Then the interval $(m, m+N)$ is contained in the interval (6.4) and thus, choosing for μ the value ν given by (1.4), the requirement (6.4) is not violated. But then we get, using (7.1) too,

$$|Z(\mu)| \geqslant \left(\frac{c_{13}k\log k}{8e(1+\log a+c_{13}k\log k)}\right)^{c_{13}k\log k}\frac{c_{12}}{2k^7}\cdot\frac{1}{c_{13}k\log k}.$$

Taking in account (1.7) and choosing c_1 in (1.5) sufficiently large we get, using also the second half of (1.5) and (7.3),

$$|Z(\mu)| > \left(\frac{c_{14}}{\log a}\right)^{c_{13}k\log k} > \left(\frac{1}{1+6\delta}\right)^{m} 3(c_5+c_{11}+1)\frac{\log k}{\delta^2}$$
$$> 3(c_5+c_{11}+1)\frac{\log k}{\delta^2}\left(\frac{1}{1+6\delta}\right)^{\mu}$$

indeed.

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Rational dependence in finite sets of numbers

by

E. G. Straus (Los Angeles, Cal.)

J. Mikusiński and A. Schinzel ([1]) proved that, in a finite set of points on the real line so that every distance except the maximal one occurs more than once, all distances are commensurable. This theorem was discussed in the Undergraduate Research Program in Mathematics at UCLA under the author's direction; and the proof developed there leads to a generalization which was conjectured (and proved for the case $m' = 2$ in [2]):

Theorem. *Let $x_1, x_2, \ldots, x_n$ be real numbers and let m be the dimension of the vector space, V, spanned by $\{x_i-x_j | i, j = l, \ldots, n\}$ over the rationals. Let m' be the dimension of the rational vector space, V', spanned only by those x_i-x_j for which $x_i-x_j \neq x_k-x_l$ whenever $(i, j) \neq (k, l)$. Then $m' = m$.*

Proof. Assume, without loss of generality, that $x_1 = 0$ and let $\eta_1, \eta_2, \ldots, \eta_{m'}$, be a basis for V', and $\eta_1, \ldots \eta_{m'}, \ldots, \eta_m$ be a basis for V.

Decompose the n-tuple $X = (x_1, \ldots, x_n)$ into $\sum_{s=1}^{m} X^{(s)}\eta_s$ where the $X^{(s)}$ are n-tuples of rationals, not all of them 0. By this construction we have $x_i^{(s)} = x_j^{(s)}$ for all $s > m'$ whenever x_i-x_j is attained for a unique pair (i, j). Whenever $x_i-x_j = x_k-x_l$ we obviously have $x_i^{(s)}-x_j^{(s)} = x_k^{(s)}-x_l^{(s)}$ for all $s = 1, \ldots, m$.

Now assume $m > m'$ and let $X(t) = X+tX^{(m)}$, t real. There may be a finite number of choices of t so that $x_i(t) = x_j(t)$ for some $i \neq j$ (at most one choice for each pair (i, j)), we exclude all those t. Obviously

$$x_i-x_j = x_k-x_l \Rightarrow x_i(t)-x_j(t) = x_k(t)-x_l(t)$$

for all t. Since $X^{(m)} \neq 0$ the elements of $X(t)$ become unbounded as $t \to \infty$ while $x_1(t) = 0$ for all t. In particular, for t sufficiently large, $\max_i x_i(t) - \min_i x_i(t) = x_j(t)-x_k(t) > x_n-x_1$. Thus $x_j(t)-x_k(t)$ is a unique difference among the $x_i(t)$ and therefore x_j-x_k is a unique difference among the x_i. But we have

$$x_j(t)-x_k(t) = x_j-x_k+t(x_j^{(m)}-x_k^{(m)}) > x_n-x_1$$

which implies $x_j^{(m)} - x_k^{(m)} \neq 0$. In other words

$$x_j - x_k = \sum_{s=1}^{m} (x_j^{(s)} - x_k^{(s)}) \eta_s, \qquad x_j^{(m)} - x_k^{(m)} \neq 0$$

contrary to the hypothesis that $x_j - x_k$ is a linear combination with rational coefficients of $\eta_1, \ldots, \eta_{m'}$ alone.

We remark that the assumption that the x_i are real numbers was purely for convenience of the argument and we might just as well have said that they are elements of a vector space over an arbitrary field of characteristic 0 and then let m and m' be the corresponding dimensions over that field.

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UNIVERSITY OF CALIFORNIA, LOS ANGELES

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Über ein Problem von Erdös und Moser

von

A. Sárközy und E. Szemerédi (Budapest)

Es seien $a_1, a_2, \ldots, a_n$ beliebige reelle Zahlen, für die $0 < a_1 < a_2 < \ldots < a_n$. Bezeichnen wir die Lösungszahl von

$$(1) \qquad \sum_{i=1}^{n} \varepsilon_i a_i = t; \qquad \varepsilon_i = 0, \text{ oder } 1$$

mit $f(t)$. Erdös und Moser bewiesen (s. [1]), daß

$$\max_{0 \leqslant t < +\infty} f(t) < c_1 \frac{2^n}{n^{3/2}} \log^{3/2} n$$

($c_1, c_2, \ldots$ werden positive Konstanten bedeuten), und vermuteten, daß

$$\max_{0 \leqslant t < +\infty} f(t) < c_2 \frac{2^n}{n^{3/2}}$$

(es ist leicht zu sehen, daß für $a_1 = 1, a_2 = 2, \ldots, a_n = n$ wir haben $\max_{t=0,1,2,\ldots,n^2} f(t) > c_3(2^n/n^{3/2})$). In dieser Arbeit werden wir diese Vermutung bewiesen.

Satz. *Es sei $\varepsilon > 0$ eine beliebige Zahl. Dann ist für $n > n_0(\varepsilon)$*

$$\max_{0 \leqslant t < +\infty} f(t) < (1+\varepsilon) \frac{8}{\sqrt{\pi}} \cdot \frac{2^n}{n^{3/2}}.$$

Beweis. Wir brauchen das folgende Lemma, das eine modifizierte und schwähere Gestalt eines Satzes von Katona (s. [1]) ist:

Lemma. *Es sei A eine beliebige Menge, und $A = B \cup C$, $B \cap C = 0$. Bezeichnen wir die Anzahl der Elemente von B, bzw. C mit b, bzw. c. Es seien $M_1, M_2, \ldots, M_l$ Teilmengen von A, und $l \geqslant 2^b \binom{c}{[c/2]} + 1$. Dann existieren Teilmengen M_u und M_v, für die*

$$(2) \qquad M_u \cap B = M_v \cap B$$