} = 0$ if $j \neq m$, $E_{kj}E_{mn} = E_{kn}$ if $j = m$, and $E_{jk}^* = E_{kj}$, we have

$$\begin{aligned}\|F\|_{S_4}^4 &= \operatorname{tr}\Big[\Big(\sum_{j,k=1}^{N}(b_{jk}E_{jk}\chi_{jj}\psi_{kk})^* \sum_{m,n} b_{mn}E_{mn}\chi_{mm}\psi_{nn}\Big)^2\Big] \\ &= \operatorname{tr}\Big[\Big(\sum_{j,k,n}\overline{b_{jk}}b_{jn}E_{kn}|\chi_{jj}|^2\overline{\psi_{kk}}\psi_{nn}\Big)^2\Big] \\ &= \sum_{j,k,n,r=1}^{N}\overline{b_{jk}}b_{jn}\overline{b_{rn}}b_{rk}|\chi_{jj}|^2|\chi_{rr}|^2|\psi_{kk}|^2|\psi_{nn}|^2.\end{aligned}$$

After substituting for the coefficients b_{jk} etc. we see that

$$\|F\|_{L^4(G,S_4)}^4 = \sum_{j,k,n,r} N^{-1}\overline{u_{jk}}u_{jn}\overline{u_{rn}}u_{rk}\int_G |\chi_{jj}|^2|\chi_{rr}|^2|\psi_{kk}|^2|\psi_{nn}|^2\,dg.$$

Now

$$\int_G |\chi_{jj}|^2|\chi_{rr}|^2|\psi_{kk}|^2|\psi_{nn}|^2\,dg = \int_{SU(N)}|\chi_{jj}|^2|\chi_{rr}|^2 dg_1 \int_{SU(M)}|\psi_{kk}|^2|\psi_{nn}|^2\,dg_2,$$

and these integrals depend on whether or not $j = r$ and/or $k = n$. Thus we write

$$\|F\|_{L^4(G,S_4)}^4 = N^{-1}\sum_{k,n=1}^{N}\int_{SU(M)}|\psi_{kk}|^2|\psi_{nn}|^2(I + I')\,dg_2$$

where

$$\begin{aligned} I &= \sum_{j=1}^{N}|u_{jk}|^2|u_{jn}|^2\int_{SU(N)}|\chi_{jj}|^4\,dg_1 = \frac{2}{N^2(N+1)}, \\ I' &= \sum_{j\neq r}\overline{u_{jk}}u_{jn}\overline{u_{rn}}u_{rk}\int_{SU(N)}|\chi_{jj}|^2|\chi_{rr}|^2\,dg_1.\end{aligned}$$

(The calculation of I follows from Lemma 4.1(i) and the fact that $|u_{jk}| = N^{-1/2}$.) To calculate I', we use the fact that (u_{jk}) is unitary so

$$\sum_{j\neq r}\overline{u_{jk}}u_{jn}\overline{u_{rn}}u_{rk} = \sum_{j=1}^{N}\overline{u_{jk}}u_{jn}(\delta_{kn} - \overline{u_{jn}}u_{jk}) = \delta_{kn} - N^{-1}$$

where $\delta_{kn} = 1$ if $k = n$ and 0 else. Consequently, Lemma 4.1(ii) implies

$$I' = \frac{1}{N^2-1}(\delta_{kn} - N^{-1}).$$

Applying Lemma 4.1 again to evaluate $\int_{SU(M)} |\psi_{kk}|^2|\psi_{nn}|^2 dg_2$, we deduce that there is a constant c_1, independent of N, M, such that

$$\|F\|^4_{L^4(S_4)} \le c_1 N^{-2} M^{-2}. \tag{4.2}$$

Similar arguments establish that

$$\|T_\phi(F)\|^4_{S_4} = \sum_{j,k,n,r} \overline{c_{jk}} c_{jn} \overline{c_{rn}} c_{rk} \int_G |\chi_{jj}|^2|\chi_{rr}|^2|\psi_{kk}|^2|\psi_{nn}|^2$$

where $c_{jk} = a_{jk}b_{jk} = N^{-3/4}$. After applying Lemma 4.1 one final time we see that there is a constant $c_2 > 0$ (independent of N, M) such that $\|T_\phi(F)\|^4_{L^4(S_4)} \ge c_2 N^{-1} M^{-2}$. This bound, coupled with (4.2), certainly implies (4.1). To summarize, we have just proven that $T_\phi \notin M_4^{\mathrm{cb}}(G)$ and hence E is not Λ_4^{cb}. Moreover, the nestedness of the spaces $M_p^{\mathrm{cb}}(G)$ implies the following:

THEOREM 4.2. *Let $G = \prod_j SU(n_j)$. If $n_j \to \infty$, the group G admits a set of representations of unbounded degree that is Λ_p for all $1 < p < \infty$, but not Λ_p^{cb} for any $p \ge 4$. Further, $M_p^{\mathrm{cb}}(G) \subsetneq M_p(G)$ for all $p \ge 4$.*

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