ON A GENERALIZATION OF UNIFORMITIES

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The aim of the present paper is to introduce and investigate a notion which is a generalization of the notion of uniformity on a given set. It is known that by means of uniformities one can describe all properties of completely regular topologies on a given set, including the theory of compactifications, which coincides with the theory of completions of corresponding totally bounded uniformities (Smirnov [11] and Isbell [5]). Our generalization provides a possibility of obtaining several kinds of extensions of arbitrary topological spaces. We will show a close connection between H -closed extension of a given Hausdorff space (Katětov [6]) and completion of a totally bounded uniformity in our sense. The theory of extremally disconnected resolutions (Iliadis [4]) can be also described in our theory. Recently also Fedorčuk [3] found an approach to the theory of resolutions by means of a certain kind of generalized uniformities, but his uniformities depend on the topology of a given set.

It is known that theories of uniform spaces in the sense of neighbourhoods of the diagonal and in the sense of coverings are equivalent. The theory of uniform spaces in the sense of neighbourhoods of the diagonal was extended by Császár [2] and Pervin [9]. They showed that every topological space has a *quasi-uniformity* (i.e., a family of sets satisfying conditions of a uniformity, except for that of symmetry), which induces the original topology. Our theory is not equivalent to the theory of quasi-uniform spaces. Also a generalization due to Morita [8] and Rinow [10] by means of coverings does not coincide with that of ours.

Proofs of theorems, which are analogous to the corresponding theorems for uniformities, will be sketched only or wholly omitted.

1. Preliminaries. Let X be a set. If $P \subset 2^X$, then $\bigcup P$ denotes the union of all members of P . An *intersection* of P_1 and P_2 is the family $P_1 \wedge P_2 = \{u_1 \cap u_2 : u_1 \in P_1, u_2 \in P_2\}$. If $M \subset X$, then the *star* of M with respect to P is the set

$$\text{st}(M, P) = \bigcup \{u \in P : u \cap M \neq \emptyset\}.$$

The star of one point set $\{x\}$ is denoted by $\text{st}(x, P)$.

If for every $x \in X$ there exists $u_2 \in P_2$ such that $\text{st}(x, P_1) \subset u_2$, then we say that the family P_1 is a *pointwise-star-refinement* of the family P_2 , in symbols $P_1 \succ_1 P_2$. Symbols $P_1 \succ P_2$ mean that the family P_1 is a refinement of the family P_2 .

A family P_1 is called a *star refinement* of a family P_2 , in symbol $P_1 \succ_* P_2$, iff for every $u_1 \in P_1$ there exists $u_2 \in P_2$ such that $\text{st}(u_1, P_1) \subset u_2$.

We say that a family P is a *pointwise n -star-refinement* of family Q iff there exists a finite sequence $P_0 \succ_1 P_1 \succ_1 \dots \succ_1 P_n$ such that $P = P_0$ and $Q = P_n$; in symbols $P \succ_n Q$.

Now we shall state three lemmas easy to prove.

1.1. If elements of a family P are mutually disjoint, then $P \succ_* P$.

1.2. If $P_1 \succ_2 P_2$, then for every $x \in X$ there exists $u \in P_2$ such that $\text{st}[\text{st}(x, P_1)] \subset u$.

1.3. If $P_1 \succ_2 P_2$, then $P_1 \succ_* P_2$.

2. Axioms of f -uniformity. Let X be a given set. A family $\mathcal{U} \subset 2^{2^X}$ is called an *f -uniformity* on the set X if the following conditions are satisfied:

F1. If $x \in X$, then there exists $P \in \mathcal{U}$ such that $x \in \bigcup P$.

F2. If for every $x \in \bigcup P$ there exists $P_x \in \mathcal{U}$ such that $x \in \bigcup P_x$ and $P_x \succ P$, then $P \in \mathcal{U}$.

F3. If $P_1, P_2 \in \mathcal{U}$ and $x \in \bigcup (P_1 \wedge P_2)$, then there exists $P \in \mathcal{U}$ such that $x \in \bigcup P$ and $P \succ_* P_1 \wedge P_2$.

F4. For every $x \in X$ and for every $P_1, P_2 \in \mathcal{U}$, if $x \in \bigcup P_1$, then there is $\text{st}(x, P_1) \cap \bigcup P_2 \neq \emptyset$.

Elements of f -uniformities will be called *fullfilments*.

If we assume that every element of f -uniformity $\mathcal{U}$ is a covering of X , then axiom F4 is superfluous and axioms F1-F3 coincide with the axioms for uniformities (cf. Isbell [5]):

C1. Family $\mathcal{U}$ is non-empty.

C2. If $P_1 \in \mathcal{U}$ and $P_1 \succ P_2$, then $P_2 \in \mathcal{U}$.

C3. If $P_1, P_2 \in \mathcal{U}$, then there exists $P \in \mathcal{U}$ such that $P \succ_* P_1 \wedge P_2$.

A pair $(X, \mathcal{U})$ consisting of a set X and an f -uniformity $\mathcal{U}$ on the set X is called an *f -uniform space*.

3. Bases and subbases of f -uniformities. A family $\mathcal{B} \subset \mathcal{U}$ is called a *base* of an f -uniformity $\mathcal{U}$ if, for every $P \in \mathcal{U}$ and every $x \in \bigcup P$, there exists $Q \in \mathcal{B}$ such that $x \in \bigcup Q$ and $Q \succ_* P$.

Let us notice that

3.1. Every base of an f -uniformity $\mathcal{U}$ satisfies conditions F1, F3 and F4.

It is easy to notice that

3.2. If a family $\mathcal{B} \subset 2^{2^X}$ satisfies conditions F1, F3 and F4, then the family $\mathcal{U} = \{P \subset 2^X: \text{for every } x \in \bigcup P \text{ there exists } Q \in \mathcal{B} \text{ such that } x \in \bigcup Q \text{ and } Q \succ_* P\}$ is an f -uniformity for which $\mathcal{B}$ is a base.

In such a case we say that $\mathcal{U}$ is induced by the base $\mathcal{B}$.

A family $\mathcal{B}' \subset \mathcal{U}$ is called a *subbase* of an f -uniformity $\mathcal{U}$ if the family of all finite intersections $P_1 \wedge \dots \wedge P_n$, where $P_i \in \mathcal{B}'$ and $i = 1, \dots, n$, is a base of the f -uniformity $\mathcal{U}$.

3.3. Let a family $\mathcal{B}' \subset 2^{2^X}$ satisfy conditions F1 and

F3a. If $P_1 \in \mathcal{B}'$ and $x \in \bigcup P_1$, then there exists $P \in \mathcal{B}'$ such that $x \in \bigcup P$ and $P \succ_* P_1$.

F4a. For every $x \in X$ and for all finite intersections $P_1 \wedge \dots \wedge P_n$ and $Q_1 \wedge \dots \wedge Q_m$, where $P_i, Q_j \in \mathcal{B}'$, $i = 1, \dots, n$ and $j = 1, \dots, m$, if $x \in \bigcup (P_1 \wedge \dots \wedge P_n)$, then $\bigcup (Q_1 \wedge \dots \wedge Q_m) \cap \text{st}(x, P_1 \wedge \dots \wedge P_n) \neq \emptyset$.

Then the family $\mathcal{B}$ of all finite intersections $P_1 \wedge \dots \wedge P_n$, where $P_i \in \mathcal{B}'$ and $i = 1, \dots, n$, satisfies conditions F1, F3 and F4, and, according to 3.1, it induces an f -uniformity $\mathcal{U}$ called *f -uniformity induced by $\mathcal{B}'$* .

Of course, $\mathcal{B}'$ is a subbase of the f -uniformity $\mathcal{U}$.

4. A topology induced by an f -uniformity. In this section we show that every f -uniformity on a set X induces a topology on this set and, conversely, every topology on a set X induces an f -uniformity on this set.

4.1. If $\mathcal{U}$ is an f -uniformity on a set X , then $T_{\mathcal{U}} = \{G \subset X: \text{for every } x \in G \text{ there exists } P \in \mathcal{U} \text{ such that } x \in \bigcup P \text{ and } \text{st}(x, P) \subset G\}$ is a topology on X .

$T_{\mathcal{U}}$ is called the *topology induced by $\mathcal{U}$* .

4.2. The interior of a set $A \subset X$ in the topology $T_{\mathcal{U}}$ is the set $\{x \in X: \text{st}(x, P) \subset A \text{ for some } P \in \mathcal{U} \text{ such that } x \in \bigcup P\}$.

Proofs of 4.1 and 4.2 are analogous to proofs of the corresponding theorems in [5].

4.3. If $P \in \mathcal{U}$ and $x \in \bigcup P$, then $x \in \text{int st}(x, P)$.

Every family of sets of the form $\text{int st}(x, P)$, where $x \in X$, $P \in \mathcal{B}$ and $\mathcal{B}$ is a base of the uniformity $\mathcal{U}$, forms a base for the topology $T_{\mathcal{U}}$.

From the axiom F4 it follows that

4.4. The unions of all fullfilments from a given f -uniformity $\mathcal{U}$ are open and dense in the topology $T_{\mathcal{U}}$, i.e., if $P \in \mathcal{U}$, then $\overline{\bigcup P} = X$ and $\text{int} \bigcup P = \bigcup P$.

4.5. If $\mathcal{B}$ is a base for an f -uniformity $\mathcal{U}$, then the family $\mathcal{B}^0 = \{P^0: P \in \mathcal{B}\}$, where $P^0 = \{\text{int } u: u \in P\}$, is a base for the f -uniformity $\mathcal{U}$ and for

every $P \in \mathcal{B}$ there is $\bigcup P = \bigcup P^0$. Elements of the fullfilments from $\mathcal{B}^0$ form a base for the topology $T_{\mathcal{U}}$.

Proof. It is obvious that $P^0 \succ P$. It suffices to show that $P^0 \in \mathcal{U}$. Let $x \in \bigcup P^0$. There exists $P_1 \in \mathcal{U}$ such that $x \in \bigcup P_1$ and $P_1 \succ_2 P$. From 1.2 and 4.2 it follows that $P_1 \succ P^0$. According to F2, $P^0 \in \mathcal{U}$. The remaining part of the theorem follows from 4.3 and F3.

4.6. *If T is a topology on X , then it is induced by an f -uniformity (not necessary unique) having a base consisting of finite fullfilments the elements of which are mutually disjoint sets.*

Proof. Let $T_0 \subset T$ be a base for the topology T . We define the f -uniformity $\mathcal{U}$ as the f -uniformity induced by subbase $\mathcal{B}'$:

$$\mathcal{B}' = \{P_u : u \in T_0\}, \quad \text{where } P_u = \{u, X - \bar{u}\}.$$

We shall verify that the family $\mathcal{B}'$ satisfies conditions from 3.3. Condition F1 is obvious and condition F3a follows from 1.1. Now let us verify that F4a is satisfied. Let us notice that the unions $\bigcup P_u, u \in T_0$, are open and dense in the topology T . This implies that a union of each open intersection $P_1 \wedge \dots \wedge P_n$, where $P_i \in \mathcal{B}'$ for $i = 1, \dots, n$, is open and dense in the topology T . Hence if $x \in \bigcup (Q_1 \wedge \dots \wedge Q_m)$, then

$$\text{st}(x, Q_1 \wedge \dots \wedge Q_m) \cap \bigcup (P_1 \wedge \dots \wedge P_n) \neq \emptyset$$

for every $P_i, Q_j \in \mathcal{B}'$, $i = 1, \dots, n$ and $j = 1, \dots, m$.

The topologies T and $T_{\mathcal{U}}$ are equal. Indeed, for every finite sequence $P_1, \dots, P_n$, where $P_i \in \mathcal{B}'$, $\text{st}(x, P_1 \wedge \dots \wedge P_n)$ belongs to T . Hence $T_{\mathcal{U}} \subset T$. Let $u \in T$ and $x \in u$. There exists $u_0 \in T_0$ such that $x \in u_0 \subset u$. We have $x \in \text{st}(x, P_{u_0}) = u_0 \subset u$. Thus $T \subset T_{\mathcal{U}}$.

5. The greatest f -uniformities inducing a given topology. Let T be a topology on a set X .

5.1. *There exists the greatest f -uniformity in the set of f -uniformities inducing topology T on X .*

Proof. Let M be the set of f -uniformities inducing topology T . In the same way as in 4.6 we verify that family $\mathcal{B} = \bigcup \{\mathcal{U} : \mathcal{U} \in M\}$ satisfies conditions of 3.3. It is easy to notice that the f -uniformity induced by the subbase $\mathcal{B}'$ is the greatest among f -uniformities inducing topology T .

In the same way one can prove that

5.2. *There exists the greatest f -uniformity in the set of f -uniformities inducing topology T and having bases consisting of finite fullfilments.*

This f -uniformity will be called the *greatest totally bounded f -uniformity*.

5.3. *If a family P contains a finite subfamily Q such that*

$$\bigcup P = \bigcup \{\text{int } u : u \in Q\} \quad \text{and} \quad X = \overline{\bigcup Q},$$

then P is an element of the greatest totally bounded f -uniformity.

Proof. For every $x \in \bigcup Q$ there exists Q_x such that $x \in \bigcup Q_x$, $Q_x \succ_* Q$ and every element from Q_x is open in topology T .

Indeed, let $Q = \{u_1, \dots, u_n\}$ and let us assume that $x \in \text{int } u_1$. Then we define

$$Q_x = \{v_1, \dots, v_n\}, \quad \text{where } v_1 = \text{int } u_1, \dots, v_n = \text{int } u_n - \overline{(u_1 \cup \dots \cup u_{n-1})}.$$

The elements of Q_x are mutually disjoint sets, $X = \overline{\bigcup Q_x}$ and $Q_x \succ_* Q$. Let $\mathcal{U}$ be the greatest totally bounded f -uniformity inducing topology T . A family $\mathcal{B} = \mathcal{U} \cup \{Q_x : x \in \bigcup Q\}$ satisfies conditions of 3.3 and induces a totally bounded f -uniformity $\mathcal{U}^*$. It is easy to see that the topology T is induced by $\mathcal{U}^*$. Thus we have $\mathcal{U}^* \subset \mathcal{U}$. Now according to F2 we conclude that $P \in \mathcal{U}$, because for every $x \in \bigcup P$ there exists $Q_x \in \mathcal{U}$ such that $x \in \bigcup Q_x$ and $Q_x \succ_* P$.

5.4. *If $\bigcup P = \bigcup \{\text{int } u : u \in P\}$ and $X = \overline{\bigcup \{\text{int } u : u \in P\}}$, then P belongs to the greatest f -uniformity.*

Proof. Let $\mathcal{U}$ be the greatest f -uniformity inducing the topology T . It suffices to show that for every $x \in \bigcup P$ there exists a family P_x consisting of mutually disjoint open sets such that $x \in \bigcup P_x$ and $X = \overline{\bigcup P_x}$. Then it can be proved in a similar to 5.3 that $P_x \in \mathcal{U}$ and, in consequence, that $P \in \mathcal{U}$.

Let $x \in \bigcup P$. There exists $u \in P$ such that $x \in \text{int } u$. Let S be a set with inclusion ordering such that if $Q \in S$, then 1° $\text{int } u \in Q$, 2° $Q \succ P$, 3° elements of Q are mutually disjoint.

Let $S_0 \subset S$ be a chain. Notice that $\bigcup S_0 \in S$. Now, by Kuratowski-Zorn Lemma, it follows that there exists in S a maximal element P_x . To complete the proof it suffices to show that $X = \overline{\bigcup P_x}$. Suppose that $X \neq \overline{\bigcup P_x}$. Since $X = \overline{\bigcup P}$, we infer that $\bigcup P - \overline{\bigcup P_x} \neq \emptyset$. There exists an open set $u_0 \neq \emptyset$ such that $u_0 \subset \bigcup P - \overline{\bigcup P_x}$ and $u_0 \subset u_1$, where $u_1 \in P$. The family $P_x \cup \{u_0\}$ belongs to S and is greater than P_x , but this contradicts the fact that P_x is a maximal element in S .

It follows by this proof that

5.5. *A set of elements, which are families consisting of open mutually disjoint sets and having dense unions, is a base for the greatest f -uniformity.*

6. Axioms of separation. An f -uniformity $\mathcal{U}$ on a set X is called a T_0 f -uniformity iff for every pair of different points $x, y \in X$ there exists $P \in \mathcal{U}$ such that

(T_0) $x \in \bigcup P$ or $y \in \bigcup P$ and $\text{st}(x, P) \cap \text{st}(y, P) = \emptyset$.

It is called a T_1 *f-uniformity* iff for every pair of different points $x, y \in X$ there exists $P \in \mathcal{U}$ such that

(T_1) $x, y \in \bigcup P$ and $y \notin \text{st}(x, P)$.

It is called a T_2 *f-uniformity* iff for every pair of different points $x, y \in X$ there exist $P, P' \in \mathcal{U}$ such that

(T_2) $x \in \bigcup P, y \in \bigcup P'$ and $\text{st}(x, P) \cap \text{st}(y, P') = \emptyset$.

It is called a T_3 *f-uniformity* iff it is a T_1 *f-uniformity* and

(T_3) for every $Q \in \mathcal{U}$ and $x \in \bigcup Q$ there exists $Q' \in \mathcal{U}$ such that $x \in \text{st}(x, Q') \subset \text{st}(x, Q)$.

It can be proved that

6.1. An *f-uniformity* $\mathcal{U}$ on a set X is a T_i *f-uniformity* iff the topology $T_{\mathcal{U}}$ is T_i for $i = 0, 1, 2, 3$.

If elements of $\mathcal{U}$ are coverings, then conditions (T_0)-(T_3) are equivalent.

7. Maps. Let $(X, \mathcal{U})$ and $(Y, \mathcal{V})$ be *f-uniform spaces* and let $g: X \rightarrow Y$ be a map. We introduce additional notation:

$$\bar{P} = \{\bar{u}: u \in P\}, \quad g(P) = \{g(u): u \in P\}, \quad g^{-1}(Q) = \{g^{-1}(v): v \in Q\},$$

and $\delta(x, y) < P$ means that there exists $u \in P$ such that $(x, y) \in u$.

A map $g: X \rightarrow Y$ is called *uniform relative to the f-uniformities* $\mathcal{U}$ and $\mathcal{V}$ (in symbols, $g: (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$) iff for every $Q \in \mathcal{V}$, $g^{-1}(Q) \in \mathcal{U}$.

7.1. Let $B_{\mathcal{U}}, B_{\mathcal{V}}$ be bases of *f-uniformities* $\mathcal{U}$ and $\mathcal{V}$. The following conditions are equivalent:

(a) $g: (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ is a uniform map.

(b) For every $Q \in B_{\mathcal{V}}$ and for every $g(x) \in \bigcup Q$ there exists $P \in B_{\mathcal{U}}$ such that $x \in \bigcup P$ and $g(P) \supset Q$.

(c) For every $Q \in B_{\mathcal{V}}$ and for every $g(x) \in \bigcup Q$ there exists $P \in B_{\mathcal{U}}$ such that $x \in \bigcup P$ and

(*) for every $x, y \in X$, if $\delta(x, y) < P$, then $\delta[g(x), g(y)] < Q$.

The proof is not difficult.

From 4.5 we infer

7.2. If $g: (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ is uniform, then the map $g: (X, T_{\mathcal{U}}) \rightarrow (Y, T_{\mathcal{V}})$ is continuous.

From 4.4 it follows

7.3. If $g: (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ is uniform, then for every $Q \in \mathcal{V}$, $g^{-1}(\bigcup Q)$ is open and dense in the space $(X, T_{\mathcal{U}})$.

Conversely, theorem 5.4 implies

7.4. If $g: (X, T) \rightarrow (Y, L)$ is a continuous map and, for each $P \in \mathcal{V}$, $g^{-1}(\bigcup P)$ is open and dense in (X, T) , then $g: (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ is a uniform

map, where $\mathcal{U}$ is the greatest f -uniformity inducing the topology T and $\mathcal{V}$ is an f -uniformity inducing the topology L .

If $g: (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ is a uniform map and $\mathcal{U}, \mathcal{V}$ are the greatest totally bounded f -uniformities, then the map $g: (X, T_{\mathcal{U}}) \rightarrow (Y, T_{\mathcal{V}})$ is continuous and skeletal (Mioduszewski and Rudolf [7]), i.e., $\text{int } g^{-1}(\bar{v}) \subset \overline{g^{-1}(v)}$ for every open v in $T_{\mathcal{V}}$.

Indeed, $\{v, Y - \bar{v}\} \in \mathcal{V}$, whence

$$X = \overline{g^{-1}[v \cup (Y - \bar{v})]}, \quad X = \overline{g^{-1}(v)} \cup \overline{X - g^{-1}(\bar{v})}, \quad X - \overline{X - g^{-1}(\bar{v})} \subset \overline{g^{-1}(v)},$$

i.e., $\text{int } g^{-1}(\bar{v}) \subset \overline{g^{-1}(v)}$.

Example. Let D be the discrete topology and T the natural topology on the set of real numbers and let $\mathcal{U}_D$ and $\mathcal{U}_T$ be the greatest f -uniformities inducing the topologies D and T , respectively. From 4.4 it follows that elements of $\mathcal{U}_D$ are coverings. The identity map is continuous but not uniform.

An important role for extension of uniform maps play maps called Θ -uniform.

A map $g: (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ is called Θ -uniform iff for every $x \in X$ and for every $Q \in \mathcal{V}$ such that $g(x) \in \bigcup Q$ there exists $P \in \mathcal{U}$ such that $x \in \bigcup P$ and for every pair of points $x, y \in X$ if $\delta(x, y) < P$, then $\delta[g(x), g(y)] < \bar{Q}$.

Let us state without proof the following proposition:

7.5. *If $g: (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ is a Θ -uniform map and $\mathcal{V}$ is T_3 f -uniformity, then the map $g: (X, T_{\mathcal{U}}) \rightarrow (Y, T_{\mathcal{V}})$ is continuous.*

8. Cauchy filters. Let $(X, \mathcal{U})$ be an f -uniform space. A filter ξ is called *Cauchy filter* iff for every $P \in \mathcal{U}$, $P \cap \xi$ is non-empty.

It can be proved in a way analogous to that in Bourbaki's book that

8.1. *For every Cauchy filter ξ there exists a Cauchy filter ξ_0 , called the minimal Cauchy filter of ξ , such that for each Cauchy filter ξ_1 , if $\xi_1 \subset \xi$, then $\xi_0 \subset \xi_1$.*

The filter ξ_0 is induced by the base $\xi'_0 = \{\text{st}(A, P): A \in \xi', P \in \mathcal{B}\}$, where ξ' is a base of ξ and $\mathcal{B}$ is a base of $\mathcal{U}$.

8.2. *A family $\xi''_0 = \{\text{st}(u, P): u \in P \cap \xi, P \in \mathcal{B}\}$, where $\mathcal{B}$ is a base of $\mathcal{U}$, is a base of the minimal Cauchy filter ξ_0 .*

Let $\eta(x) = \{\text{st}(x, P) \neq \emptyset: P \in \mathcal{U}\}$. A point $x \in X$ belongs to the limit of Cauchy filter ξ , $x \in \lim \xi$, iff $\eta(x) \subset \xi$.

8.3. *Let ξ be a Cauchy filter. The following conditions are equivalent:*

- (a) $x \in \lim \xi$;
- (b) $x \in \lim \xi_0$, where ξ_0 is the minimal Cauchy filter of ξ ;
- (c) $x \in \bigcap \{\bar{A}: A \in \xi\}$.

8.4. *If $\mathcal{U}$ is a T_2 f -uniformity, then each limit of Cauchy filter contains at most one point.*

If $\mathcal{U}$ is a T_2 covering f -uniformity, then each point $x \in X$ is a limit of exactly one minimal Cauchy filter with the base $\eta(x)$. This is not true if $\mathcal{U}$ is not covering f -uniformity.

Example. Let $X = \{0, 1, \frac{1}{2}, \frac{1}{3}, \dots\}$ be a subspace of the space of real numbers with the natural topology and let $\mathcal{U}$ be the greatest f -uniformity inducing the topology on X . A family $P = \{\{1\}, \{\frac{1}{2}\}, \dots\}$ is an element of $\mathcal{U}$. There is no Cauchy filter ξ such that $0 = \lim \xi$.

Indeed, this follows from the definition of Cauchy filter and from 8.3 that there exists $u \in P$, $u = \{1/n\}$, such that $0 \in \bar{u}$, but $0 \notin \bigcup \{\{1/n\} : n = 1, 2, \dots\}$.

Example. Let $X = \{-1, -\frac{1}{2}, \dots, 0, \dots, \frac{1}{2}, 1\}$ and let $\mathcal{U}$ be an f -uniformity induced by the base

$$\mathcal{B} = \{P_n : n = 1, 2, \dots\} \cup \{Q_n : n = 1, 2, \dots\},$$

where

$$P_n = \left\{ \{-1\}, \left\{-\frac{1}{2}\right\}, \dots, \left\{-\frac{1}{n}\right\}, \left(-\frac{1}{n}, \frac{1}{n}\right) - \{0\}, \left\{\frac{1}{n}\right\}, \dots, \left\{\frac{1}{2}\right\}, \{1\} \right\}$$

and

$$Q_n = \left\{ \{-1\}, \left\{-\frac{1}{2}\right\}, \dots, \left\{-\frac{1}{n}\right\}, \left(-\frac{1}{n}, \frac{1}{n}\right), \left\{\frac{1}{n}\right\}, \dots, \left\{\frac{1}{2}\right\}, \{1\} \right\}.$$

Let ξ_1, ξ_2 be the filters induced by

$$\xi'_1 = \left\{ \left(0, \frac{1}{n}\right) \cap X : n = 1, 2, \dots \right\} \cup \left\{ \left(-\frac{1}{n}, \frac{1}{n}\right) \cap X : n = 1, 2, \dots \right\},$$

$$\xi'_2 = \left\{ \left(-\frac{1}{n}, 0\right) \cap X : n = 1, 2, \dots \right\} \cup \left\{ \left(-\frac{1}{n}, \frac{1}{n}\right) \cap X : n = 1, 2, \dots \right\}.$$

It can be verified that (a) $\xi_1 \neq \xi_2$, (b) ξ_1 and ξ_2 are minimal Cauchy filters, (c) $0 = \lim \xi_1 = \lim \xi_2$, and (d) $\eta(0)$ is not a base of Cauchy filter.

8.5. If $\mathcal{U}$ contains a base consisting of locally finite fullfilments, then every point $x \in X$ is a limit of Cauchy filter.

Proof. By Kuratowski-Zorn Lemma it follows that $\eta(x)$ is contained in a maximal filter ξ , the elements of which have non-empty interiors. Let P be a locally finite fullfilment. Since P is locally finite and ξ is a maximal filter, there exists $u \in P$, $\text{int } u \neq \emptyset$, such that $u \in \xi$. This implies that ξ is a Cauchy filter.

Let us notice

8.6. If $x \in \bigcap \{\bigcup P : P \in \mathcal{U}\}$, then $\eta(x)$ is a base for a Cauchy filter.

9. Complete f -uniform spaces. An f -uniform space $(X, \mathcal{U})$ is *complete* iff every Cauchy filter has a non-empty limit.

9.1. Every f -uniform space $(X, \mathcal{U})$ is embeddable in a uniform and dense manner into a complete f -uniform space $(\tilde{X}, \tilde{\mathcal{U}})$ such that if $\mathcal{U}$ is totally bounded, then $\tilde{\mathcal{U}}$ is totally bounded, and if $\mathcal{U}$ is T_i , then $\tilde{\mathcal{U}}$ is T_i for $i = 0, 1, 2$.

Proof. Let X_0 be a family of all minimal Cauchy filters having empty limits. We define on the set $\tilde{X} = X \cup X_0$ an f -uniformity $\tilde{\mathcal{U}}$ induced by the base $\tilde{\mathcal{B}} = \{\tilde{P} : P \in \mathcal{B}\}$, where $\mathcal{B}$ is a base of $\mathcal{U}$ and $\tilde{P} = \{\tilde{u} : u \in P\}$, $\tilde{u} = u \cup \{\xi \in X_0 : u \in \xi\}$. The f -uniformity $\tilde{\mathcal{U}}$ does not depend on the choice of a base $\mathcal{B}$ in $\mathcal{U}$, because if $P_1 \succ P_2$, then $\tilde{P}_1 \succ \tilde{P}_2$.

A. The family $\tilde{\mathcal{U}}$ satisfies conditions F1-F4.

F1. From the definition of $\tilde{\mathcal{U}}$ and from the fact that elements of X_0 are Cauchy filters it follows that $X_0 \subset \bigcup \tilde{P}$ for every $P \in \mathcal{U}$. Hence $X = \bigcup \{\bigcup \tilde{P} : \tilde{P} \in \tilde{\mathcal{U}}\}$.

F2 is obvious.

F3. Let us notice that if $P_1, P_2 \in \mathcal{B}$, then $\tilde{P}_1 \wedge \tilde{P}_2 \in \tilde{\mathcal{U}}$; hence it suffices to show that for every $\tilde{P} \in \tilde{\mathcal{B}}$ and for every $x \in \bigcup \tilde{P}$ there exists $P_1 \in \mathcal{B}$ such that $\tilde{P}_1 \succ_* \tilde{P}$ and $x \in \bigcup \tilde{P}_1$; if $y \in X$, then

$$\text{st}(y, \tilde{P}_1) = \bigcup \{\tilde{u} : y \in u \in P_1\} \subset \tilde{v} \in \tilde{P},$$

where $v \in P$ is such that $\text{st}(y, P_1) \subset v$; if $y \in X_0$, then there exists $u_0 \in P_1 \cap y$, i.e., $y \in \tilde{u}_0$ and

$$\begin{aligned} \text{st}(y, \tilde{P}_1) &= \bigcup \{\tilde{u} : y \in \tilde{u}, u \in P_1\} \subset \bigcup \{\tilde{u} : u \cap u_0 \neq \emptyset, u \in P_1\} \\ &\subset [\text{st}(u_0, \tilde{P}_1)]^\sim \subset \tilde{v}, \end{aligned}$$

where $\tilde{v}$ is such that $v \in P$ and $\text{st}(u_0, P_1) \subset v$. We put $P_1 \succ_2 P$.

F4. Let $P_1, P_2 \in \mathcal{B}$ and $x \in \bigcup \tilde{P}_1$; if $x \in X$, then

$$\emptyset \neq \text{st}(x, P_1) \subset \bigcup P_2 \subset \text{st}(x, \tilde{P}_1) \cap \bigcup \tilde{P}_2;$$

if $x \in X_0$, then for every $\tilde{P} \in \tilde{\mathcal{U}}$, $x \in \bigcup \tilde{P}$ and hence $\emptyset \neq \text{st}(x, \tilde{P}_1) \cap \bigcup \tilde{P}_2$.

B. The map $i: X \subset \tilde{X}$ is a uniform embedding. Indeed, for every $\tilde{P} \in \tilde{\mathcal{B}}$ we have

$$\tilde{P} \wedge \{X\} = \{\tilde{u} \cap X : \tilde{u} \in \tilde{P}\} = \{u : u \in P\} = P \in \mathcal{B} \subset \mathcal{U}.$$

C. The topological space $(X, T_{\mathcal{U}})$ is a dense subspace of $(\tilde{X}, T_{\tilde{\mathcal{U}}})$. Indeed, for every $\xi \in X_0$ and for every $\tilde{P} \in \tilde{\mathcal{B}}$ there is $\emptyset \neq u = X \cap \tilde{u} \subset X \cap \text{st}(\xi, \tilde{P})$, where $u \in P$ is such that $u \in \xi$.

D. The space $(\tilde{X}, \tilde{\mathcal{U}})$ is complete. Let $\bar{\xi}$ be a Cauchy filter. According to 8.3, it may be assumed that $\bar{\xi}$ is a minimal Cauchy filter. From 8.2 it follows that interiors of elements of $\bar{\xi}$ are non-empty and hence the family $\xi = \{A \cap X : A \in \bar{\xi}\}$ is a filter. It is easy to see that ξ is also Cauchy filter in $(X, \mathcal{U})$. Let $\xi_0 \subset \xi$ be the minimal Cauchy filter of ξ . If there exists $x \in X$ such that $x \in \lim \xi_0$, then it is obvious that $x \in \lim \bar{\xi}$. If $\emptyset = \lim \xi_0$, then $\xi_0 \in \lim \bar{\xi}$. Let $u_0 \in P$ be such that $u_0 \in \xi_0$ and $\tilde{u}_0 \in \bar{\xi}$. But if $u_0 \in \xi_0$, then $\xi_0 \in \tilde{u}_0$ and $\tilde{u}_0 \subset \text{st}(\xi_0, \tilde{P})$, whence $\xi_0 \in \lim \bar{\xi}$.

E. If $\mathcal{U}$ is T_i , then $\tilde{\mathcal{U}}$ is T_i f -uniformity, $i = 0, 1, 2$. We give the proof for $i = 2$ (proofs in other cases are similar).

Let $x_1, x_2 \in X$ and $x_1 \neq x_2$. Examine separate cases.

(e₁) $x_1, x_2 \in X$. There exist $P_1, P_2 \in \mathcal{B}$ such that $x_1 \in \bigcup P_1, x_2 \in \bigcup P_2$ and $\text{st}(x_1, P_1) \cap \text{st}(x_2, P_2) = \emptyset$. Suppose $\xi \in \text{st}(x_1, \tilde{P}_1) \cap \text{st}(x_2, \tilde{P}_2)$. Then there exist $u_1 \in P_1$ and $u_2 \in P_2$ such that $x_1 \in u_1, x_2 \in u_2$, and $u_1, u_2 \in \xi$; a contradiction with $u_1 \cap u_2 = \emptyset$.

(e₂) $x_1 \in X$ and $x_2 \in X_0$. Since $x_1 \notin \lim x_2$, there exist, according to 8.2 and 8.3c, $P_1, P_2 \in \mathcal{U}$ such that $x_1 \in \bigcup P_1$ and $\text{st}(x_1, P_1) \cap \text{st}(u, P_2) = \emptyset$, where $u \in x_2 \cap P_2$. From the definition of $\tilde{\mathcal{U}}$ it follows that $\text{st}(x_1, \tilde{P}_1) \cap \text{st}(x_2, \tilde{P}_2) = \emptyset$.

(e₃) $x_1, x_2 \in X_0$. From 8.2 it follows that there exist $u_1 \in P_1, u_2 \in P_2 \in \mathcal{U}$ such that $\text{st}(u_1, P_1) \cap \text{st}(u_2, P_2) = \emptyset$ and $u_1 \in x_1 \cap P_1, u_2 \in x_2 \cap P_2$.

Suppose there exists $\xi \in \text{st}(x_1, \tilde{P}_1) \cap \text{st}(x_2, \tilde{P}_2)$. This implies the existence of $u'_1 \in P_1, u'_2 \in P_2$ such that $u'_1 \in x_1, u'_2 \in x_2$, and $u'_1 \cap u'_2 \in \xi$. From $u_1 \cap u'_1 \neq \emptyset, u_2 \cap u'_2 \neq \emptyset$ and $u'_1 \cap u'_2 \neq \emptyset$, it follows that $\text{st}(u_1, P_1) \cap \text{st}(u_2, P_2) \neq \emptyset$; a contradiction.

9.2. Remark. In cases (e₂) and (e₃) we have not referred to the fact that $\mathcal{U}$ is $T_i, i = 0, 1, 2$.

The space $(X_0, \mathcal{U}_0)$, where $\mathcal{U}_0 = \{\{X_0\} \wedge \tilde{P} : \tilde{P} \in \tilde{\mathcal{U}}\}$, is called a *remainder* of the completion of a space $(X, \mathcal{U})$.

9.3. The remainder X_0 is a completely regular space.

More precisely, the f -uniformity $\mathcal{U}_0$ on the set X_0 is T_2 covering uniformity.

Proof. Let us notice that $X_0 \subset \bigcap \{\bigcup \tilde{P} : \tilde{P} \in \tilde{\mathcal{U}}\}$. From Remark 9.2 and from (e₃) of the proof of 9.1 it follows that $\mathcal{U}_0$ is a T_2 f -uniformity.

Let A be a dense set in a space $(X, T_{\mathcal{U}})$. It can be verified that $\mathcal{U}_A = \{\{A\} \wedge P : P \in \mathcal{U}\}$ induces an f -uniformity on the set A .

9.4. Let $(X, \mathcal{U})$ be an f -uniform space, $(Y, \mathcal{V})$ — a T_2 complete f -uniform space, and let $(A, \mathcal{U}_A)$ be a dense subspace of $(X, \mathcal{U})$ such that

$$X - A \subset \bigcap \{\bigcup P : P \in \mathcal{U}\}.$$

Then every uniform map $g : (A, \mathcal{U}_A) \rightarrow (Y, \mathcal{V})$ has Θ -uniform extension $g^* : (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$.

Proof of the theorem is similar to the proof of a corresponding theorem for uniformities.

10. f -compact spaces. A filter ξ is called *open* iff interior of every element of ξ is non-empty.

10.1. Every open filter ξ is contained in an open filter $\text{int } \xi$ with a base consisting of sets of the form $\text{int } A$, where $A \in \xi$.

10.2. Every open filter ξ is contained in a maximal open filter.

Proof follows from the Kuratowski-Zorn Lemma.

A space (X, T) is called *f -compact* if for every family $S \subset T$ there exists a finite family $S_0 \subset S$ such that if $X = \bigcup S$, then $X = \bigcup \{\bar{G} : G \in S_0\}$.

It is not difficult to prove (Alexandroff and Urysohn [1]) that

10.3. A space (X, T) is *f -compact* iff for each open filter ξ there is $\bigcap \{\bar{A} : A \in \xi\} \neq \emptyset$.

10.4. If an *f -uniformity* $\mathcal{U}$ on a set X is totally bounded and complete, then the space $(X, T_{\mathcal{U}})$ is *f -compact*.

Proof. Let $\mathcal{B} \subset \mathcal{U}$ be a base consisting of finite fullfilments, elements of which are open in $(X, T_{\mathcal{U}})$, and let ξ be an open filter. We prove that each maximal filter $\xi_{\max} \supset \xi$ is a Cauchy filter. Hence $\emptyset \neq \lim \xi_{\max} \subset \lim \xi$ and from 10.3 it follows that $(X, T_{\mathcal{U}})$ is *f -compact*.

Let $P \in \mathcal{B}$. Since $\bigcup P$ is dense, for each $A \in \xi_{\max}$ there is $A \cap \bigcup P \neq \emptyset$. From the fact that $\xi_{\max}$ is maximal and $\bigcup P$ is open it follows that $\bigcup P \in \xi_{\max}$. But P is finite, hence there exists $u \in P$ such that $u \in \xi_{\max}$ and this means that $\xi_{\max}$ is a Cauchy filter.

10.5. If a space (X, T) is *f -compact*, then every *f -uniformity* inducing topology T is complete.

Proof. Let ξ_0 be a minimal Cauchy filter. From 8.2 it follows that ξ_0 is open. According to 10.3 and 8.3c, it is obvious that $\lim \xi_0 \neq \emptyset$.

10.6. Every topological space (X, T) can be embedded into *f -compact* space $(\tilde{X}, \tilde{T})$ such that if (X, T) is T_i , then $(\tilde{X}, \tilde{T})$ is T_i , $i = 0, 1, 2$.

Proof. It follows from 4.6 that there exists a totally bounded *f -uniformity* $\mathcal{U}$ inducing topology T . According to 9.1, there exists an embedding $i: (X, \mathcal{U}) \subset (\tilde{X}, \tilde{\mathcal{U}})$, where $\tilde{\mathcal{U}}$ is totally bounded and complete. Theorem 10.4 implies that a space $(\tilde{X}, T_{\tilde{\mathcal{U}}})$ is *f -compact*. We put $\tilde{T} = T_{\tilde{\mathcal{U}}}$.

Let us notice that completion of a covering uniformity is a covering uniformity. Thus completion of a covering totally bounded uniformity leads to a compactification.

11. A cover of $(X, \mathcal{U})$. An *f -uniformity* $\mathcal{U}$ on a set X is called *q -uniformity* iff $\mathcal{U}$ is T_2 and each point $x \in X$ is a limit of a Cauchy filter. Theorem

8.5 gives a partial answer to the question whether a T_2 f -uniformity is a q -uniformity.

Let $\mathcal{U}$ be a q -uniformity and let $\hat{X}$ be a family of all convergent minimal Cauchy filters in $(X, \mathcal{U})$. On the set $\hat{X}$ we define a covering uniformity $\hat{\mathcal{U}}$ induced by the base $\hat{\mathcal{B}} = \{\hat{P} : P \in \mathcal{B}\}$, where $\mathcal{B}$ is a base of $\mathcal{U}$, $\hat{P} = \{\hat{u} : u \in P\}$, and $\hat{u} = \{\xi \in \hat{X} : u \in \xi\}$. The family $\hat{\mathcal{U}}$ does not depend on a choice of base $\mathcal{B} \subset \mathcal{U}$, because, for every $P_1, P_2 \in \mathcal{U}$, if $P_1 \succ P_2$, then $\hat{P}_1 \succ \hat{P}_2$. Between the sets $\hat{X}$ and X there exists a natural map

$$n: \hat{X} \xrightarrow{\text{onto}} X$$

defined as $n(\xi) = x$ iff $x = \lim \xi$, $\xi \in \hat{X}$.

The map $n: \hat{X} \rightarrow X$ is called the *cover map* and the space $(\hat{X}, \hat{\mathcal{U}})$ is called the *cover* of a space $(X, \mathcal{U})$.

11.1. *The family $\hat{\mathcal{U}}$ is a T_2 covering uniformity on the set $\hat{X}$.*

Proof. From the definition of a Cauchy filter it follows that every $\hat{P} \in \hat{\mathcal{B}}$ is a covering of X . Thus family $\hat{\mathcal{U}}$ is non-empty and each element of $\hat{\mathcal{U}}$ is a covering. Hence conditions C1 and C2 are satisfied.

Let us check condition C3. Let $\hat{P}_1, \hat{P}_2 \in \hat{\mathcal{B}}$. Condition F3 implies the existence of $P \in \mathcal{B}$ such that $P \succ P_1 \wedge P_2$. Let us notice that $\hat{P} \succ \hat{P}_1 \wedge \hat{P}_2 = \widehat{P_1 \wedge P_2}$. Now, it suffices to show that for every $\hat{P} \in \hat{\mathcal{B}}$ there exists $\hat{P}_1 \in \hat{\mathcal{B}}$ such that $\hat{P}_1 \succ_* \hat{P}$. It suffices to take $P_1 \in \mathcal{B}$ such that $P_1 \succ_2 P$. Indeed, if $\xi_1, \xi_2 \in \text{st}(\xi, P_1)$, then there exist $\hat{u}_1, \hat{u}_2 \in P_1$ such that $\xi_1 \in \hat{u}_1$, $\xi_2 \in \hat{u}_2$, and $\xi \in \hat{u}_1 \cap \hat{u}_2$, i.e. $u_i \in \xi$, $i = 1, 2$. Let us take $\hat{u} \in \hat{P}_1$ such that $\xi \in u$. Since $u, u_1, u_2 \in \xi$, $u_1 \cap u \cap u_2 \neq \emptyset$. This implies that $u_1 \cup u_2 \subset \text{st}(u, P_1)$. There exists $v \in P$ such that $\text{st}(u, P_1) \subset v$. The last inclusion implies that $v \in \xi$ and $v \in \xi_i$, $i = 1, 2$, whence $\xi, \xi_1, \xi_2 \in v$. This means that $\text{st}(\xi, P_1) \subset v$, thus $\hat{P}_1 \succ_* \hat{P}$.

The proof that $\hat{\mathcal{U}}$ is T_2 is the same as the proof of 9.1 in case (e_3) .

11.2. *The cover map $n: (\hat{X}, \hat{\mathcal{U}}) \rightarrow (X, \mathcal{U})$ is Θ -uniform.*

Proof. Let us notice that for every $\hat{u} \in \hat{P} \in \hat{\mathcal{B}}$ there is $n(\hat{u}) \subset \bar{u}$, and hence for each $\hat{P} \in \hat{\mathcal{B}}$, $n(\hat{P}) \succ \bar{P}$, where $\mathcal{B}$ is a base of $\mathcal{U}$. Since $\hat{\mathcal{U}}$ is a covering uniformity, condition $n(\hat{P}) \succ P$ implies that n is Θ -uniform map.

From 7.5 it follows that

11.3. *If $\mathcal{U}$ is T_3 , then the cover map $n: (\hat{X}, T_{\hat{\mathcal{U}}}) \rightarrow (X, T_{\mathcal{U}})$ is continuous.*

11.4. *Let $\mathcal{U}$ and $\mathcal{V}$ be q -uniformities and let $g: (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ be a uniform map. Then there exists a uniform map $\hat{g}: (\hat{X}, \hat{\mathcal{U}}) \rightarrow (\hat{Y}, \hat{\mathcal{V}})$ such that the diagram*

$$\begin{array}{ccc}
 (\hat{X}, \hat{\mathcal{U}}) & \xrightarrow{\hat{g}} & (\hat{Y}, \hat{\mathcal{V}}) \\
 \downarrow n_{\mathcal{U}} & & \downarrow n_{\mathcal{V}} \\
 (X, \mathcal{U}) & \xrightarrow{g} & (Y, \mathcal{V})
 \end{array}$$

is commutative.

Proof. Let $\xi \in \hat{X}$. We define $\hat{g}(\xi)$ to be the minimal Cauchy filter of a filter $\eta = \{g(A) : A \in \xi\}$. The map $\hat{g}$ is well defined, because η is a Cauchy filter. The diagram is commutative, since if $x = \lim \xi$, then $g(x) = \lim \hat{g}(\xi)$.

To prove that $\hat{g}$ is a uniform map it suffices to show that for every $\hat{Q} \in \hat{\mathcal{V}}$ there exists $\hat{P} \in \hat{\mathcal{U}}$ such that if $\delta(\xi_1, \xi_2) < \hat{P}$, then $\delta[\hat{g}(\xi_1), \hat{g}(\xi_2)] < \hat{Q}$. Let $Q_1 \in \mathcal{V}$ and $Q_1 \succ_2 Q$. The map g is uniform, hence there exists $P \in \mathcal{U}$ such that $g(P) \succ Q_1$. Let $\xi_1, \xi_2 \in \hat{u} \in \hat{P}$, i.e. $u \in \xi_1$ and $u \in \xi_2$. There exists $v_1 \in Q_1$ such that $g(u) \subset v_1$. From 8.2 and the definition of g it follows that $\text{st}(v_1, Q_1) \in g(\xi_i)$, $i = 1, 2$. There exists $v \in Q$ such that $\text{st}(v_1, Q_1) \subset v$, hence $v \in g(\xi_i)$, $i = 1, 2$, and this implies that $\hat{g}(\xi_1), \hat{g}(\xi_2) \in \hat{v}$.

It may be shown that $\hat{g}$ is unique.

11.5. If $\mathcal{U}$ is the greatest totally bounded f -uniformity on X , then the topology $T_{\hat{\mathcal{U}}}$ on $\hat{X}$ is extremally disconnected.

Proof. Let $\hat{A} = \{\xi \in \hat{X} : A \in \xi\}$. The operation $\wedge$ has following properties:

- (a) if $A \in T_{\mathcal{U}}$, then $\hat{A} \in T_{\hat{\mathcal{U}}}$,
 - (b) for every $S \subset T_{\mathcal{U}}$, a set $\bigcup \{\hat{A} : A \in S\}$ is dense in $\widehat{\bigcup \{A : A \in S\}}$.
- The proof of (a) and (b) is not difficult and we omit it.

Let $A \in T_{\hat{\mathcal{U}}}$. From 4.5 it can be deduced that

$$A = \bigcup \{\hat{u} : u \in S\}, \quad \text{where } S \subset T_{\mathcal{U}}.$$

The set A is dense in $A_1 = \widehat{\bigcup \{u : u \in S\}}$. Let $G = \bigcup \{u : u \in S\}$. It is obvious that $G \in T_{\mathcal{U}}$ and $\hat{G} = A_1$. From 5.3 it follows that $\{G, X - \bar{G}\} \in \mathcal{U}$. Hence $\{\hat{G}, \widehat{X - \bar{G}}\} \in \hat{\mathcal{U}}$ and $\hat{X} = \hat{G} \cup \widehat{X - \bar{G}}$. Since $G \cap (X - \bar{G}) = \emptyset$, there is $\hat{G} \cap \widehat{X - \bar{G}} = \emptyset$. From the latter equality and from (a) it follows that A_1 is closed and open in $T_{\hat{\mathcal{U}}}$ and that the closure of A is equal to A_1 . Thus $T_{\hat{\mathcal{U}}}$ is extremally disconnected.

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