STUDIA MATHEMATICA, T. XXXVII. (1971)

On locally uniformly convex and differentiable norms in certain non-separable Banach spaces

by

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In the present paper we prove that a Banach space which has a weakly compact fundamental set (in particular, every reflexive Banach space) admits an equivalent locally uniformly convex norm.

We begin with some notation and definitions.

$m(\Lambda)$ will denote the space of all bounded real-valued functions defined on Λ with the norm

$$\|x\| = \sup_{a\in\Lambda} |x(a)|.$$

$c_0(\Lambda)$ will denote the linear closed subspace of $m(\Lambda)$ consisting of all x in $m(\Lambda)$ such that for every $\varepsilon > 0$ the set $\{a\colon |x(a)| > \varepsilon\}$ is finite. An equivalent norm in $c_0(\Lambda)$ is defined by the formula (Day [9])

$$J(x) = \sup\Big[\sum_{i=1}^{k} 2^{-i} x^2(a_i)\Big]^{1/2}, \tag{1}$$

where the supremum is taken over all finite subsets $\{a_i\}_1^k \subset \Lambda$.

In the sequel we shall assume that the set Λ is well-ordered by the relation $<$; the smallest element of the set $\{\beta\colon \beta\in\Lambda, a<\beta\}$ will be denoted by $a+1$.

$C(K)$ will denote the space of all real-valued continuous functions defined on a compact space K with the norm

$$\|x\| = \max_{t\in K} |x(t)|.$$

Let $\mathscr{S}$ be a σ-algebra of subsets of a set S and let μ be a measure defined on $\mathscr{S}$. The symbol $L_p(S, \mathscr{S}, \mu)$ will denote the space of equivalence classes of μ-measurable p-summable functions defined on S with the norm

$$\|x\| = \Big[\int_S |x(t)|^p d\mu\Big]^{1/p}.$$

A Banach lattice is called an *AL-space* if $x \wedge y = 0$ implies $\|x+y\| = \|x-y\|$, and $x, y \geqslant 0$ implies $\|x+y\| = \|x\|+\|y\|$.

If Z is a subset of a Banach space X, then sp Z will denote the closed linear span of Z.

The weight of X, i. e. the smallest cardinal number of a dense subset of X, will be denoted by dens X.

If $\|\cdot\|$ is the norm in X, then $\|\cdot\|^*$ will denote the norm in the conjugate space X^*.

By the *gradient* of the norm we mean the operator which assigns to an element $x \in X$ with $\|x\| = 1$ a functional $x^* \in X^*$ such that $x^*(x) = \|x^*\|^* = 1$.

A Banach space is called *strictly convex* if the conditions $\|x\| = \|y\| = 1$ and $\|x+y\| = 2$ imply $x = y$.

A Banach space is called *locally uniformly convex* if the conditions $\|x_k\| = \|x\| = 1$ and $\lim_{k\to\infty} \|x_k + x\| = 2$ imply $\lim_{k\to\infty} \|x_k - x\| = 0$.

A Banach space is called *weakly 2-rotund* if for each sequence (x_n) with $\|x_n\| = 1$ $(n = 1, 2, \ldots)$ the condition $\lim_{n,m\to\infty} \|x_n + x_m\| = 2$ implies that (x_n) is a weak Cauchy sequence.

The norm of a Banach space X is called *differentiable in the sense of Fréchet* if for each $x \in X$ with $\|x\| = 1$ we have

$$\lim_{\tau\to 0} \frac{1}{\tau}(\|x+\tau y\| + \|x-\tau y\| - 2) = 0,$$

and the convergence is uniform with respect to $y \in X$, $\|y\| \leqslant 1$.

Clarkson [5] has shown that every separable Banach space is isomorphic to a strictly convex Banach space. Kadec [11] and [12] generalized the result of Clarkson and proved that every separable Banach space is isomorphic to a locally uniformly convex Banach space. Day [9] has shown that these results cannot be extended to the non-separable case: if Λ is uncountable, then $m(\Lambda)$ is not isomorphic to any strictly convex Banach space. However, Day [9] has proved that if a Banach space X satisfies the condition

(*) there exists a one-to-one bounded linear operator T transforming X into $c_0(\Gamma)$ for some Γ,

then X is isomorphic to a strictly convex Banach space; he used the norm (1). Day [10] has shown that every AL-space X satisfies (*). Amir and Lindenstrauss [1] have proved that every Banach space X containing a weakly compact fundamental subset satisfies (*). It has been proved in [17] that if X has an unconditional basis (uncountable, in general), then X is isomorphic to a locally uniformly convex Banach space (for the definitions of bases and unconditional bases in non-separable Banach spaces, see, e. g., [4] and [17], respectively). In the present note the result of [17] is generalized.

The following lemma will be needed in the sequel; the proof of it is a simple modification of some argument given in [17].

LEMMA. *Suppose that all terms of the sequence (transfinite, in general) $\{p_\alpha\}_{\alpha\in\Lambda}$ are either positive-homogeneous sublinear functionals or linear functionals defined on a linear space X. Let the operator Q, defined by $Qx = \{p_\alpha(x)\}_{\alpha\in\Lambda}$, transforms X into the space $c_0(\Lambda)$. Then $J(Qx)$ is a positive-homogeneous sublinear functional. If*

$$J(Qx) = J(Qx_k) = \lim_{k\to\infty} \tfrac{1}{2} J[Q(x_k + x)],$$

then

$$\lim_{k\to\infty} \|Qx_k - Qx\|_{c_0(\Lambda)} = 0.$$

PROPOSITION 1. *Suppose that a Banach space X satisfies (*) and that there exists a sequence (transfinite, in general) of bounded linear operators $T_\alpha\colon X\to X$ $(\alpha \in \Lambda)$ satisfying the following conditions:*

(i) *for each $x \in X$ and $\varepsilon > 0$ the set*

$$\Lambda(x, \varepsilon) = \{\alpha\colon \|T_{\alpha+1}x - T_\alpha x\| > \varepsilon(\|T_{\alpha+1}\| + \|T_\alpha\|)\}$$

is finite;

(ii) *for each $x \in X$*

$$x \in Y_x = \operatorname{sp}\Big[(\|T_1 x\| T_1 X) \cup \bigcup_{\alpha\in\Lambda(x)} (T_{\alpha+1} - T_\alpha)X\Big],$$

where $\Lambda(x) = \bigcup_{\varepsilon>0} \Lambda(x, \varepsilon)$;

(iii) dens sp$[(T_{\alpha+1} - T_\alpha)X] \leqslant$ dens $T_1 X = \aleph_0$.

Then X is isomorphic to a locally uniformly convex Banach space.

Proof. Suppose that $\{e_i\}_1^\infty$ and $\{e_i^\alpha\}_1$ are dense in T_1X and $(T_{\alpha+1} - T_\alpha)X$, respectively. Let $\mathfrak{A}_n$ denotes the family of all subsets of Λ containing at most n elements. We define the functionals:

$$E_A^{(n)}(x) = \inf_{a_i, a_i^{(\alpha)}} \Big\| x - \sum_{\alpha\in A} \sum_{i=1}^{n} a_i^{(\alpha)} e_i^\alpha - \sum_{i=1}^{n} a_i e_i \Big\|$$

$$\Big(A \in \bigcup_{n=1}^{\infty} \mathfrak{A}_n;\ a_i, a_i^{(\alpha)} \text{ are real numbers}\Big),$$

$$t_\alpha(x) = (\|T_{\alpha+1}\| + \|T_\alpha\|)^{-1} \|T_{\alpha+1}x - T_\alpha x\|,$$

$$F_A(x) = \sum_{\alpha\in A} t_\alpha(x) \qquad \Big(A \in \bigcup_{n=1}^{\infty} \mathfrak{A}_n\Big),$$

$$G_n(x) = \sup_{A\in\mathfrak{A}_n} [E_A^{(n)}(x) + nF_A(x)], \qquad G_0(x) = \|x\|.$$

Let $\Delta = \{0, -1, -2, \ldots\} \cup \Lambda \cup \Gamma$, where Γ is the set appearing in (*) (we assume that the sets $\{0, -1, -2, \ldots\}$, Λ, and Γ are disjoint). Let Q

be the operator which assigns to each element $x \in X$ an element $Qx \in c_0(\Lambda)$ defined in the following way:

$$Qx(\delta) = \begin{cases} 2^\delta G_{-\delta}(x) & \text{for } \delta = 0, -1, -2, \ldots, \\ t_\delta(x) & \text{for } \delta \in \Lambda, \\ Tx(\delta) & \text{for } \delta \in \Gamma. \end{cases}$$

We introduce an equivalent norm by the formula

$$|||x||| = J(Qx). \tag{2}$$

If $|||x_k||| = |||x||| = 1, \lim\limits_{k\to\infty} |||x_k + x||| = 2$, then, by Lemma,

$$\lim_{k\to\infty} G_n(x_k) = G_n(x) \quad (n = 0, 1, 2, \ldots), \tag{3}$$

$$\lim_{k\to\infty} t_\alpha(x_k) = t_\alpha(x) \quad (\alpha \in \Lambda), \tag{4}$$

$$\lim_{k\to\infty} \|Tx_k - Tx\|_{c_0(\Gamma)} = 0. \tag{5}$$

It follows from (5) that in order to prove Proposition 1 it is enough to show that $\{x_k\}_1^\infty$ is compact. Given $\varepsilon > 0$, we find

$$B \in \bigcup_{n=1}^\infty \mathfrak{A}_n, \quad B \subset \Lambda(x),$$

and an m such that $E_B^{(m)}(x) < \varepsilon/3$. Let $\Lambda_B(x)$ denotes the set $\{\alpha\colon t_\alpha(x) < \min\limits_{\beta\in B} t_\beta(x)\}$. Let j be the number of the elements of the set $\Lambda(x) \setminus \Lambda_B(x)$, and let

$$b = \min_{\alpha\in\Lambda_B(x), \beta\in B} [t_\beta(x) - t_\alpha(x)].$$

Let

$$n > \max\left\{m, j\frac{\varepsilon + 3\|x\|}{3b}\right\}.$$

We find $A \in \mathfrak{A}_n$ such that

$$G_n(x) - [E_A^{(n)}(x) + nF_A(x)] \leqslant \frac{\varepsilon}{3}. \tag{6}$$

Observe that $B \subset A$. Indeed, otherwise we could take a set $D \in \mathfrak{A}_n \setminus \mathfrak{A}_{n-1}$ such that $t_\alpha(x) \leqslant t_\beta(x)$ for all $\alpha \in \Lambda \setminus D$ and $\beta \in D$, and hence

$$G_n(x) - [E_A^{(n)}(x) + nF_A(x)] \geqslant E_D^{(n)}(x) + nF_D(x) - [E_A^{(n)}(x) + F_A(x)]$$

$$\geqslant nb - \|x\| > \frac{\varepsilon}{3}$$

would contradict (6).

It follows from (3), (4) and (6) that there exists a positive integer M such that $k > M$ implies $E_A^{(n)}(x_k) < \varepsilon$. Hence some (compact) ball in the finite-dimensional space

$$\operatorname{sp}[\{x_k\}_1^M \cup \{e_i\}_1^n \cup (\bigcup_{\alpha\in A} \{e_i^\alpha\}_1^n)]$$

is an ε-net for $\{x_k\}_1^\infty$.

COROLLARY 1. *Every Banach space with a basis (uncountable, in general) is isomorphic to a locally uniformly convex Banach space.*

PROPOSITION 2. *Suppose that a Banach space X contains a weakly compact fundamental subset. Then there exists a sequence (transfinite, in general) of linear projections $P_\gamma\colon X \to X$ $(\gamma \leqslant \lambda)$ such that $\|P_\gamma\| = 1$, $P_\gamma P_\xi = P_\xi P_\gamma = P_\xi$ for $\xi < \gamma$, $P_\gamma x \in \operatorname{sp}\{P_{\xi+1} x\}_{\xi<\gamma}$, the set $\{\gamma\colon \|P_{\gamma+1}x - P_\gamma x\| > \varepsilon\}$ is finite for all $x \in X, \varepsilon > 0$, $\operatorname{dens} P_1 X = \aleph_0$, $\operatorname{dens} P_\gamma X < \operatorname{dens} X$ for $\gamma < \lambda$, $P_\lambda X = X$.*

The proof of Proposition 2 is essentially that given in [1].

THEOREM 1. *If a Banach space X contains a weakly compact fundamental subset U (in particular, if is a reflexive space), then X is isomorphic to a locally uniformly convex Banach space.*

Proof. It suffices to construct operators $T_\alpha\colon X \to X$ $(\alpha \in \Lambda)$ satisfying conditions (i)-(iii) of Proposition 1. We shall proceed by transfinite induction with respect to dens X.

If dens $X = \aleph_0$, then the identity operator satisfies the desired conditions. Let dens $X = \aleph$ and suppose that the theorem is true for each cardinal number less than $\aleph$. There exist projections P_γ $(\gamma \leqslant \lambda)$ with the properties mentioned in Proposition 2. Obviously $\operatorname{dens}[(P_{\gamma+1} - P_\gamma)X] < \aleph$, and $(P_{\gamma+1} - P_\gamma)U$ is a weakly compact subset of $(P_{\gamma+1} - P_\gamma)X$. By the inductive hypothesis there exists a sequence $\{S_\beta^\gamma\}_{\beta\in\Lambda_\gamma}$ of operators mapping $(P_{\gamma+1} - P_\gamma)X$ into itself and satisfying the conditions of Proposition 1. Let Λ denotes the set of all pairs (γ, β), where $\beta \in \Lambda_\gamma \cup \{0\}$. If $\alpha \in \Lambda$, then α' and α'' will denote the first and the second index of α, respectively. We assume that $\alpha_1 > \alpha_2$ if either $\alpha_1' > \alpha_2'$ or $\alpha_1' = \alpha_2'$ and $\alpha_1'' > \alpha_2''$. We define the operators T_α $(\alpha \in \Lambda)$ in the following way:

$$T_\alpha = S_{\alpha''}^{\alpha'}(P_{\alpha'+1} - P_{\alpha'}) + P_{\alpha'} \quad (S_0^{\alpha'} = 0).$$

It is obvious that $\operatorname{dens}\operatorname{sp}[(T_{\alpha+1} - T_\alpha)X] \leqslant \aleph_0$. Since $T_\alpha x = S_{\alpha''}^{\alpha'} x$ for all x in $(P_{\alpha'+1} - P_{\alpha'})X$, we have $\|T_\alpha\| \geqslant \|S_{\alpha''}^{\alpha'}\|$. The sets

$$\{\gamma\colon \|P_{\gamma+1}x - P_\gamma x\| > \varepsilon\},$$

$$\{\beta\colon \beta \in \Lambda_\gamma, \|(S_{\beta+1}^\gamma - S_\beta^\gamma)(P_{\ +1}x - P_\gamma x)\| > \varepsilon(\|S_{\beta+1}^\gamma\| + \|S_\beta^\gamma\|)\}$$

are finite, hence, by the inequality

$$\frac{\|T_{\alpha+1}x-T_\alpha x\|}{\|T_{\alpha+1}\|+\|T_\alpha\|}\leqslant \max\left\{\|P_{\alpha'+1}x-P_{\alpha'}x\|,\ \frac{\|(S^{\alpha'}_{\alpha''+1}-S^{\alpha'}_{\alpha''})(P_{\alpha'+1}x-P_{\alpha'}x)\|}{\|S^{\alpha'}_{\alpha''+1}\|+\|S^{\alpha'}_{\alpha''}\|}\right\},$$

the set $\Lambda(x, \beta)$ is also finite.

Now we shall show by induction that $P_\gamma x \in Y_x$ for all γ. Suppose that $P_\xi x \in Y_x$ for all $\xi < \gamma$. If there is an η such $\eta+1 = \gamma$, then $P_\gamma x \in Y_x$ because $P_\eta x \in Y_x$ and $(P_\gamma - P_\eta)x \in Y_x$. Let γ have no preceding index. Then $P_{\xi+1}x \in Y_x$ for all $\xi < \gamma$. It follows from Proposition 2 that $P_\gamma x \in \operatorname{sp}\{P_{\xi+1}x\}_{\xi<\gamma}$. Hence $P_\gamma x \in Y_x$ for all γ. On the other hand, $P_\lambda x = x$ and, consequently, $x \in Y_x$. This completes the proof.

COROLLARY 2. *If K is an Eberlein compact space (i. e, K is homeomorphic to a weakly compact subset of a Banach space), then $C(K)$ is isomorphic to a locally uniformly convex Banach space.*

Proof. Amir and Lindenstrauss [1] have shown that $C(K)$ contains a weakly compact fundamental subset.

COROLLARY 3. *Banach space is reflexive iff it is isomorphic to a weakly 2-rotund.*

Proof. Cudia [7] has observed that in any reflexive space the local uniform convexity implies the weak uniform convexity which itself implies reflexivity.

COROLLARY 4. *In every reflexive Banach space there exists an equivalent norm which is differentiable in the sense of Fréchet.*

Proof. Lovalia [15] has shown that if X^* is locally uniformly convex, then the norm of X is differentiable in the sense of Fréchet.

COROLLARY 5. *Every reflexive Banach space is a strong differentiability space* (see [3] for the definition).

The proof follows from Corollary 4 and from the results of Asplund [3].

COROLLARY 6. *In every reflexive Banach space X there exists an equivalent norm $|||\cdot|||$ such that the gradient of this norm is a homeomorphism of $\{x \in X\colon |||x||| = 1\}$ onto $\{x^* \in X^*\colon |||x^*|||^* = 1\}$.*

Proof. It follows from the results of Asplund [2] and from Corollary 3 that there exists an equivalent norm $|||\cdot|||$ in X such that the spaces X and X^*, with the norms $|||\cdot|||$ and $|||\cdot|||^*$, respectively, are locally uniformly convex. The gradient of the norm $|||:|||$ is a homeomorphism (cf. [7]).

COROLLARY 7. *Every weakly compact convex set is the closed convex hull of its strongly exposed points* (see [14] for the definition).

The proof follows from Theorem 1 and from the results of Lindenstrauss [14].

COROLLARY 8. *Every weakly compact set is dentable* (see [16] for the definition).

The proof follows from Theorem 1 and from the results of Rieffel [16].

COROLLARY 9. *In a reflexive space a set K is weakly compact iff it is an intersection of a family of finite union of closed balls.*

The proof follows from Theorem 1 and from the results of Corson and Lindenstrauss [6].

Remark. The Corollaries 7, 8, and 9 were reported to me by Prof. J. Lindenstrauss to whom I express my thanks.

THEOREM 2. *Every AL-space is isomorphic to a locally uniformly convex Banach space.*

Proof. First we shall show that if μ is a finite measure, then the set χ of all μ-measurable characteristic functions defined on S is weakly sequentially compact in $L_1(S, \mathscr{S}, \mu)$. It is easy to see that χ is weakly sequentially compact in $L_2(S, \mathscr{S}, \mu)$. Since $L_1(S, \mathscr{S}, \mu)$ is weakly sequentially complete and $L_1^*(S, \mathscr{S}, \mu) \subset L_2^*(S, \mathscr{S}, \mu)$ (see, e. g., [8], ch. IV.6), the set χ is also weakly sequentially compact in $L_1(S, \mathscr{S}, \mu)$.

By the theorem of Eberlein-Šmulian, the weak closure of χ is weakly compact. Note that χ is fundamental in $L_1(S, \mathscr{S}, \mu)$ (see, e. g., [8], III. 3. 8).

It follows from [13] that in every AL-space X there exist projections P_γ $(\gamma \leqslant \lambda)$ such that $\|P_\gamma\| = 1$, $P_\gamma P_\xi = P_\xi P_\gamma = P_\xi$ for $\xi < \gamma$, the set $\{\gamma\colon \|P_{\gamma+1}x - P_\gamma x\| > \varepsilon\}$ is finite for all $x \in X$, $\varepsilon > 0$, $P_\gamma x \in \operatorname{sp}\{P_{\xi+1}x\}_{\xi<\gamma}$, $P_\lambda X = X$; moreover, there exist μ, $\mathscr{S}$, S, μ_γ, $\mathscr{S}_\gamma$, S such that the spaces P_1X, $(P_{\gamma+1} - P_\gamma)X$ are isometrically isomorphic to $L_1(S, \mathscr{S}, \mu)$ and $L_1(S_\gamma, \mathscr{S}_\gamma, \mu_\gamma)$ respectively (μ and μ_γ are finite). Now it is easy to construct operators T_α satisfying the conditions of Proposition 1.

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Reçu par la Rédaction le 23. 12. 1969

STUDIA MATHEMATICA, T. XXXVII. (1971)

On permanently singular elements in commutative m-convex locally convex algebras

by

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1. Introduction. All algebras in this paper are assumed to be commutative. By a *superalgebra* of a topological algebra A we mean any topological algebra having a subalgebra topologically isomorphic to A and having the same unit element provided A possessed one. If k is a class of topological algebras, then an element $x \in A \in k$ is said to be *permanently singular* in the class k (or shortly, k-singular) if for any superalgebra A_1 of A, belonging to the class k, x is singular in A_1. In this paper we deal with the classes of topological algebras, locally convex algebras and multiplicatively convex algebras denoted respectively by $\mathscr{T}$, $\mathscr{LC}$, $\mathscr{M}$.

In paper [1] Arens give a characterization of permanently singular elements in the class of Banach algebras. He proved that an element $x \in A \in \mathscr{B}$ ($\mathscr{B}$-class of Banach algebras) is $\mathscr{B}$-singular if and only if it is a topological divisor of zero (and, consequently, if and only if it is $\mathscr{T}$--singular). In this paper we study a concept of $\mathscr{M}$-singularity and we show that none of these statements is true for multiplicatively convex locally convex algebras (shortly m-convex algebras). In section 3, we give a characterization of $\mathscr{M}$-singularity and show by an example that there are $\mathscr{M}$-singular elements, which are even $\mathscr{T}$-singular but are not topological divisors of zero. In section 4 we give our main result stating that there are $\mathscr{M}$-singular elements which are non-$\mathscr{LC}$-singular. This example shows also that all non-zero elements of the radical of a certain m-convex algebra can be invertible in some its locally convex extension. Moreover, both algebras are B_0-algebras (= complete metric $\mathscr{LC}$-algebras).

We cannot give characterization of $\mathscr{LC}$-singularity but we pose a conjecture about it (cf. Problem 1).

2. Prerequisites. By a *topological algebra* we mean a topological linear space over complex or real scalars in which is defined a jointly

* This work was done during the author's stay in Aarhus.