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Blow-up of a nonlocal p-Laplacian evolution equation with critical initial energy

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Abstract. This paper is concerned with the initial boundary value problem for a nonlocal p-Laplacian evolution equation with critical initial energy. In the framework of the energy method, we construct an unstable set and establish its invariance. Finally, the finite time blow-up of solutions is derived by a combination of the unstable set and the concavity method.

1. Introduction. In this paper, we study the following initial boundary value problem for a nonlocal p-Laplacian evolution equation:

$$u_t - \Delta_p u = |u|^{q-1}u - \frac{1}{|\Omega|}\int_\Omega |u|^{q-1}u\,dx, \qquad x \in \Omega,\ t > 0, \tag{1.1}$$

$$u(x,0) = u_0(x) \not\equiv 0, \qquad \int_\Omega u_0\,dx = 0, \qquad x \in \Omega, \tag{1.2}$$

$$|\nabla u|^{p-2}\frac{\partial u}{\partial n} = 0, \qquad x \in \partial\Omega,\ t > 0, \tag{1.3}$$

where Ω is a bounded domain of $\mathbb{R}^N$ with a smooth boundary $\partial\Omega$, $|\Omega|$ denotes the Lebesgue measure of Ω, $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$, and p and q satisfy

$$\text{(A)} \qquad p \ge 2, \quad p-1 < q < \infty \text{ if } N \le p, \quad p-1 < q < \frac{Np}{N-p} - 1 \text{ if } N > p.$$

Equation (1.1) arises in fluid mechanics, biology, population dynamics, and combustion theory (see [2, 3, 8, 20, 26]). In fluid mechanics, (1.1) is a slow diffusion equation and can be used to describe the slow diffusion of

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concentration of non-Newtonian flow in a porous medium. So it is also called a non-Newtonian filtration equation.

In [7], it was shown that certain solutions of problem (1.1)–(1.3) with $E(u_0) < d$ (where $E(u_0)$ is the initial energy) blow up in finite time, where

$$d = \frac{q+1-p}{p(q+1)} C^{-\frac{p(q+1)}{q+1-p}}, \tag{1.4}$$

and C is the best Sobolev embedding constant from $W^{1,p}(\Omega)$ into $L^{q+1}(\Omega)$.

In addition, many equations related to (1.1) have been investigated. In [9], the nonlocal semilinear parabolic equation

$$u_t - \Delta u = |u|^{q-1}u - \frac{1}{|\Omega|} \int_\Omega |u|^{q-1}u \, dx \tag{1.5}$$

with the homogeneous Neumann boundary condition was studied, and a blow-up result for certain solutions with $E(u_0) < d$ was established, where d is defined by (1.4) and C is the best Sobolev embedding constant from $W^{1,2}(\Omega)$ into $L^{q+1}(\Omega)$.

In [18], Niculescu and Roventa investigated a nonlocal parabolic equation with general nonlinearities f,

$$u_t - \Delta u = f(|u|) - \frac{1}{|\Omega|} \int_\Omega f(|u|) \, dx, \tag{1.6}$$

and obtained the finite time blow-up of solutions, but restricted to $E(u_0) \le 0$. Niculescu and Roventa [19] also considered blow-up of solutions for a nonlocal p-Laplacian evolution equation with general nonlinearities f,

$$u_t - \Delta_p u = f(|u|) - \frac{1}{|\Omega|} \int_\Omega f(|u|) \, dx \tag{1.7}$$

under the same constraint that $E(u_0) \le 0$.

Obviously, although (1.6) and (1.7) are generalizations of (1.5) and (1.1) respectively, the energy range for blow-up of solutions to (1.5) and (1.1) is wider. The main results of [7, 9] mentioned above describe the finite time blow-up of solutions with $E(u_0) < d$, that is, with subcritical initial energy. In contrast, we are here mainly interested in the finite time blow-up of solutions for problem (1.1)–(1.3) with $E(u_0) = d$, that is, with critical initial energy. To our knowledge, much less effort has been devoted to this case.

The definition and local existence of weak solutions for problem (1.1)–(1.3) were stated in [7]. Our main results in this paper are the following

THEOREM 1.1. *Let $u(t)$ be a solution of problem* (1.1)–(1.3) *with p and q satisfying* (A). *Assume that $E(u_0) = d$ and $u_0 \in \mathcal{U}$, where d and $\mathcal{U}$ will be* *stated later. Then the solution $u(t)$ of problem* (1.1)–(1.3) *blows up in finite time.*

Theorem 1.1 shows that the solutions of problem (1.1)–(1.3) with critical initial energy also blow up in finite time.

COROLLARY 1.2. *If in Theorem* 1.1, $E(u_0) = d$ *is replaced by* $E(u_0) \le d$, *then the conclusion still holds.*

Thus, the blow-up result of [7] is extended to the case $E(u_0) \le d$. In other words, the energy range for blow-up of solutions to problem (1.1)–(1.3) is extended to $E(u_0) \le d$.

This paper is organized as follows. In Section 2 we recall some basic facts and obtain the energy identity associated to problem (1.1)–(1.3). Moreover, by calculating the mountain pass level d, we modify the unstable set $\mathcal{U}$ developed by Payne and Sattinger [21] and establish its invariance under the flow of problem (1.1)–(1.3) with critical initial energy, which is a technical innovation of this paper and plays an essential role in the proofs of our main results. In Section 3 we prove the main results by the concavity method [12, 13, 23, 30]. The relationship between initial energy and finite time blow-up of solutions is further showed in the proofs.

2. Preliminary results. We start by introducing some notation that will be used throughout this paper. For the standard $L^p(\Omega)$ space we write $\|u\|_p = \|u\|_{L^p(\Omega)}$, $\|u\| = \|u\|_{L^2(\Omega)}$, and $(u,v) = \int_\Omega uv\,dx$.

Noting that the nonlocal source term

$$\frac{1}{|\Omega|}\int_\Omega |u|^{q-1}u\,dx$$

is equivalent to a constant, we find that the integral on the right hand side of (1.1) is zero. Hence,

$$\frac{d}{dt}\int_\Omega u\,dx = \int_\Omega u_t\,dx = \int_\Omega \Delta_p u\,dx = 0,$$

that is, the integral of u over Ω is conserved. Therefore, by the initial condition (1.2), we conclude that each solution u of problem (1.1)–(1.3) satisfies

$$\int_\Omega u\,dx = 0.$$

We now consider the energy functional $E : W^{1,p}(\Omega) \to \mathbb{R}$ defined by

$$E(u) = \frac{1}{p}\|\nabla u\|_p^p - \frac{1}{q+1}\|u\|_{q+1}^{q+1}.$$

Clearly, E is of class C^1 over $W^{1,p}(\Omega)$, and critical points of E are solutions of the following stationary problem associated to problem (1.1)–(1.3):

$$
(2.1)\qquad \begin{cases} -\Delta_p u = |u|^{q-1}u - \dfrac{1}{|\Omega|}\displaystyle\int_\Omega |u|^{q-1}u\,dx & \text{in } \Omega, \\ |\nabla u|^{p-2}\dfrac{\partial u}{\partial n} = 0 & \text{on } \partial\Omega. \end{cases}
$$

Note that assumption (A) is a compactness condition for the embedding $W^{1,p}(\Omega) \hookrightarrow L^{q+1}(\Omega)$. According to the critical point theory, it is a necessary and sufficient condition for the validity of the Palais–Smale condition. In view of the Mountain Pass Theorem of Ambrosetti and Rabinowitz [1], problem (2.1) admits a positive mountain pass solution whose energy d can be characterized by

$$
d = \inf_{u \in W^{1,p}(\Omega)\setminus\{0\}} \sup_{\lambda>0} E(\lambda u).
$$

The number d is called the mountain pass level; it is generally estimated by constructing the variational problem in order to investigate the global well-posedness of solutions for various nonlinear evolution equations (see e.g. ([4–6, 10, 11, 14–17, 21, 22, 24, 27–29]). Indeed, once the value of d is obtained, the problem concerned will be simplified. This is also inspired by the above-mentioned papers.

Now we are in a position to obtain the expression of d by applying the idea of Vitillaro [25] where a different purpose was achieved.

Definition 2.1.

$$
d = \max_{y\in[0,\infty)} g(y),
$$

where

$$
y = \|\nabla u\|_p, \quad g(y) = \frac{1}{p}y^p - \frac{C^{q+1}}{q+1}y^{q+1},
$$

and C is the best Sobolev constant for the embedding inequality $\|u\|_{q+1} \le C\|\nabla u\|_p$, i.e.

$$
C = \sup_{u\in W^{1,p}(\Omega)\setminus\{0\}} \frac{\|u\|_{q+1}}{\|\nabla u\|_p}.
$$

Lemma 2.2.

$$
d = \frac{q+1-p}{p(q+1)} C^{-\frac{p(q+1)}{q+1-p}}.
$$

Proof. Let $g'(y_0) = 0$. By Definition 2.1 we get

$$
y_0 = C^{-\frac{q+1}{q+1-p}}.
$$

It is easy to see that $g(y)$ is strictly increasing for $0 \le y < y_0$, strictly decreasing for $y > y_0$, $\lim_{y\to\infty} g(y) = -\infty$, and

$$
d = g(y_0) = \frac{q+1-p}{p(q+1)} C^{-\frac{p(q+1)}{q+1-p}}. \quad \blacksquare
$$

Next, we define the following *unstable set* by using d.

Definition 2.3.

$$\mathcal{U} = \left\{ u \in W^{1,p}(\Omega) \;\middle|\; \|\nabla u\|_p > \left(\frac{p(q+1)}{q+1-p} d \right)^{1/p} \right\}.$$

Clearly, $W^{1,p}(\Omega) = \mathcal{U} \cup \mathcal{U}^c$, where

$$\mathcal{U}^c = \left\{ u \in W^{1,p}(\Omega) \;\middle|\; \|\nabla u\|_p \le \left(\frac{p(q+1)}{q+1-p} d \right)^{1/p} \right\}$$

and

$$\partial\mathcal{U} = \partial\mathcal{U}^c = \left\{ u \in W^{1,p}(\Omega) \;\middle|\; \|\nabla u\|_p = \left(\frac{p(q+1)}{q+1-p} d \right)^{1/p} \right\}.$$

Here we mention that the unstable set as defined in this paper is different from those appearing in previous work [5, 6, 10, 11, 15, 16, 21, 27–29] Definition 2.3 makes it easy to understand the structure of the unstable set.

Lemma 2.4. *Let p and q satisfy* (A).

(i) *If $\|\nabla u\|_p = (p(q+1)d/(q+1-p))^{1/p}$, then $\|\nabla u\|_p^p \ge \|u\|_{q+1}^{q+1}$.*
(ii) *If $\|\nabla u\|_p^p < \|u\|_{q+1}^{q+1}$, then $\|\nabla u\|_p > (p(q+1)d/(q+1-p))^{1/p}$.*

Proof. (i) From Lemma 2.2 and

$$\|\nabla u\|_p = \left(\frac{p(q+1)}{q+1-p} d \right)^{1/p},$$

we obtain

$$\|\nabla u\|_p = C^{-\frac{q+1}{q+1-p}}.$$

Noting that $\|\nabla u\|_p \neq 0$, we have

$$\|\nabla u\|_p^p = C^{q+1} \|\nabla u\|_p^{q+1}.$$

Hence, $\|\nabla u\|_p^p \ge \|u\|_{q+1}^{q+1}$.

(ii) From $\|\nabla u\|_p^p < \|u\|_{q+1}^{q+1}$, it follows that $\|\nabla u\|_p \neq 0$ and

$$\|\nabla u\|_p^p < \|u\|_{q+1}^{q+1} \le C^{q+1} \|\nabla u\|_p^{q+1}.$$

A simple calculation yields

$$\|\nabla u\|_p > C^{-\frac{q+1}{q+1-p}}.$$

Furthermore, by Lemma 2.2 we obtain

$$\|\nabla u\|_p > \left(\frac{p(q+1)}{q+1-p} d \right)^{1/p}. \ \blacksquare$$

Let $u(t)$ be a solution of problem (1.1)–(1.3) with $u_0 \in W^{1,p}(\Omega)$. From Green's formula and the Neumann–Robin boundary condition (1.3) we obtain

$$\begin{aligned}\frac{dE(u(t))}{dt} &= (|\nabla u|^{p-2}\nabla u, \nabla u_t) - (|u|^{q-1}u, u_t)\\ &= \int_{\partial\Omega} \frac{\partial u}{\partial n}|\nabla u|^{p-2}u_t\,ds - (\Delta_p u, u_t) - (|u|^{q-1}u, u_t)\\ &= -(\Delta_p u + |u|^{q-1}u, u_t)\\ &= -\Big(u_t + \frac{1}{|\Omega|}\int_\Omega |u|^{q-1}u\,dx, u_t\Big)\\ &= -\|u_t\|^2 - \frac{1}{|\Omega|}\int_\Omega |u|^{q-1}u\,dx \int_\Omega u_t\,dx = -\|u_t\|^2.\end{aligned}$$

Therefore, $E(u)$ is decreasing in time. Moreover, by integrating both sides of the above equality over $[0,t]$, for all t we have the identity

$$\int_0^t \|u_t(\tau)\|^2\,d\tau + E(u(t)) = E(u_0). \tag{2.2}$$

Lemma 2.5. *Let $u(t)$ be a solution of problem* (1.1)–(1.3) *with p and q satisfying* (A). *Assume that $u_0 \in W^{1,p}(\Omega)$ and $E(u_0) \le d$. Then $\|\nabla u\|_p^p < \|u\|_{q+1}^{q+1}$ if and only if $\|\nabla u\|_p > (p(q+1)d/(q+1-p))^{1/p}$, for all $0 \le t < \infty$.*

Proof. From $E(u_0) \le d$, (2.2) and

$$\begin{aligned}E(u) &= \frac{1}{p}\|\nabla u\|_p^p - \frac{1}{q+1}\|u\|_{q+1}^{q+1}\\ &= \frac{q+1-p}{p(q+1)}\|\nabla u\|_p^p + \frac{1}{q+1}(\|\nabla u\|_p^p - \|u\|_{q+1}^{q+1}),\end{aligned}$$

it follows that

$$\frac{q+1-p}{p(q+1)}\|\nabla u\|_p^p + \frac{1}{q+1}(\|\nabla u\|_p^p - \|u\|_{q+1}^{q+1}) \le d, \tag{2.3}$$

for all $0 \le t < \infty$. If $\|\nabla u\|_p > (p(q+1)d/(q+1-p))^{1/p}$, then by (2.3) we get $\|\nabla u\|_p^p < \|u\|_{q+1}^{q+1}$. On the other hand, if $\|\nabla u\|_p^p < \|u\|_{q+1}^{q+1}$, then we conclude from Lemma 2.4(ii) that $\|\nabla u\|_p > (p(q+1)d/(q+1-p))^{1/p}$. ∎

Lemma 2.6. *Let $u(t)$ be a solution of problem* (1.1)–(1.3) *with p and q satisfying* (A). *Assume that $u_0 \in \mathcal{U}$ and $E(u_0) = d$. Then $u(t) \in \mathcal{U}$ for all $0 < t < \infty$.*

Proof. Suppose that $u(t) \notin \mathcal{U}$ for some $0 < t < \infty$. Then from $u_0 \in \mathcal{U}$ and the continuity of $u(t)$, we see that there exists the first time $t_0 \in (0,\infty)$ such that $u(t_0) \in \partial\mathcal{U}$ and $u(t) \in \mathcal{U}$ for all $0 \le t < t_0$. Hence, by recalling

Definition 2.3, we get

$$\|\nabla u(t_0)\|_p = \left(\frac{p(q+1)}{q+1-p}d\right)^{1/p}, \tag{2.4}$$

and for all $0 \le t < t_0$ we have

$$\|\nabla u(t)\|_p > \left(\frac{p(q+1)}{q+1-p}d\right)^{1/p}. \tag{2.5}$$

From (2.4) and Lemma 2.4(i) we obtain $\|\nabla u(t_0)\|_p^p \ge \|u(t_0)\|_{q+1}^{q+1}$. Consequently,

$$\begin{aligned} E(u(t_0)) &= \frac{1}{p}\|\nabla u(t_0)\|_p^p - \frac{1}{q+1}\|u(t_0)\|_{q+1}^{q+1} \\ &\ge \frac{q+1-p}{p(q+1)}\|\nabla u(t_0)\|_p^p = d. \end{aligned} \tag{2.6}$$

On the other hand, for $0 \le t < \infty$, let

$$M(t) = \frac{1}{2}\int_0^t \|u(\tau)\|^2\, d\tau.$$

Then $M'(t) = \frac{1}{2}\|u(t)\|^2$ and

$$\begin{aligned} M''(t) = (u, u_t) &= \left(u, \Delta_p u + |u|^{q-1}u - \frac{1}{|\Omega|}\int_\Omega |u|^{q-1}u\, dx\right) \\ &= (u, \Delta_p u) + (u, |u|^{q-1}u) - \frac{1}{|\Omega|}\int_\Omega |u|^{q-1}u\, dx \int_\Omega u\, dx \\ &= -\|\nabla u\|_p^p + \|u\|_{q+1}^{q+1}. \end{aligned}$$

Combining this with (2.5) and Lemma 2.5, we conclude that $M''(t) > 0$ for $0 \le t < t_0$. As a consequence,

$$M'(t) > M'(0) = \tfrac{1}{2}\|u_0\|^2 \ge 0$$

for $0 < t < t_0$. Finally, $M(t_0) > M(0) = 0$, i.e.

$$\int_0^{t_0} \|u(\tau)\|^2\, d\tau > 0, \tag{2.7}$$

which together with

$$\int_0^{t_0} \|u(\tau)\|^2\, d\tau + E(u(t_0)) = E(u_0) = d$$

gives $E(u(t_0)) < d$. Obviously, this contradicts (2.6). ∎

Clearly, Lemma 2.6 shows the invariance of $\mathcal{U}$ under the flow of problem (1.1)–(1.3) with critical initial energy.

COROLLARY 2.7. *If in Lemma 2.6, $E(u_0) = d$ is replaced by $E(u_0) \le d$, then the conclusion still holds.*

Proof. In view of the proof of Lemma 2.6, it is easy to obtain (2.6). Moreover, by considering the same auxiliary function $M(t)$ we get (2.7) and

$$\int_0^{t_0} \|u(\tau)\|^2\, d\tau + E(u(t_0)) = E(u_0) \le d.$$

Consequently, $E(u(t_0)) < d$, which contradicts (2.6). ∎

3. Proofs of main results

Proof of Theorem 1.1. Let T be the maximal existence time of $u(t)$. We will prove $T < \infty$. Indeed, suppose $T = \infty$. Let

$$M(t) = \frac{1}{2}\int_0^t \|u(\tau)\|^2\, d\tau.$$

Then $M'(t) = \frac{1}{2}\|u(t)\|^2$ and

$$M''(t) = (u, u_t) = -\|\nabla u\|_p^p + \|u\|_{q+1}^{q+1}. \tag{3.1}$$

From (2.2) and

$$E(u(t)) = \frac{q+1-p}{p(q+1)}\|\nabla u\|_p^p + \frac{1}{q+1}(\|\nabla u\|_p^p - \|u\|_{q+1}^{q+1}),$$

it follows that

$$\|\nabla u\|_p^p - \|u\|_{q+1}^{q+1} = -\frac{q+1-p}{p}\|\nabla u\|_p^p + (q+1)E(u_0) - (q+1)\int_0^t \|u_t(\tau)\|^2\, d\tau.$$

Combining this with (3.1), we get

$$M''(t) = \frac{q+1-p}{p}\|\nabla u\|_p^p - (q+1)E(u_0) + (q+1)\int_0^t \|u_t(\tau)\|^2\, d\tau. \tag{3.2}$$

From $u_0 \in \mathcal{U}$, Lemma 2.6 and Definition 2.3, for all $0 < t < \infty$ we may write

$$\|\nabla u\|_p > \left(\frac{p(q+1)}{q+1-p}d\right)^{1/p},$$

which together with (3.2) and $E(u_0) = d$ yields

$$M''(t) > (q+1)\int_0^t \|u_t(\tau)\|^2\, d\tau \tag{3.3}$$

for all $0 < t < \infty$. Hence, there exists a $t_0 > 0$ such that $M'(t) \ge M'(t_0) > 0$ and $M(t) \ge M'(t_0)(t - t_0) + M(t_0)$ for $t \ge t_0$. Consequently,

$$\lim_{t\to\infty} M(t) = \infty.$$

On the other hand, from (3.3) and the Cauchy–Schwarz inequality we deduce that

$$M(t)M''(t) > \frac{q+1}{p}\int_0^t \|u(\tau)\|^2\,d\tau \int_0^t \|u_t(\tau)\|^2\,d\tau$$
$$\geq \frac{q+1}{p}\Big(\int_0^t (u,u_t)\,d\tau\Big)^2 = \frac{q+1}{p}(M'(t)-M'(0))^2.$$

Thus, there exists an $\alpha > 0$ such that

$$M(t)M''(t) > (1+\alpha)(M'(t))^2, \quad t \geq t_0.$$

Therefore, $M^{-\alpha}(t)$ is concave on $[t_0,\infty)$, $M^{-\alpha}(t) > 0$, and $\lim_{t\to\infty} M(t) = \infty$. This is a contradiction. ∎

Proof of Corollary 1.2. According to the proof of Theorem 1.1, it is easy to get (3.2). We conclude from $u_0 \in \mathcal{U}$ and Corollary 2.7 that $u(t) \in \mathcal{U}$ for all $0 < t < \infty$. Combining this with Definition 2.3, (3.2) and $E(u_0) \leq d$, we obtain (3.3). The remainder of the proof is the same as that of Theorem 1.1. ∎

REMARK 3.1. Note that (1.5) could be regarded as the special case of (1.1) when $p = 2$. Therefore, the idea of this paper can also be utilized to consider blow-up of (1.5) with critical initial energy.

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