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JORDAN SUPERDERIVATIONS AND JORDAN TRIPLE SUPERDERIVATIONS OF SUPERALGEBRAS

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Abstract. We study Jordan (θ,θ)-superderivations and Jordan triple (θ,θ)-superderivations of superalgebras, using the theory of functional identities in superalgebras. As a consequence, we prove that if $A = A_0 \oplus A_1$ is a prime superalgebra with $\deg(A_1) \geq 9$, then Jordan superderivations and Jordan triple superderivations of A are superderivations of A, and generalized Jordan superderivations and generalized Jordan triple superderivations of A are generalized superderivations of A.

1. Introduction. Let A be an associative algebra. A Jordan derivation d of A is a linear mapping from A into itself satisfying $d(x^2) = d(x)x + xd(x)$ for all $x \in A$. In the 1950's Herstein [15] proved that if A is a prime ring of characteristic different from 2, then any Jordan derivation of A is a derivation of A. In 1988, Brešar [8] studied Jordan derivations on a 2-torsion free semiprime ring. Fošner [13] extended Herstein's theorem to superalgebras and proved that a Jordan superderivation on a prime associative superalgebra whose even part is noncommutative is a superderivation. Later, Fošner [14] considered Jordan superderivations on semiprime superalgebras.

The concept of a generalized derivation was introduced by Brešar [11]. Generalized derivations on prime rings were studied by Hvala [16]. The following definition is a common generalization of Jordan derivations and generalized derivations. A linear mapping $f : A \to A$ is called a *generalized Jordan derivation* if there exists a Jordan derivation $d : A \to A$ such that

$$f(xy + yx) = f(x)y + yd(x) + xd(y) + f(y)x, \quad x, y \in A.$$

Jing and Lu [17] considered generalized Jordan derivations of prime rings and standard operator algebras. Their results were extended to semiprime rings by Vukman [20] who proved that every generalized Jordan derivation of a 2-torsion free semiprime ring is a generalized derivation.

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The following two notions correspond to Jordan derivations and generalized Jordan derivations. A linear mapping $J : A \to A$ is called a *Jordan triple derivation* if

$$J(xyx) = J(x)yx + xJ(y)x + xyJ(x), \quad x, y \in A.$$

A linear mapping $G : A \to A$ is called a *generalized Jordan triple derivation* if there exists a Jordan triple derivation J of A such that

$$G(xyx) = G(x)yx + xJ(y)x + xyJ(x), \quad x, y \in A.$$

Brešar [9] proved that every Jordan triple derivation on a 2-torsion free semiprime ring is a derivation. It turns out that every Jordan derivation on a 2-torsion free ring is a Jordan triple derivation. Jing and Lu [17] proved that every generalized Jordan triple derivation on a prime ring is a generalized derivation. Liu and Shiue [18] proved that Jordan (θ, ϕ)-derivations and Jordan triple (θ, ϕ)-derivations are (θ, ϕ)-derivations, and generalized Jordan (θ, ϕ)-derivations and generalized Jordan triple (θ, ϕ)-derivations are generalized (θ, ϕ)-derivations on a 2-torsion free semiprime ring.

On the other hand, a functional identity can be described as an identical relation involving elements in a ring together with functions. The goal when studying a functional identity is to describe the form of these functions or to determine the structure of the ring admitting the FI in question. The theory of functional identities in rings originated from results on commuting mappings [7]. The name “functional identity” was introduced by Brešar [10]. The crucial tool in the theory of functional identities in rings is the notion of d-free set, which was developed by Beidar and Chebotar [4, 5]. Making use of the theory of functional identities in rings, Herstein's conjectures on Lie mappings in rings have been settled [1, 2, 3]. Subsequently, Wang [22] established the theory of functional identities in superalgebras and gave the definition of d-superfree sets. As an application, Wang [23] described Lie superhomomorphisms from the set of skew elements of a superalgebra with superinvolution into a unital superalgebra. For functional identities and d-superfree sets of superalgebras we refer the reader to [12], [21] and [22].

In this paper, we study Jordan (θ, θ)-superderivations, generalized Jordan (θ, θ)-superderivations, Jordan triple (θ, θ)-superderivations and generalized Jordan triple (θ, θ)-superderivations of superalgebras, using the theory of functional identities in superalgebras. As a consequence, we prove that if $A = A_0 \oplus A_1$ is a prime superalgebra with $\deg(A_1) \geq 9$, then Jordan superderivations and Jordan triple superderivations of A are superderivations of A, and generalized Jordan superderivations and generalized Jordan triple superderivations of A are generalized superderivations of A.

2. Preliminaries. Throughout the paper, by an algebra we shall mean an algebra over a fixed unital commutative ring Φ. We assume without further mention that $1/2 \in \Phi$.

An associative algebra A over Φ is said to be an *associative superalgebra* if there exist two Φ-submodules A_0 and A_1 of A such that $A = A_0 \oplus A_1$ and $A_iA_j \subseteq A_{i+j}$, $i, j \in Z_2$. A superalgebra is called *trivial* if $A_1 = 0$. The elements of A_i are homogeneous of degree i and we write $|a_i| = i$ for all $a_i \in A_i$. For a superalgebra A, we define $\sigma : A \to A$ by $(a_0 + a_1)^\sigma = a_0 - a_1$, then σ is an automorphism of A such that $\sigma^2 = 1$. On the other hand, for an algebra A, if there exists an automorphism σ of A such that $\sigma^2 = 1$, then A becomes a superalgebra $A = A_0 \oplus A_1$, where $A_i = \{x \in A \mid x^\sigma = (-1)^i x\}$, $i = 0, 1$. A superalgebra A is called *prime* if $aAb = 0$ implies $a = 0$ or $b = 0$, whenever at least one of the elements a and b is homogeneous.

An element $x \in A_0 \cup A_1$ is said to be *algebraic* over C of degree $\leq n$ if there exist $c_0, c_1, \ldots, c_n \in C$, not all zero and such that $\sum_{i=0}^{n} c_i x^{n-i} = 0$. The element x is said to be algebraic over C of degree n if it is algebraic over C of degree $\leq n$ and is not algebraic over C of degree $\leq n-1$. By $\deg(x)$ we shall mean the degree of x over C (if x is algebraic over C) or ∞ (if x is not algebraic over C). Given a nonempty subset $S \subseteq A_0 \cup A_1$, we set

$$\deg(S) = \sup\{\deg(x) \mid x \in S\}.$$

Montaner [19] found that a prime superalgebra A is not necessarily a prime algebra but a semiprime algebra. Hence one can define the maximal right ring of quotients Q of A, and some useful properties of Q can be found in [6]. By [6, Proposition 2.5.3] σ can be uniquely extended to Q. Therefore Q is also a superalgebra. Moreover, we can show that Q is a prime superalgebra.

For any $x, y \in A_0 \cup A_1$, we consider the Jordan superproduct

$$x \circ_s y = xy + (-1)^{|x|\,|y|} yx$$

and the Lie superproduct

$$[x, y]_s = xy - (-1)^{|x|\,|y|} yx.$$

Accordingly, $a \circ_s b = a_0 \circ_s b_0 + a_1 \circ_s b_0 + a_0 \circ_s b_1 + a_1 \circ_s b_1$ and $[a, b]_s = [a_0, b_0]_s + [a_1, b_0]_s + [a_0, b_1]_s + [a_1, b_1]_s$, where $a = a_0 + a_1$, $b = b_0 + b_1$.

The following definitions will be needed throughout the paper.

Let A be a superalgebra and let θ be an automorphism of A. For $i \in \{0, 1\}$, a (θ, θ)-*superderivation of degree* i is a Φ-linear mapping $d_i : A \to A$ which satisfies $d_i(A_j) \subseteq A_{i+j}$ for $j \in Z_2$ and

$$d_i(ab) = d_i(a)\theta(b) + (-1)^{i|a|} \theta(a) d_i(b)$$

for all $a, b \in A_0 \cup A_1$. If $d = d_0 + d_1$, then d is a (θ, θ)-*superderivation.*

For $i \in \{0,1\}$. A Φ-linear mapping $g_i : A \to A$ is called a *generalized (θ,θ)-superderivation of degree i* if $g_i(A_j) \subseteq A_{i+j}$, $j \in Z_2$, and

$$g_i(xy) = g_i(x)\theta(y) + (-1)^{i|x|}\theta(x)d_i(y)$$

for all $x, y \in A_0 \cup A_1$, where d_i is a (θ,θ)-superderivation of degree i. If $g = g_0 + g_1$, then g is called a *generalized (θ,θ)-superderivation.*

The following definition is an extension of Jordan derivations.

Definition 2.1. Let A be a superalgebra and let θ be an automorphism of A. For $i \in \{0,1\}$, a Φ-linear mapping $\alpha_i : A \to A$ is called a *Jordan (θ,θ)-superderivation of degree i* if $\alpha_i(A_j) \subseteq A_{i+j}$, $j \in Z_2$, and

$$\alpha_i(x \circ_s y) = \alpha_i(x) \circ_s \theta(y) + (-1)^{i|x|}\theta(x) \circ_s \alpha_i(y)$$

for all $x, y \in A_0 \cup A_1$. If $\alpha = \alpha_0 + \alpha_1$, then α is called a *Jordan (θ,θ)-superderivation.*

According to the concepts of generalized Jordan derivations and generalized (θ,θ)-superderivations, we will give the concept of generalized Jordan (θ,θ)-superderivations.

Definition 2.2. Let A be a superalgebra and let θ be an automorphism of A. For $i \in \{0,1\}$, a Φ-linear mapping $\phi_i : A \to A$ is called a *generalized Jordan (θ,θ)-superderivation of degree i* if $\phi_i(A_j) \subseteq A_{i+j}$, $j \in Z_2$, and

$$\begin{aligned}\phi_i(x \circ_s y) = \phi_i(x)\theta(y) &+ (-1)^{i|y|+|x|\,|y|}\theta(y)\alpha_i(x)\\ &+ (-1)^{i|x|}\theta(x)\alpha_i(y) + (-1)^{|x|\,|y|}\phi_i(y)\theta(x)\end{aligned}$$

for all $x, y \in A_0 \cup A_1$, where α_i is a Jordan (θ,θ)-superderivation of degree i. If $\phi = \phi_0 + \phi_1$, then ϕ is called a *generalized Jordan (θ,θ)-superderivation.*

In trivial superalgebras, the concept of Jordan (θ,θ)-superderivations (resp., generalized Jordan (θ,θ)-superderivations) coincides with that of Jordan (θ,θ)-derivations (resp., generalized Jordan (θ,θ)-derivations).

Definition 2.3. Let A be a superalgebra and let θ be an automorphism of A. For $i \in \{0,1\}$, a Φ-linear mapping $\beta_i : A \to A$ is called a *Jordan triple (θ,θ)-superderivation of degree i* if $\beta_i(A_j) \subseteq A_{i+j}$, $j \in Z_2$, and

$$\begin{aligned}\beta_i(x \circ_s y \circ_s z) = \beta_i(x) \circ_s \theta(y) \circ_s \theta(z) &+ (-1)^{i|x|}\theta(x) \circ_s \beta_i(y) \circ_s \theta(z)\\ &+ (-1)^{i(|x|+|y|)}\theta(x) \circ_s \theta(y) \circ_s \beta_i(z)\end{aligned}$$

for all $x, y, z \in A_0 \cup A_1$. If $\beta = \beta_0 + \beta_1$, then β is called a *Jordan triple (θ,θ)-superderivation.*

Definition 2.4. Let A be a superalgebra and let θ be an automorphism of A. For $i \in \{0,1\}$, a Φ-linear mapping $\xi_i : A \to A$ is called a *generalized*

Jordan triple (θ,θ)-superderivation of degree i if $\xi_i(A_j) \subseteq A_{i+j}$, $j \in Z_2$, and

$$\begin{aligned}
&\xi_i(x \circ_s y \circ_s z)\\
&\quad = \xi_i(x)\theta(y)\theta(z) + (-1)^{|x|\,|y|}\xi_i(y)\theta(x)\theta(z)\\
&\qquad + (-1)^{|x|\,|z|+|y|\,|z|}\xi_i(z)\theta(x)\theta(y) + (-1)^{|x|\,|y|+|x|\,|z|+|y|\,|z|}\xi_i(z)\theta(y)\theta(x)\\
&\qquad + (-1)^{i|x|}\theta(x)\beta_i(y)\theta(z) + (-1)^{i|y|+|x|\,|y|}\theta(y)\beta_i(x)\theta(z)\\
&\qquad + (-1)^{i|z|+|x|\,|y|+|x|\,|z|+|y|\,|z|}\theta(z)\beta_i(y)\theta(x)\\
&\qquad + (-1)^{i|z|+|x|\,|z|+|y|\,|z|}\theta(z)\beta_i(x)\theta(y) + (-1)^{i(|x|+|y|)}\theta(x)\theta(y)\beta_i(z)\\
&\qquad + (-1)^{i(|x|+|y|)+|x|\,|y|}\theta(y)\theta(x)\beta_i(z)\\
&\qquad + (-1)^{i(|x|+|z|)+|x|\,|z|+|y|\,|z|}\theta(z)\theta(x)\beta_i(y)\\
&\qquad + (-1)^{i(|y|+|z|)+|x|\,|y|+|x|\,|z|+|y|\,|z|}\theta(z)\theta(y)\beta_i(x)
\end{aligned}$$

for all $x, y, z \in A_0 \cup A_1$, where β_i is a Jordan triple (θ,θ)-superderivation of degree i. If $\xi = \xi_0 + \xi_1$, then ξ is called a *generalized Jordan triple (θ,θ)-superderivation.*

It is clear that Jordan triple (θ,θ)-superderivations (resp., generalized Jordan triple (θ,θ)-superderivations) of trivial superalgebras are Jordan triple (θ,θ)-derivations (resp., generalized Jordan triple (θ,θ)-derivations).

3. Jordan superderivations. The following identity will be used frequently:

(3.1) $$[[x,y]_s,z]_s = x \circ_s (y \circ_s z) - (-1)^{|x|\,|y|} y \circ_s (x \circ_s z), \qquad x,y,z \in A_0 \cup A_1.$$

LEMMA 3.1. *Let A be a prime superalgebra with maximal right ring of quotients Q and extended centroid C. Suppose that θ is an automorphism of A and $\alpha_i : A \to A$ is a Jordan (θ,θ)-superderivation of degree $i \in \{0,1\}$. If A is a 4-superfree subset of Q, then $\alpha_i : A \to A$ is a (θ,θ)-superderivation of degree i.*

Proof. Define $\delta : A \times A \to Q$ by

$$\delta(x,y) = \alpha_i(xy) - \alpha_i(x)\theta(y) - (-1)^{i|x|}\theta(x)\alpha_i(y)$$

for all $x, y \in A_0 \cup A_1$. Note that $\delta(x_1,y_0) = -\delta(y_0,x_1)$ and $\delta(x_1,y_1) = \delta(y_1,x_1)$.

Applying α_i to (3.1), we get

(3.2) $$\alpha_i([[x_1,y_1]_s,z_0]_s) = \alpha_i(x_1 \circ_s (y_1 \circ_s z_0)) + \alpha_i(y_1 \circ_s (x_1 \circ_s z_0)).$$

By [19, Lemma 1.2] and [18, Corollary 1], $\alpha_i|A_0$ is a (θ,θ)-derivation. So

$$\begin{aligned}&\alpha_i([[x_1,y_1]_s,z_0]_s)\\&\qquad=[\alpha_i([x_1,y_1]_s),\theta(z_0)]_s+\theta(x_1)\theta(y_1)\alpha_i(z_0)+\theta(y_1)\theta(x_1)\alpha_i(z_0)\\&\qquad\quad-\alpha_i(z_0)\theta(x_1)\theta(y_1)-\alpha_i(z_0)\theta(y_1)\theta(x_1)\end{aligned}$$

and

$$\begin{aligned}&\alpha_i(x_1\circ_s(y_1\circ_s z_0))+\alpha_i(y_1\circ_s(x_1\circ_s z_0))\\&\ =\alpha_i(x_1)\circ_s\theta(y_1\circ_s z_0)+(-1)^i\theta(x_1)\circ_s\alpha_i(y_1\circ_s z_0)\\&\quad+\alpha_i(y_1)\circ_s\theta(x_1\circ_s z_0)+(-1)^i\theta(y_1)\circ_s\alpha_i(x_1\circ_s z_0)\\&\ =\alpha_i(x_1)\circ_s\theta(y_1z_0+z_0y_1)+(-1)^i\theta(x_1)\circ_s(\alpha_i(y_1)\circ_s\theta(z_0))\\&\quad+\theta(x_1)\circ_s(\theta(y_1)\circ_s\alpha_i(z_0))+\alpha_i(y_1)\circ_s\theta(x_1z_0+z_0x_1)\\&\quad+(-1)^i\theta(y_1)\circ_s(\alpha_i(x_1)\circ_s\theta(z_0))+\theta(y_1)\circ_s(\theta(x_1)\circ_s\alpha_i(z_0))\\&\ =\alpha_i(x_1)\theta(y_1)\theta(z_0)+\alpha_i(x_1)\theta(z_0)\theta(y_1)-(-1)^i\theta(y_1)\theta(z_0)\alpha_i(x_1)\\&\quad-(-1)^i\theta(z_0)\theta(y_1)\alpha_i(x_1)+(-1)^i\theta(x_1)\alpha_i(y_1)\theta(z_0)+(-1)^i\theta(x_1)\theta(z_0)\alpha_i(y_1)\\&\quad-\alpha_i(y_1)\theta(z_0)\theta(x_1)-\theta(z_0)\alpha_i(y_1)\theta(x_1)+\theta(x_1)\theta(y_1)\alpha_i(z_0)\\&\quad+(-1)^i\theta(x_1)\alpha_i(z_0)\theta(y_1)-(-1)^i\theta(y_1)\alpha_i(z_0)\theta(x_1)-\alpha_i(z_0)\theta(y_1)\theta(x_1)\\&\quad+\alpha_i(y_1)\theta(x_1)\theta(z_0)+\alpha_i(y_1)\theta(z_0)\theta(x_1)-(-1)^i\theta(x_1)\theta(z_0)\alpha_i(y_1)\\&\quad-(-1)^i\theta(z_0)\theta(x_1)\alpha_i(y_1)+(-1)^i\theta(y_1)\alpha_i(x_1)\theta(z_0)+(-1)^i\theta(y_1)\theta(z_0)\alpha_i(x_1)\\&\quad-\alpha_i(x_1)\theta(z_0)\theta(y_1)-\theta(z_0)\alpha_i(x_1)\theta(y_1)+\theta(y_1)\theta(x_1)\alpha_i(z_0)\\&\quad+(-1)^i\theta(y_1)\alpha_i(z_0)\theta(x_1)-(-1)^i\theta(x_1)\alpha_i(z_0)\theta(y_1)-\alpha_i(z_0)\theta(x_1)\theta(y_1)\end{aligned}$$

for all $x_1,y_1\in A_1$, $z_0\in A_0$. Comparing the above relations, we have

$$\begin{aligned}[\alpha_i([x_1,y_1]_s),\theta(z_0)]_s&=\alpha_i(x_1)\theta(y_1)\theta(z_0)+(-1)^i\theta(x_1)\alpha_i(y_1)\theta(z_0)\\&\quad+\alpha_i(y_1)\theta(x_1)\theta(z_0)+(-1)^i\theta(y_1)\alpha_i(x_1)\theta(z_0)\\&\quad-\theta(z_0)\alpha_i(x_1)\theta(y_1)-(-1)^i\theta(z_0)\theta(x_1)\alpha_i(y_1)\\&\quad-\theta(z_0)\alpha_i(y_1)\theta(x_1)-(-1)^i\theta(z_0)\theta(y_1)\alpha_i(x_1).\end{aligned}$$

Therefore

$$[\delta(x_1,y_1)+\delta(y_1,x_1),\theta(z_0)]_s=0$$

for all $x_1,y_1\in A_1$, $z_0\in A_0$. From $\delta(x_1,y_1)=\delta(y_1,x_1)$, we obtain

$$[\delta(x_1,y_1),\theta(z_0)]_s=0. \tag{3.3}$$

Since A is a 4-superfree subset of Q, [22, Theorem 3.8] implies that

$$\begin{aligned}\delta(x_1,y_1)&=\lambda_1\theta(x_1)\theta(y_1)\\&\quad+\lambda_2\theta(y_1)\theta(x_1)+\mu_1(x_1)\theta(y_1)+\mu_2(y_1)\theta(x_1)+\nu_1(x_1,y_1),\end{aligned}\tag{3.4}$$

where $\lambda_1,\lambda_2\in C+C\omega$, $\mu_1,\mu_2:A_1\to C+C\omega$, $\nu_1:A_1\times A_1\to C+C\omega$. Substituting (3.4) into (3.3), we find that

- the coefficient of $\theta(x_1)\theta(y_1)\theta(z_0)$ in (3.3) is λ_1;
- the coefficient of $\theta(y_1)\theta(x_1)\theta(z_0)$ in (3.3) is λ_2;
- the coefficient of $\theta(y_1)\theta(z_0)$ in (3.3) is $\mu_1(x_1)$;
- the coefficient of $\theta(x_1)\theta(z_0)$ in (3.3) is $\mu_2(y_1)$.

In view of 4-superfreeness of A, [22, Theorem 3.7] now forces that

$$\lambda_1 = \lambda_2 = \mu_1 = \mu_2 = 0.$$

So $\delta(x_1, y_1) = \nu_1(x_1, y_1)$ and $\delta(y_1, x_1) = \nu_1(x_1, y_1)$. Therefore

$$\begin{aligned}\alpha_i([x_1, y_1]_s) &= \alpha_i(x_1y_1 + y_1x_1)\\ &= \alpha_i(x_1y_1) + \alpha_i(y_1x_1)\\ &= [\alpha_i(x_1), \theta(y_1)]_s + (-1)^i[\theta(x_1), \alpha_i(y_1)]_s + 2\nu_1(x_1, y_1).\end{aligned}$$

Again applying α_i to (3.1), we get

$$\alpha_i([[x_1, y_0]_s, z_1]_s) = \alpha_i(x_1 \circ_s (y_0 \circ_s z_1)) - \alpha_i(y_0 \circ_s (x_1 \circ_s z_1)).$$

Extending the above expression, we have

$$\begin{aligned}&\alpha_i([[x_1, y_0]_s, z_1]_s)\\ &\quad= [\alpha_i([x_1, y_0]_s), \theta(z_1)]_s + (-1)^i[\theta([x_1, y_0]_s), \alpha_i(z_1)]_s + 2\nu_1([x_1, y_0]_s, z_1)\\ &\quad= [\alpha_i([x_1, y_0]_s), \theta(z_1)]_s + (-1)^i\theta(x_1)\theta(y_0)\alpha_i(z_1) - (-1)^i\theta(y_0)\theta(x_1)\alpha_i(z_1)\\ &\qquad+ \alpha_i(z_1)\theta(x_1)\theta(y_0) - \alpha_i(z_1)\theta(y_0)\theta(x_1) + 2\nu_1([x_1, y_0]_s, z_1)\end{aligned}$$

and

$$\begin{aligned}&\alpha_i(x_1 \circ_s (y_0 \circ_s z_1)) - \alpha_i(y_0 \circ_s (x_1 \circ_s z_1))\\ &\quad= \alpha_i(x_1) \circ_s \theta(y_0 \circ_s z_1) + (-1)^i\theta(x_1) \circ_s \alpha_i(y_0 \circ_s z_1)\\ &\qquad- \alpha_i(y_0) \circ_s \theta(x_1 \circ_s z_1) - \theta(y_0) \circ_s \alpha_i(x_1 \circ_s z_1)\\ &\quad= \alpha_i(x_1) \circ_s \theta(y_0 \circ_s z_1) + (-1)^i\theta(x_1) \circ_s (\alpha_i(y_0) \circ_s \theta(z_1))\\ &\qquad+ (-1)^i\theta(x_1) \circ_s (\theta(y_0) \circ_s \alpha_i(z_1)) - \alpha_i(y_0) \circ_s (\theta(x_1) \circ_s \theta(z_1))\\ &\qquad- \theta(y_0) \circ_s (\alpha_i(x_1) \circ_s \theta(z_1)) - (-1)^i\theta(y_0) \circ_s (\theta(x_1) \circ_s \alpha_i(z_1))\\ &\quad= \alpha_i(x_1)\theta(y_0)\theta(z_1) + \alpha_i(x_1)\theta(z_1)\theta(y_0) - (-1)^i\theta(y_0)\theta(z_1)\alpha_i(x_1)\\ &\qquad- (-1)^i\theta(z_1)\theta(y_0)\alpha_i(x_1) + (-1)^i\theta(x_1)\alpha_i(y_0)\theta(z_1) + \theta(x_1)\theta(z_1)\alpha_i(y_0)\\ &\qquad- \alpha_i(y_0)\theta(z_1)\theta(x_1) - (-1)^i\theta(z_1)\alpha_i(y_0)\theta(x_1) + (-1)^i\theta(x_1)\theta(y_0)\alpha_i(z_1)\\ &\qquad+ (-1)^i\theta(x_1)\alpha_i(z_1)\theta(y_0) - \theta(y_0)\alpha_i(z_1)\theta(x_1) - \alpha_i(z_1)\theta(y_0)\theta(x_1)\\ &\qquad- \alpha_i(y_0)\theta(x_1)\theta(z_1) + \alpha_i(y_0)\theta(z_1)\theta(x_1) - \theta(x_1)\theta(z_1)\alpha_i(y_0)\\ &\qquad+ \theta(z_1)\theta(x_1)\alpha_i(y_0) - \theta(y_0)\alpha_i(x_1)\theta(z_1) + (-1)^i\theta(y_0)\theta(z_1)\alpha_i(x_1)\\ &\qquad- \alpha_i(x_1)\theta(z_1)\theta(y_0) + (-1)^i\theta(z_1)\alpha_i(x_1)\theta(y_0) - (-1)^i\theta(y_0)\theta(x_1)\alpha_i(z_1)\\ &\qquad+ \theta(y_0)\alpha_i(z_1)\theta(x_1) - (-1)^i\theta(x_1)\alpha_i(z_1)\theta(y_0) + \alpha_i(z_1)\theta(x_1)\theta(y_0)\end{aligned}$$

for all $x_1, z_1 \in A_1$, $y_0 \in A_0$. Comparing the above expressions, we obtain

$$\begin{aligned}
&[\alpha_i([x_1, y_0]_s), \theta(z_1)]_s + 2\nu_1([x_1, y_0]_s, z_1)\\
&\qquad = \alpha_i(x_1)\theta(y_0)\theta(z_1) + (-1)^i\theta(x_1)\alpha_i(y_0)\theta(z_1) - \alpha_i(y_0)\theta(x_1)\theta(z_1)\\
&\qquad\quad - \theta(y_0)\alpha_i(x_1)\theta(z_1) + (-1)^i\theta(z_1)\alpha_i(x_1)\theta(y_0) + \theta(z_1)\theta(x_1)\alpha_i(y_0)\\
&\qquad\quad - (-1)^i\theta(z_1)\alpha_i(y_0)\theta(x_1) - (-1)^i\theta(z_1)\theta(y_0)\alpha_i(x_1)\\
&\qquad = [\alpha_i(x_1)\theta(y_0) + (-1)^i\theta(x_1)\alpha_i(y_0), \theta(z_1)]_s\\
&\qquad\quad - [\alpha_i(y_0)\theta(x_1) + \theta(y_0)\alpha_i(x_1), \theta(z_1)]_s.
\end{aligned}$$

So $[\delta(x_1, y_0)-\delta(y_0, x_1), \theta(z_1)]_s = -2\nu_1([x_1, y_0]_s, z_1)$. As $\delta(x_1, y_0) = -\delta(y_0, x_1)$, we have

$$[\delta(x_1, y_0), \theta(z_1)]_s = -\nu_1([x_1, y_0]_s, z_1). \tag{3.5}$$

By [22, Theorem 3.8], we have

$$\begin{aligned}
\delta(x_1, y_0) = {}& \lambda'_1\theta(x_1)\theta(y_0) + \lambda'_2\theta(y_0)\theta(x_1)\\
&+ \mu'_1(x_1)\theta(y_0) + \mu'_2(y_0)\theta(x_1) + \nu_2(x_1, y_0),
\end{aligned} \tag{3.6}$$

where $\lambda'_1, \lambda'_2 \in C + C\omega$, $\mu'_1 : A_1 \to C + C\omega$, $\mu'_2 : A_0 \to C + C\omega$, and $\nu_2 : A_1 \times A_0 \to C + C\omega$. Substituting (3.6) into (3.5), we find that

- the coefficient of $\theta(x_1)\theta(y_0)\theta(z_1)$ in (3.5) is λ'_1;
- the coefficient of $\theta(y_0)\theta(x_1)\theta(z_1)$ in (3.5) is λ'_2;
- the coefficient of $\theta(y_0)\theta(z_1)$ in (3.5) is $\mu'_1(x_1)$;
- the coefficient of $\theta(x_1)\theta(z_1)$ in (3.5) is $\mu'_2(y_0)$.

Again applying [22, Theorem 3.7], we get

$$\lambda'_1 = \lambda'_2 = \mu'_1 = \mu'_2 = 0.$$

Therefore $\delta(x_1, y_0) = \nu_2(x_1, y_0)$ and $\delta(x_0, y_1) = -\delta(y_1, x_0) = -\nu_2(y_1, x_0)$.

We shall now compute $\alpha_i(x_1y_0z_1)$ in two different ways. On the one hand,

$$\begin{aligned}
\alpha_i(x_1y_0z_1) &= \alpha_i(x_1y_0)\theta(z_1) + (-1)^i\theta(x_1)\theta(y_0)\alpha_i(z_1) + \nu_1(x_1y_0, z_1)\\
&= \big(\alpha_i(x_1)\theta(y_0) + (-1)^i\theta(x_1)\alpha_i(y_0) + \nu_2(x_1, y_0)\big)\theta(z_1)\\
&\quad + (-1)^i\theta(x_1)\theta(y_0)\alpha_i(z_1) + \nu_1(x_1y_0, z_1).
\end{aligned}$$

On the other hand,

$$\begin{aligned}
\alpha_i(x_1y_0z_1) &= \alpha_i(x_1)\theta(y_0z_1) + (-1)^i\theta(x_1)\alpha_i(y_0z_1) + \nu_1(x_1, y_0z_1)\\
&= \alpha_i(x_1)\theta(y_0)\theta(z_1) + (-1)^i\theta(x_1)\big(\alpha_i(y_0)\theta(z_1)\\
&\qquad + \theta(y_0)\alpha_i(z_1) - \nu_2(z_1, y_0)\big) + \nu_1(x_1, y_0z_1)
\end{aligned}$$

for all $x_1, z_1 \in A_1$, $y_0 \in A_0$. Comparing both expressions, we get

$$\nu_2(x_1, y_0)\theta(z_1) + \nu_1(x_1y_0, z_1) = -(-1)^i\theta(x_1)\nu_2(z_1, y_0) + \nu_1(x_1, y_0z_1).$$

By [22, Theorem 3.7], we have $\nu_2(x_1, y_0) = 0$. So $\delta(x_1, y_0) = \delta(y_0, x_1) = 0$.

Computing $\alpha_i(x_1y_1z_1)$ in two different ways, we have

$$\begin{aligned}\alpha_i(x_1y_1z_1) &= \alpha_i(x_1)\theta(y_1z_1) + (-1)^i\theta(x_1)\alpha_i(y_1z_1)\\ &= \alpha_i(x_1)\theta(y_1)\theta(z_1) + (-1)^i\theta(x_1)\big(\alpha_i(y_1)\theta(z_1)\\ &\qquad + (-1)^i\theta(y_1)\alpha_i(z_1) + \nu_1(y_1,z_1)\big)\end{aligned}$$

and

$$\begin{aligned}\alpha_i(x_1y_1z_1) &= \alpha_i(x_1y_1)\theta(z_1) + \theta(x_1y_1)\alpha_i(z_1)\\ &= \big(\alpha_i(x_1)\theta(y_1) + (-1)^i\theta(x_1)\alpha_i(y_1) + \nu_1(x_1,y_1)\big)\theta(z_1)\\ &\quad + \theta(x_1)\theta(y_1)\alpha_i(z_1)\end{aligned}$$

for all $x_1, y_1, z_1 \in A_1$. Comparing the above relations, we get

$$(-1)^i\theta(x_1)\nu_1(y_1,z_1) = \nu_1(x_1,y_1)\theta(z_1).$$

By [22, Theorem 3.7], we have $\nu_1(x_1,y_1) = 0$. It follows that $\delta(x_1,y_1) = 0$. Therefore α_i is a (θ,θ)-superderivation of degree i. ∎

By [22, Theorem 4.16] and Lemma 3.1, we have

Theorem 3.2. *Let $A = A_0 \oplus A_1$ be a prime superalgebra with maximal right ring of quotients Q and extended centroid C. Suppose that θ is an automorphism of A and $\alpha : A \to A$ is a Jordan (θ,θ)-superderivation. If* $\deg(A_1) \geq 9$, *then $\alpha : A \to A$ is a (θ,θ)-superderivation.*

The remainder of this section will be devoted to the study of generalized Jordan (θ,θ)-superderivations.

Lemma 3.3. *Let A be a prime superalgebra with maximal right ring of quotients Q and extended centroid C. Suppose that θ is an automorphism of A and $\phi_i : A \to A$ is a generalized Jordan (θ,θ)-superderivation of degree $i \in \{0,1\}$. If A is a* 4*-superfree subset of Q, then $\phi_i : A \to A$ is a generalized (θ,θ)-superderivation of degree i.*

Proof. By definition, we set

$$\begin{aligned}\phi_i(x \circ_s y) &= \phi_i(x)\theta(y) + (-1)^{i|y|+|x|\,|y|}\theta(y)\alpha_i(x)\\ &\quad + (-1)^{i|x|}\theta(x)\alpha_i(y) + (-1)^{|x|\,|y|}\phi_i(y)\theta(x)\end{aligned}$$

and $\pi : A \times A \to Q$ to satisfy

$$\pi(x,y) = \phi_i(xy) - \phi_i(x)\theta(y) - (-1)^{i|x|}\theta(x)\alpha_i(y)$$

for all $x, y \in A_0 \cup A_1$, where α_i is a Jordan (θ,θ)-superderivation of degree $i \in \{0,1\}$. Note that $\pi(x_1,y_0) = -\pi(y_0,x_1)$ and $\pi(x_1,y_1) = \pi(y_1,x_1)$.

Applying ϕ_i to (3.1), we get

$$\phi_i([[x_1,y_1]_s,z_0]_s) = \phi_i(x_1 \circ_s (y_1 \circ_s z_0)) + \phi_i(y_1 \circ_s (x_1 \circ_s z_0)). \tag{3.7}$$

By [18, Corollary 2], $\phi_i|A_0$ is a generalized (θ,θ)-derivation. Since α_i is a

(θ,θ)-superderivation of degree i, we obtain

$$\begin{aligned}\phi_i([[x_1,y_1]_s,z_0]_s) &= \phi_i([x_1,y_1]_s)\theta(z_0)+\theta([x_1,y_1]_s)\alpha_i(z_0)\\ &\quad -\phi_i(z_0)\theta([x_1,y_1]_s)-\theta(z_0)\alpha_i([x_1,y_1]_s)\\ &= \phi_i([x_1,y_1]_s)\theta(z_0)+\theta(x_1)\theta(y_1)\alpha_i(z_0)+\theta(y_1)\theta(x_1)\alpha_i(z_0)\\ &\quad -\phi_i(z_0)\theta(x_1)\theta(y_1)-\phi_i(z_0)\theta(y_1)\theta(x_1)\\ &\quad -\theta(z_0)\big(\alpha_i(x_1)\theta(y_1)+(-1)^i\theta(x_1)\alpha_i(y_1)+\alpha_i(y_1)\theta(x_1)\\ &\qquad\qquad +(-1)^i\theta(y_1)\alpha_i(x_1)\big)\end{aligned}$$

and

$$\begin{aligned}&\phi_i(x_1\circ_s(y_1\circ_s z_0))+\phi_i(y_1\circ_s(x_1\circ_s z_0))\\ &=\phi_i(x_1)\theta(y_1\circ_s z_0)-(-1)^i\theta(y_1\circ_s z_0)\alpha_i(x_1)+(-1)^i\theta(x_1)\alpha_i(y_1\circ_s z_0)\\ &\quad -\phi_i(y_1\circ_s z_0)\theta(x_1)+\phi_i(y_1)\theta(x_1\circ_s z_0)-(-1)^i\theta(x_1\circ_s z_0)\alpha_i(y_1)\\ &\quad +(-1)^i\theta(y_1)\alpha_i(x_1\circ_s z_0)-\phi_i(x_1\circ_s z_0)\theta(y_1)\\ &=\phi_i(x_1)\theta(y_1z_0+z_0y_1)-(-1)^i\theta(y_1z_0+z_0y_1)\alpha_i(x_1)\\ &\quad +(-1)^i\theta(x_1)\big(\alpha_i(y_1)\theta(z_0)+(-1)^i\theta(y_1)\alpha_i(z_0)+\alpha_i(z_0)\theta(y_1)+\theta(z_0)\alpha_i(y_1)\big)\\ &\quad -\big(\phi_i(y_1)\theta(z_0)+\theta(z_0)\alpha_i(y_1)+(-1)^i\theta(y_1)\alpha_i(z_0)+\phi_i(z_0)\theta(y_1)\big)\theta(x_1)\\ &\quad +\phi_i(y_1)\theta(x_1z_0+z_0x_1)-(-1)^i\theta(x_1z_0+z_0x_1)\alpha_i(y_1)\\ &\quad +(-1)^i\theta(y_1)\big(\alpha_i(x_1)\theta(z_0)+(-1)^i\theta(x_1)\alpha_i(z_0)+\alpha_i(z_0)\theta(x_1)+\theta(z_0)\alpha_i(x_1)\big)\\ &\quad -\big(\phi_i(x_1)\theta(z_0)+\theta(z_0)\alpha_i(x_1)+(-1)^i\theta(x_1)\alpha_i(z_0)+\phi_i(z_0)\theta(x_1)\big)\theta(y_1)\end{aligned}$$

for all $x_1,y_1\in A_1$, $z_0\in A_0$. Comparing the above relations, we have

$$\begin{aligned}\phi_i([x_1,y_1]_s)\theta(z_0) &= \phi_i(x_1)\theta(y_1)\theta(z_0)+(-1)^i\theta(x_1)\alpha_i(y_1)\theta(z_0)\\ &\quad +\phi_i(y_1)\theta(x_1)\theta(z_0)+(-1)^i\theta(y_1)\alpha_i(x_1)\theta(z_0).\end{aligned}$$

Therefore

$$\big(\pi(x_1,y_1)+\pi(y_1,x_1)\big)\theta(z_0)=0.$$

Since $\pi(x_1,y_1)=\pi(y_1,x_1)$, we obtain

$$\pi(x_1,y_1)\theta(z_0)=0.$$

Since A is a 4-superfree subset of Q, [22, Theorems 3.8 and 3.7] imply that $\pi(x_1,y_1)=0$. Therefore $\phi_i(x_1y_1)=\phi_i(x_1)\theta(y_1)+(-1)^i\theta(x_1)\alpha_i(y_1)$.

Again applying ϕ_i to (3.1), we get

$$\phi_i([[x_1,y_0]_s,z_1]_s)=\phi_i(x_1\circ_s(y_0\circ_s z_1))-\phi_i(y_0\circ_s(x_1\circ_s z_1))$$

for all $x_1,z_1\in A_1$, $y_0\in A_0$. Extending the above expression, we have

$$\begin{aligned}\phi_i([x_1,y_0]_s)\theta(z_1) &= \phi_i(x_1)\theta(y_0)\theta(z_1)+(-1)^i\theta(x_1)\alpha_i(y_0)\theta(z_1)\\ &\quad -\phi_i(y_0)\theta(x_1)\theta(z_1)-\theta(y_0)\alpha_i(x_1)\theta(z_1).\end{aligned}$$

So $(\pi(x_1, y_0) - \pi(y_0, x_1))\theta(z_1) = 0$. Since $\pi(x_1, y_0) = -\pi(y_0, x_1)$, we have

$$\pi(x_1, y_0)\theta(z_1) = 0.$$

By [22, Theorems 3.8 and 3.7], we get $\pi(x_1, y_0) = \pi(y_0, x_1) = 0$. So ϕ_i is a generalized (θ, θ)-superderivation of degree i. ∎

By [22, Theorem 4.16] and Lemma 3.3, we have

THEOREM 3.4. *Let $A = A_0 \oplus A_1$ be a prime superalgebra with maximal right ring of quotients Q and extended centroid C. Suppose that θ is an automorphism of A and $\phi : A \to A$ is a generalized Jordan (θ, θ)-superderivation. If* $\deg(A_1) \geq 9$*, then $\phi : A \to A$ is a generalized (θ, θ)-superderivation.*

In particular, when $\theta = 1$ in Theorems 3.2 and 3.4, we have

COROLLARY 3.5. *Let $A = A_0 \oplus A_1$ be a prime superalgebra with maximal right ring of quotients Q and extended centroid C. Suppose $\alpha : A \to A$ is a Jordan superderivation. If* $\deg(A_1) \geq 9$*, then $\alpha : A \to A$ is a superderivation.*

COROLLARY 3.6. *Let $A = A_0 \oplus A_1$ be a prime superalgebra with maximal right ring of quotients Q and extended centroid C. Suppose that $\phi : A \to A$ is a generalized Jordan superderivation. If* $\deg(A_1) \geq 9$*, then $\phi : A \to A$ is a generalized superderivation.*

4. Jordan triple superderivations. In the section, we will be concerned with Jordan triple superderivations and generalized Jordan triple superderivations. The following identity will be used frequently:

$$[x, [y, z]_s]_s = x \circ_s y \circ_s z - (-1)^{|y|\,|z|} x \circ_s z \circ_s y, \qquad x, y, z \in A_0 \cup A_1. \tag{4.1}$$

LEMMA 4.1. *Let A be a prime superalgebra with maximal right ring of quotients Q and extended centroid C. Suppose that θ is an automorphism of A and $\beta_i : A \to A$ is a Jordan triple (θ, θ)-superderivation of degree $i \in \{0, 1\}$. If A is a 4-superfree subset of Q, then $\beta_i : A \to A$ is a (θ, θ)-superderivation of degree i.*

Proof. As in the proof of Lemma 3.1, we define $\delta : A \times A \to Q$ by

$$\delta(x, y) = \beta_i(xy) - \beta_i(x)\theta(y) - (-1)^{i|x|}\theta(x)\beta_i(y)$$

for all $x, y \in A_0 \cup A_1$. Moreover, $\delta(x_1, y_0) = -\delta(y_0, x_1)$ and $\delta(x_1, y_1) = \delta(y_1, x_1)$.

Applying β_i to (4.1), we get

$$\beta_i([x_0, [y_1, z_1]_s]_s) = \beta_i(x_0 \circ_s y_1 \circ_s z_1) + \beta_i(x_0 \circ_s z_1 \circ_s y_1). \tag{4.2}$$

By [18, Theorem 1], $\beta_i|A_0$ is a (θ,θ)-derivation. So

$$\begin{aligned}\beta_i([x_0,[y_1,z_1]_s]_s) &= \beta_i(x_0)\theta(y_1)\theta(z_1)+\beta_i(x_0)\theta(z_1)\theta(y_1)-\theta(y_1)\theta(z_1)\beta_i(x_0)\\ &\quad -\theta(z_1)\theta(y_1)\beta_i(x_0)+[\theta(x_0),\beta_i([y_1,z_1]_s)]_s\end{aligned}$$

and

$$\begin{aligned}&\beta_i(x_0\circ_s y_1\circ_s z_1)+\beta_i(x_0\circ_s z_1\circ_s y_1)\\ &\quad=\beta_i(x_0)\circ_s\theta(y_1)\circ_s\theta(z_1)+\theta(x_0)\circ_s\beta_i(y_1)\circ_s\theta(z_1)\\ &\qquad+(-1)^i\theta(x_0)\circ_s\theta(y_1)\circ_s\beta_i(z_1)+\beta_i(x_0)\circ_s\theta(z_1)\circ_s\theta(y_1)\\ &\qquad+\theta(x_0)\circ_s\beta_i(z_1)\circ_s\theta(y_1)+(-1)^i\theta(x_0)\circ_s\theta(z_1)\circ_s\beta_i(y_1)\\ &\quad=\beta_i(x_0)\theta(y_1)\theta(z_1)+(-1)^i\theta(y_1)\beta_i(x_0)\theta(z_1)-(-1)^i\theta(z_1)\beta_i(x_0)\theta(y_1)\\ &\qquad-\theta(z_1)\theta(y_1)\beta_i(x_0)+\theta(x_0)\beta_i(y_1)\theta(z_1)+\beta_i(y_1)\theta(x_0)\theta(z_1)\\ &\qquad-(-1)^i\theta(z_1)\theta(x_0)\beta_i(y_1)-(-1)^i\theta(z_1)\beta_i(y_1)\theta(x_0)+(-1)^i\theta(x_0)\theta(y_1)\beta_i(z_1)\\ &\qquad+(-1)^i\theta(y_1)\theta(x_0)\beta_i(z_1)-\beta_i(z_1)\theta(x_0)\theta(y_1)-\beta_i(z_1)\theta(y_1)\theta(x_0)\\ &\qquad+\beta_i(x_0)\theta(z_1)\theta(y_1)+(-1)^i\theta(z_1)\beta_i(x_0)\theta(y_1)-(-1)^i\theta(y_1)\beta_i(x_0)\theta(z_1)\\ &\qquad-\theta(y_1)\theta(z_1)\beta_i(x_0)+\theta(x_0)\beta_i(z_1)\theta(y_1)+\beta_i(z_1)\theta(x_0)\theta(y_1)\\ &\qquad-(-1)^i\theta(y_1)\theta(x_0)\beta_i(z_1)-(-1)^i\theta(y_1)\beta_i(z_1)\theta(x_0)+(-1)^i\theta(x_0)\theta(z_1)\beta_i(y_1)\\ &\qquad+(-1)^i\theta(z_1)\theta(x_0)\beta_i(y_1)-\beta_i(y_1)\theta(x_0)\theta(z_1)-\beta_i(y_1)\theta(z_1)\theta(x_0)\end{aligned}$$

for all $x_0\in A_0$, $y_1,z_1\in A_1$. Comparing the above relations, we have

$$[\delta(y_1,z_1),\theta(x_0)]_s=0.$$

Analysis similar to that in the proof of Lemma 3.1 shows that $\delta(x_1,y_1)=\nu_1'(x_1,y_1)$ for all $x_1,y_1\in A_1$, where $\nu_1':A_1\times A_1\to C+C\omega$. Therefore

$$\beta_i([x_1,y_1]_s)=[\beta_i(x_1),\theta(y_1)]_s+(-1)^i[\theta(x_1),\beta_i(y_1)]_s+2\nu_1'(x_1,y_1)$$

for all $x_1,y_1\in A_1$.

Again applying β_i to (4.1), we get

$$\beta_i([x_1,[y_1,z_0]_s]_s)=\beta_i(x_1\circ_s y_1\circ_s z_0)-\beta_i(x_1\circ_s z_0\circ_s y_1)$$

for all $x_1,y_1\in A_1$ and $z_0\in A_0$. Extending the above expression, we have

$$[\theta(x_1),\delta(y_1,z_0)]_s=-(-1)^i\nu_1'(x_1,[y_1,z_0]_s).$$

By [22, Theorems 3.8 and 3.7], we get

$$\delta(x_1,y_0)=\nu_2'(x_1,y_0)\quad\text{and}\quad\delta(x_0,y_1)=-\delta(y_1,x_0)=-\nu_2'(y_1,x_0)$$

for all $x_0,y_0\in A_0$ and $x_1,y_1\in A_1$, where $\nu_2':A_1\times A_0\to C+C\omega$. Computing $\beta_i(x_1y_0z_1)$ and $\beta_i(x_1y_1z_1)$ in two different ways, we have

$$\nu_1'(x_1,y_1)=\nu_2'(x_1,y_0)=0\quad\text{and}\quad\delta(x_1,y_0)=\delta(y_0,x_1)=\delta(x_1,y_1)=0$$

for all $x_1,y_1\in A_1$ and $y_0\in A_0$. So β_i is a (θ,θ)-superderivation of degree i. ∎

By [22, Theorem 4.16] and the above result, we have

THEOREM 4.2. *Let $A = A_0 \oplus A_1$ be a prime superalgebra with maximal right ring of quotients Q and extended centroid C. Suppose that θ is an automorphism of A and $\beta : A \to A$ is a Jordan triple (θ, θ)-superderivation. If $\deg(A_1) \geq 9$, then $\beta : A \to A$ is a (θ, θ)-superderivation.*

Next we will study generalized Jordan triple (θ, θ)-superderivations of superalgebras.

LEMMA 4.3. *Let A be a prime superalgebra with maximal right ring of quotients Q and extended centroid C. Suppose that θ is an automorphism of A and $\xi_i : A \to A$ is a generalized Jordan triple (θ, θ)-superderivation of degree $i \in \{0, 1\}$. If A is a 4-superfree subset of Q, then $\xi_i : A \to A$ is a generalized (θ, θ)-superderivation of degree i.*

Proof. By assumption, we define $\pi : A \times A \to Q$ by

$$\pi(x, y) = \xi_i(xy) - \xi_i(x)\theta(y) - (-1)^{i|x|}\theta(x)\beta_i(y)$$

for all $x, y \in A_0 \cup A_1$, where β_i is a Jordan triple (θ, θ)-superderivation of degree $i \in \{0, 1\}$. Note that $\pi(x_1, y_0) = -\pi(y_0, x_1)$ and $\pi(x_1, y_1) = \pi(y_1, x_1)$.

Applying ξ_i to (4.1), we get

$$\xi_i([x_0, [y_1, z_1]_s]_s) = \xi_i(x_0 \circ_s y_1 \circ_s z_1) + \xi_i(x_0 \circ_s z_1 \circ_s y_1). \tag{4.3}$$

By [18, Theorem 3], $\xi_i|A_0$ is a generalized (θ, θ)-derivation. Since β_i is a (θ, θ)-superderivation of degree i, we have

$$\begin{aligned}\xi_i([x_0, [y_1, z_1]_s]_s) = {} & \xi_i(x_0)\theta(y_1)\theta(z_1) + \xi_i(x_0)\theta(z_1)\theta(y_1) + \theta(x_0)\big(\beta_i(y_1)\theta(z_1) \\ & + (-1)^i\theta(y_1)\beta_i(z_1) + \beta_i(z_1)\theta(y_1) + (-1)^i\theta(z_1)\beta_i(y_1)\big) \\ & - \xi_i([y_1, z_1]_s)\theta(x_0) - \theta(y_1)\theta(z_1)\beta_i(x_0) - \theta(z_1)\theta(y_1)\beta_i(x_0)\end{aligned}$$

and

$$\begin{aligned}&\xi_i(x_0 \circ_s y_1 \circ_s z_1) + \xi_i(x_0 \circ_s z_1 \circ_s y_1) \\ &\quad = \xi_i(x_0)\theta(y_1)\theta(z_1) + \xi_i(y_1)\theta(x_0)\theta(z_1) - \xi_i(z_1)\theta(x_0)\theta(y_1) - \xi_i(z_1)\theta(y_1)\theta(x_0) \\ &\qquad + \theta(x_0)\beta_i(y_1)\theta(z_1) + (-1)^i\theta(y_1)\beta_i(x_0)\theta(z_1) - (-1)^i\theta(z_1)\beta_i(y_1)\theta(x_0) \\ &\qquad - (-1)^i\theta(z_1)\beta_i(x_0)\theta(y_1) + (-1)^i\theta(x_0)\theta(y_1)\beta_i(z_1) + (-1)^i\theta(y_1)\theta(x_0)\beta_i(z_1) \\ &\qquad - (-1)^i\theta(z_1)\theta(x_0)\beta_i(y_1) - \theta(z_1)\theta(y_1)\beta_i(x_0) + \xi_i(x_0)\theta(z_1)\theta(y_1) \\ &\qquad + \xi_i(z_1)\theta(x_0)\theta(y_1) - \xi_i(y_1)\theta(x_0)\theta(z_1) - \xi_i(y_1)\theta(z_1)\theta(x_0) \\ &\qquad + \theta(x_0)\beta_i(z_1)\theta(y_1) + (-1)^i\theta(z_1)\beta_i(x_0)\theta(y_1) - (-1)^i\theta(y_1)\beta_i(z_1)\theta(x_0) \\ &\qquad - (-1)^i\theta(y_1)\beta_i(x_0)\theta(z_1) + (-1)^i\theta(x_0)\theta(z_1)\beta_i(y_1) + (-1)^i\theta(z_1)\theta(x_0)\beta_i(y_1) \\ &\qquad - (-1)^i\theta(y_1)\theta(x_0)\beta_i(z_1) - \theta(y_1)\theta(z_1)\beta_i(x_0)\end{aligned}$$

for all $x_0 \in A_0$, $y_1, z_1 \in A_1$. Since A is a 4-superfree subset of Q, we have $\xi_i(x_1y_1) = \xi_i(x_1)\theta(y_1) + (-1)^i\theta(x_1)\beta_i(y_1)$ for all $x_1, y_1 \in A_1$.

Likewise, applying ξ_i to (4.1), we get

$$\pi(x_1, y_0) = \pi(y_0, x_1) = 0$$

for all $x_1 \in A_1$, $y_0 \in A_0$. Therefore ξ_i is a generalized (θ, θ)-superderivation of degree i. ∎

By [22, Theorem 4.16] and the above result, we have

THEOREM 4.4. *Let $A = A_0 \oplus A_1$ be a prime superalgebra with maximal right ring of quotients Q and extended centroid C. Suppose that θ is an automorphism of A and $\xi : A \to A$ is a generalized Jordan triple (θ, θ)-superderivation. If $\deg(A_1) \geq 9$, then $\xi : A \to A$ is a generalized (θ, θ)-superderivation.*

In particular, when $\theta = 1$ in Theorems 4.2 and 4.4, we have

COROLLARY 4.5. *Let $A = A_0 \oplus A_1$ be a prime superalgebra with maximal right ring of quotients Q and extended centroid C. Suppose that $\beta : A \to A$ is a Jordan triple superderivation. If $\deg(A_1) \geq 9$, then $\beta : A \to A$ is a superderivation.*

COROLLARY 4.6. *Let $A = A_0 \oplus A_1$ be a prime superalgebra with maximal right ring of quotients Q and extended centroid C. Suppose that $\xi : A \to A$ is a generalized Jordan triple superderivation. If $\deg(A_1) \geq 9$, then $\xi : A \to A$ is a generalized superderivation.*

An easy computation shows that Jordan superderivations (resp., generalized Jordan superderivations) are Jordan triple superderivations (resp., generalized Jordan triple superderivations). We can also get Corollaries 3.5 and 3.6 from Corollaries 4.5 and 4.6.

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