are continuous functions, we have, according to (15), the following equalities

$$\sum_{j=1}^{n} f_j T'^s g_j = 0 \qquad (s = 1, 2, \ldots, n).$$

In other words the function

$$\psi(x, y) = \sum_{j=1}^{n} f_j(x) g_j(y)$$

is extinguished by the set E. Since not all function $f_1, f_2, \ldots, f_n$ vanish and $g_1, g_2, \ldots, g_n$ are linearly independent, $\psi(x, y)$ is not identically equal to 0 in the first quadrant. Thus $E \notin \mathfrak{E}_n$ and, consequently, $P_n > n$.

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A proof of Schwartz's theorem on kernels

by

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L. Schwartz has shown that every bilinear continuous functional $B(\varphi_1, \varphi_2)$ on the space $D(\Omega_1) \times D(\Omega_2)$ (see the definition below) may be represented by a linear continuous functional T on the space $D(\Omega_1 \times \Omega_2)$, i. e.

$$B(\varphi_1, \varphi_2) = T(\varphi_1 \times \varphi_2) \quad \text{for} \quad \varphi_i \in D(\Omega_i), \quad i = 1, 2, \tag{1}$$

where $(\varphi_1 \times \varphi_2)(x_1, x_2) = \varphi_1(x_1) \cdot \varphi_2(x_2)$ for $x_i \in \Omega_i$, $i = 1, 2$.

Since every such functional corresponds to a linear continuous map L of $D(\Omega_1)$ into $D'(\Omega_2)$ defined by

$$(L\varphi_1)(\varphi_2) = B(\varphi_1, \varphi_2),$$

equality (1) may be written symbolically in the form

$$L(\varphi_1)(x_2) = \int T(x_1, x_2) \varphi_1(x_1) dx_1 \quad \text{for any} \quad \varphi_1 \in D(\Omega_1) \tag{2}$$

and therefore Schwartz's theorem may be interpreted as a theorem concerning representation of linear continuous operations by kernels. The theorem is a special case of a general theorem of A. Grothendieck on topological tensor products.

The purpose of this paper is to give a simple proof of Grothendieck's theorem for a special case which often occurs in applications. The proof is based only on elementary properties of (F)-spaces ((B_0)-spaces in the Polish terminology) and (LF)-spaces.

For the convenience of the reader we shall make a short review of the properties to be used in the paper.

1. Let X be a linear space over the complex field. Given a family of seminorms $\|x\|_\alpha$ $(\alpha \in A)$ on X, we can define a topology on X taking the family of sets $\{x: \|x - x_0\|_{\alpha_i} < \varepsilon, \ i = 1, 2, \ldots, n\}$ as a fundamental system of neighbourhoods of the point x_0.

This topology is a Hausdorff topology if and only if the family of semi-norms is separating, i. e. if, for every $x \neq 0$, there is an $\alpha \in A$ such that $\|x\|_a \neq 0$. The space X with such a Hausdorff topology will be called a *locally convex space*. It is metrizable if and only if there exists a denumerable family of semi-norms $\|x\|_\beta$ $(\beta \in B)$ equivalent to the family $\|x\|_a$ $(a \in A)$ (i. e. such that the topology corresponding to it is identical with the topology corresponding to the family $\|x\|_a$ $(a \in A)$).

The locally convex space X is said to be *semi-complete* if every Cauchy sequence is convergent, i. e. if for every sequence (x_n) satisfying the condition $\lim\limits_{n,m}\|x_n - x_m\|_a = 0$ for all $a \in A$ there exists an element $x_0 \in X$ such that $\lim\limits_{n}\|x_n - x_0\|_a = 0$ for all $a \in A$.

For functions defined on a Jordan measurable set Ω and with values in a locally convex space we may define in the usual way a Riemann integral over the set Ω.

If the space X is semi-complete and $T(x)$ is a continuous linear functional on it, and if a function $x(t)$ is continuous on the closure of the set Ω, then the integrals given below exist in Riemann's sense and we have the following relation between them:

$$\Big\|\int_\Omega x(t)\,dt\Big\|_a \leqslant \int_\Omega \|x(t)\|_a dt \quad (a \in A), \qquad \int_\Omega T\big(x(t)\big)\,dt = T\Big(\int_\Omega x(t)\,dt\Big).$$

The family $\|x\|_a$ $(a \in A)$ of semi-norms is a base if the sets $\{x: \|x - x_0\|_a < \varepsilon\}$ form a fundamental system of neighbourhoods of the point x_0. If we suppose the family $\|x\|_a$ $(a \in A)$ to be a base, then a linear functional $T(x)$ is continuous on X if and only if, for an $a \in A$ and for a number M, the condition

$$|T(x)| \leqslant M\|x\|_a$$

is satisfied on X.

Now let X and Y be (F)-spaces, i. e. locally convex, metrizable, semi-complete spaces, and let $\|x\|_a$ $(a \in A)$, $\|x\|_\beta$ $(\beta \in B)$ be their denumerable bases of semi-norms. These spaces have the following properties to be used in the paper. If (T_n) is a sequence of continuous linear functionals on X such that $T_n(x)$ tends to a limit $T(x)$ for every $x \in X$, then $T(x)$ is a continuous linear functional on X. If $T(x, y)$ is a bilinear functional on the product space $X \times Y$ and if it is continuous with respect to each variable separately, then it is continuous with respect to both variables simultanuously. The functional $T(x, y)$ is continuous on $X \times Y$ if and only if

$$|T(x, y)| \leqslant M\|x\|_a\|y\|_\beta \quad \text{for all} \quad x \in X,\ y \in Y,$$

and for a number M, an $a \in A$, and a $\beta \in B$.

Definition 1. Let Ω be a compact set in the Euclidean space E_N and let $D(\Omega)$ denote the set of all complex valued functions such that they possess continuous derivatives of all orders on the space E_N and their supports are contained in the set Ω. Let us put

$$\|\varphi\|_a = \sup\{|D^a\varphi(x)|: x \in \Omega\},$$

where $a = (a_1, \ldots, a_N)$ is a multi-index and

$$D^a = \frac{\partial^{|a|}}{\partial x_1^{a_1}\ldots\partial x_N^{a_N}}, \qquad |a| = a_1 + a_2 + \ldots + a_N.$$

It is easy to verify that the set $D(\Omega)$ with the family $(\|\varphi\|_a)$ of semi-norms is an (F)-space and the family $(\|\varphi\|_a)$ is a base.

For the sake of convenience we shall use the following symbols:

$$a! = a_1!\ldots a_N!, \qquad \binom{a}{\nu} = \binom{a_1}{\nu_1}\ldots\binom{a_N}{\nu_N}, \qquad x^a = x_1^{a_1}\ldots x_N^{a_N}.$$

By λ we shall denote the multi-index $(1, 1, \ldots, 1)$. We shall write $\nu \leqslant a$ if $\nu_i \leqslant a_i$ for $i = 1, 2, \ldots, N$. We assume the following rules:

$$a + \nu = (a_1 + \nu_1, \ldots, a_N + \nu_N) \quad \text{and} \quad a - \nu = (a_1 - \nu_1, \ldots, a_N - \nu_N) \quad \text{if } \nu \leqslant a.$$

A set Z contained in a locally convex space X is said to be *bounded* if all semi-norms defining the topology in the space X are bounded on the set Z. Each compact set is bounded.

Now suppose we are given a sequence $\{X_m\}$ of (F)-spaces such that X_m is contained and closed in the space X_{m+1}. In the space $X = \bigcup\limits_m X_m$ we may introduce a locally convex topology which is the finest of all locally convex topologies on X such that all identical mappings X_m into X are continuous. Such spaces are called *(LF)-spaces*.

Each (LF)-space is semi-complete. The space X_m is closed in the space X for every $m = 1, 2, 3, \ldots$ A set Z contained in X is bounded if and only if it is contained in a space X_m and is bounded there.

A linear functional $T(x)$ is *continuous* on the space X if and only if its restriction to each space X_m is continuous on X_m.

Definition 2. Let Ω be an open set in the Euclidean space E_N and let $D(\Omega)$ be the set of all complex-valued functions such that they possess continuous derivatives of all orders on the space E_N and their supports are compact and are contained in the set Ω. Let (Ω_m) be a sequence of closed Jordan measurable sets such that $\bigcup\limits_m \Omega_m = \Omega$ and $\Omega_m \subset \operatorname{int}\Omega_{m+1}$ for $m = 1, 2, \ldots$ Then it is easily seen that $D(\Omega) = \bigcup\limits_m D(\Omega_m)$. $D(\Omega_m)$ $(m = 1, 2, \ldots)$ are (F)-spaces and $D(\Omega_m)$ is closed in the space $D(\Omega_{m+1})$. Therefore $D(\Omega)$ is an (LF)-space.

2. Let Y be an (LF)-space defined by a sequence $\{Y_m\}$ of (F)-spaces and let $|y|_\beta$ ($\beta \in B_m$) be a denumerable base of semi-norms defining the topology in the space Y_m. Let Ω, Ω_m be as in definition 2. Let H denote the set of all functions such that they are defined on the set Ω and have their values in the space Y, they possess continuous derivatives of all orders, and their supports are compact and are contained in the set Ω. Let us denote by H_m the set of all functions h belonging to H such that $h(\Omega) \subset Y_m$ and the support of h is contained in Ω_m.

Let us take any function $h \in H$. Since its support is contained in a set Ω_{m_1}, and since its set of values $h(\Omega) = h(\Omega_{m_1})$ as a continuous image of a compact set is compact, the set $h(\Omega)$ is bounded in Y. Therefore $h(\Omega)$ is thoroughly contained in a space Y_{m_2}. Put $m = \max(m_1, m_2)$. We see that $h \in H_m$ and hence $H = \bigcup_m H_m$.

Now consider the space H_m. It follows from the closedness of Y_m in the space Y that all functions $D^\alpha h(x)$, for $h \in H_m$, as mappings of Ω into Y_m are continuous. Therefore the semi-norms

$$\|h\|_\gamma = \sup\{|D^\alpha h(x)|_\beta : x \in \Omega_m\}, \qquad \gamma = (\alpha, \beta),\ \beta \in B_m,$$

are well-defined on the space H_m. Denote by Γ_m the set of all such indices γ. The family $\|h\|_\gamma$ ($\gamma \in \Gamma_m$) of seminorms is a base and the space H_m is an (F)-space. We see that H_m is closed in the space H_{m+1}. Therefore one can consider the space H as an (LF)-space.

We shall prove the following

THEOREM. *For every bilinear functional $B(\varphi, y)$ on the space $D(\Omega) \times Y$ which is continuous with respect to each variable separately there exists one and only one linear continuous functional T on the space H such that*

$$\text{(A)} \qquad B(\varphi, y) = T(\varphi \cdot y) \quad \text{for all} \quad \varphi \in D(\Omega) \text{ and } y \in Y.$$

Proof. Let us take any non-negative function $\eta(x)$ possessing continuous derivatives of all orders on the space E_N and satisfying the conditions

$$\int_{E_N} \eta(x)\,dx = 1, \qquad \eta(x) = 0 \text{ when } |x| \geqslant 1.$$

Let ε_m be any number less than the distance between the sets $E_N \setminus \Omega_{m+1}$ and Ω_m. It may be supposed that the sequence (ε_m) is decreasing and tends to zero.

Put

$$\varrho_m(x) = \frac{1}{\varepsilon_m^N}\,\eta\left(\frac{x}{\varepsilon_m}\right).$$

Then the functions $(\varrho_k * \varphi)(x) = \int_{E_N} \varrho_k(x-y)\varphi(y)\,dy$ belong to $D(\Omega_{m+1})$ for all $\varphi \in D(\Omega_m)$ and $k \geqslant m$. It is evident that the sequence $\varrho_k * \varphi$ converges to φ in the space $D(\Omega_{m+1})$.

It follows from the definition of the topology in the spaces $D(\Omega)$ and Y that $B(\varphi, y)$ is bilinear and continuous with respect to both variables simultanuously on the product space $D(\Omega_{m+1}) \times Y_k$ for every k and m. Hence there exist some M, α, and $\beta \in B_k$ such that

$$|B(\varphi, y)| \leqslant M \|\varphi\|_\alpha |y|_\beta \quad \text{for} \quad \varphi \in D(\Omega_{m+1}),\ y \in Y_k.$$

Define an abstract function $f_m(\xi)$ by the formula

$$f_m(\xi) = \varrho_m(\cdot - \xi) \quad \text{for} \quad \xi \in \Omega_m.$$

Its values lie in the space $D(\Omega_{m+1})$ and it represents a continuous mapping of the set Ω_m into the space $D(\Omega_{m+1})$. Let us take any $h \in H$. It belongs to a H_k and represents a continuous mapping $h(\xi)$ of the set Ω into Y_k. Therefore the function $B\big(\varrho_m(\cdot - \xi),\ h(\xi)\big)$ is continuous on the set Ω_m. Since the set Ω_m has been supposed to be Jordan measurable, the integral

$$T_m(h) = \int_{\Omega_m} B\big(\varrho_m(\cdot - \xi),\ h(\xi)\big)\,d\xi$$

exists in Riemann's sense. Moreover, the following inequality holds:

$$|T_m(h)| \leqslant M |\Omega_m| \sup_{\xi \in \Omega_m} \|\varrho_m(\cdot - \xi)\|_\alpha \|h\|_\gamma \quad \text{for all} \quad h \in H_k \quad (\gamma = (0, \beta)).$$

Hence the functionals T_m are linear and continuous on the space H.

Let us take any function $h(\xi) = \varphi(\xi) \cdot y$, where $\varphi \in D(\Omega_m)$, $y \in Y_m$. It is then evident that

$$T_k(\varphi \cdot y) = \int_{\Omega_k} B\big(\varrho_k(\cdot - \xi), \varphi(\xi) y\big)\,d\xi = B\Big(\int_{E_N} \varrho_k(\cdot - \xi)\varphi(\xi)\,d\xi,\ y\Big)$$
$$= B(\varrho_k * \varphi, y) \quad \text{for} \quad k \geqslant m.$$

Since $\varrho_k * \varphi$ converges to φ in the space $D(\Omega_{m+1})$ and since $B(\varphi, y)$ is continuous with respect to $\varphi \in D(\Omega_{m+1})$,

$$\lim_k T_k(\varphi \cdot y) = B(\varphi, y).$$

Therefore $T_k(\varphi \cdot y)$ tends to $B(\varphi, y)$ for all $\varphi \in D(\Omega)$ and $y \in Y$.

LEMMA 1. *For every $h \in H_{m-1}$, $\gamma \in \Gamma_m$, and $\eta > 0$ there exists an element $h_0 \in H_m$ such that $\|h - h_0\|_\gamma < \eta$, $h_0 = \varphi_1 \cdot y_1 + \varphi_2 \cdot y_2 + \ldots + \varphi_k \cdot y_k$, where $\varphi_i \in D(\Omega_m)$ and $y_i \in Y_m$.*

LEMMA 2. *The functionals (T_k) are equicontinuous on every space H_m that is for each m there exists a $\gamma \in \Gamma_m$ and an M such that*

$$|T_k(h)| \leqslant M\|h\|_\gamma \quad \text{for all} \quad h \in H_m \text{ and } k = 1, 2, \ldots$$

Let us take any $h \in H$. We shall prove that the sequence $(T_k(h))$ is convergent. Take any $\varepsilon > 0$. The element $h \in H_{m-1}$ for an m. Let M, γ be as in Lemma 2 and h_0 be the element from Lemma 1 corresponding to $\eta = \varepsilon/3M$ (it may be supposed that $M > 0$). Since the sequence $(T_k(h_0))$ is convergent, there exists an n_0 such that

$$|T_k(h) - T_s(h)| \leqslant |T_k(h-h_0)| + |T_k(h_0) - T_s(h_0)| + |T_s(h_0-h)|$$
$$\leqslant \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon \quad \text{for} \quad k, s \geqslant n_0.$$

Therefore the functionals T_k are convergent to a functional T on the whole space H. The functional T is linear and continuous on each space H_m, hence it is linear and continuous on H. It is evident that this functional satisfies the condition (A). To prove the uniqueness of the functional it suffices to show that every linear continuous functional vanishing on the functions of the form $\varphi \cdot y$ vanishes on the whole space H. Let us take any such functional U and any $h \in H$. Let $h \in H_{m-1}$ and let h_0, η be as in Lemma 1. Then $|U(h)| \leqslant |U(h-h_0)| \leqslant M\eta$ for any $\eta > 0$ and therefore $U(h) = 0$.

Now we proceed to prove the lemmas.

Proof of Lemma 1. Let us take any $\gamma = (a, \beta)$ and a function $\psi \in D(\Omega_m)$ such that $\psi(x) = 1$ on Ω_{m-1}. Put $g(x) = D^{a+\lambda}h(x)$ and

$$Q(x, t) = \sum_{0 \leqslant \nu \leqslant a} \binom{a}{\nu} \frac{(x-t)^\nu}{\nu!} D^\nu \psi(x), \qquad C = \sup\{|Q(x, t)| : x \in \Omega_m, t \in \Omega_m\}.$$

Suppose we are given any $\varepsilon > 0$. Since the function g is uniformly continuous on the space E_N, there exists a $\delta > 0$ such that

$$|g(x') - g(x'')|_\beta \leqslant \frac{\varepsilon}{C|\Omega_m|} \quad \text{if} \quad |x' - x''| < \delta.$$

Let us take any open covering $(K_1, \ldots, K_n)$ of the set Ω_{m-1} satisfying the conditions: $K_1 \subset \Omega_m$, the diameters of all sets K_1 are less than δ. Let $(\varphi_1, \ldots, \varphi_n)$ be a partition of unity corresponding to the covering, i. e. let the functions have continuous derivatives of all orders, and let the support of the function φ_i be contained in K_i, $\varphi_i(x) \geqslant 0$ on E_N, and

$$\sum_{i=1}^{n} \varphi_i(x) = 1 \quad \text{for} \quad x \in \Omega_{m-1}.$$

Choose $x_i \in K_i$ $(i = 1, \ldots, N)$. Put

$$g_0(x) = \sum_{i=1}^{n} \varphi_i(x) g(x_i).$$

It is evident that

(B) $$|g(x) - g_0(x)|_\beta = \Big|\sum_{i=1}^{n} \varphi_i(x)\big(g(x) - g_0(x)\big)\Big|_\beta \leqslant \sum_{i=1}^{n} \varphi_i(x)|g(x) - g_0(x)|_\beta$$
$$\leqslant \frac{\varepsilon}{C|\Omega_m|} \quad \text{for every} \quad x \in E_N.$$

Let us put

$$h_0(x) = \psi(x)(I^{a+\lambda}g_0)(x) = \psi(x) \int_{E(x)} \frac{(x-t)^a}{a!} g_0(t)\,dt,$$

where $E(x) = \{t: t_i \leqslant x_i \text{ for } i = 1, 2, \ldots, N\}$. It is easy to verify that

$$D^a\big(h(x) - h_0(x)\big) = D^a\big[\psi(x)\big(I^{a+\lambda}(g-g_0)\big)(x)\big] = \int_{\Omega_m(x)} Q(x, t)\big(g(t) - g_0(t)\big)\,dt,$$

where $\Omega_m(x) = E(x) \cap \Omega$. Hence we get

$$\|h - h_0\|_\gamma \leqslant |\Omega_m| C \cdot \frac{\varepsilon}{|\Omega_m| C} = \varepsilon.$$

To prove Lemma 2 we shall need

LEMMA 3. *Let $R_k = I^{a+\lambda}\varrho_k$ and let ψ be a function such that $\psi \in D(\Omega_{m+2})$ and $\psi(x) = 1$ on Ω_{m+1}. Then*

(C) $$T_k(h) = \int_{\Omega_{m-1}} B\big(\psi(\cdot)R_k(\cdot - \xi), D^{a+\lambda}h(\xi)\big)\,d\xi \text{ for all } h \in H_{m-1} \text{ and } k > m.$$

Proof. It is evident that

$$(\varrho_k * \varphi)(x) = \int_{E_N} \varrho_k(x-\xi)\varphi(\xi)\,d\xi = \psi(x) \int_{E_N} R_k(x-\xi) D^{a+\lambda}\varphi(\xi)\,d\xi$$

for $\varphi \in D(\Omega_m)$ and $k > m$. The abstract function

$$f(\xi) = \psi(\cdot)R_k(\cdot - \xi) D^{a+\lambda}\varphi(\xi)$$

having its values in the space $D(\Omega_{m+2})$ is continuous on the set Ω_{m+2}. Therefore

$$T_k(\varphi \cdot y) = B(\varrho_k * \varphi, y) = B\Big(\int_{\Omega_{m+2}} \psi(\cdot)R_k(\cdot - \xi) D^{a+\lambda}\varphi(\xi)\,d\xi, y\Big)$$
$$= \int_{\Omega_{m+2}} B\big(\psi(\cdot)R_k(\cdot - \xi), D^{a+\lambda}\varphi(\xi) y\big)\,d\xi \quad \text{for all} \quad \varphi \in D(\Omega_m),\ y \in Y,\ k > m.$$

Consider the functionals

(D) $$T'_k(h) = \int_{\Omega_{m+2}} B\big(\psi(\cdot)R_k(\cdot-\xi),\, D^{\alpha+\lambda}h(\xi)\big)\,d\xi \quad \text{for} \quad h\in H_m,\ k>m.$$

They are linear and continuous on the space H_m and $T_k(h) = T'_k(h)$ for functions of the form $h = \varphi_1\cdot y_1+\ldots+\varphi_k\cdot y_k$. Hence, by Lemma 1, they are identical on the space H_{m-1}.

It is seen that the set Ω_{m+1} in the integral (D) may be replaced by Ω_{m-1} for $h\in H_{m-1}$.

Now we can prove Lemma 2. The bilinear functional $B(\varphi, y)$, being separately continuous on the space $D(\Omega_{m+1})\times Y_{m-1}$, is continuous with respect to both variables simultaneously. Hence

$$|B(\varphi, y)| \leqslant M\|\varphi\|_\alpha |y|_\beta \quad \text{for} \quad \varphi\in D(\Omega_{m+1}),\ y\in Y_{m-1}, \text{ and an } M.$$

Therefore

$$|T_k(h)| \leqslant M \sup_{\xi\in\Omega_{m+1}} \|\psi(\cdot)R_k(\cdot-\xi)\|_\alpha |\Omega_m|\,\|h\|_\gamma \quad \text{for} \quad h\in H_{m-1} \text{ and } k>m,$$

where $\gamma = (\alpha+\lambda, \beta)$. On the other hand,

$$D^\alpha\big(\psi(x)R_k(x-\xi)\big) = \int_{U(x-\xi)} Q(x, \xi, t)\,\varrho_k(t)\,dt,$$

where

$$Q(x, \xi, t) = \sum_{0\leqslant\nu\leqslant\alpha} \binom{\alpha}{\nu} \frac{(x-\xi-t)^\nu}{\nu!} D^\nu\psi(x)$$

and $U(x) = \{t\in E(x): |t|<1\}$. Since

$$\sup_{\xi\in\Omega_{m+1}} \|\psi(\cdot)R_k(\cdot-\xi)\|_\alpha \leqslant \sup\{Q(x, \xi, t): x\in\Omega_{m+1},\ \xi\in\Omega_{m+1},\ |t|\leqslant 1\} < \infty,$$

the functionals T_k for $k>m$ are equicontinuous on the space H_{m-1}; therefore the functionals T_s for $s = 1, 2, 3, \ldots$ are equicontinuous on the space H_{m-1}. Since m is arbitrary, Lemma 2 is proved.

Remark. If we put $\Omega = \Omega_1$, $Y = D(\Omega_2)$, we get $H = D(\Omega_1\times\Omega_2)$, because it follows from the definition of the topology on H that it is identical with the topology in the space $D(\Omega_1\times\Omega_2)$.

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