Common English errors in mathematical papers

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Articles

WRONG: *The function* $-e^{-x}$ ~~*is derivative*~~ *of* e^{-x}*. The function* $-e^{-x}$ *is* ~~*a*~~ *derivative of* e^{-x}*.*

RIGHT: *The function* $-e^{-x}$ *is the derivative of* e^{-x}*.*

COMMENTS: The noun *derivative* is "countable"—it cannot occur alone, without a determiner. The sentence with the indefinite article means that $-e^{-x}$ is SOME derivative of the function e^{-x}, which does not make sense, because the derivative is uniquely determined.

WRONG: *Let* U' *be* ~~*the*~~ *linear complement of the subspace* U *in* V*.*

RIGHT: *Let* U' *be a linear complement of the subspace* U *in* V*.*

COMMENTS: There are many complements of U; if you have in mind any of them, you have to use the indefinite article. On the other hand, you can say: *Let* U' *be the linear complement of the subspace* U *in* V*, described in Remark 2*—here you are specifying WHICH complement you have in mind.

WRONG: ~~*Such operator*~~ *is defined by...*

RIGHT: *Such an operator is defined by...*

COMMENTS: The word *such*, when appearing before a singular countable noun, is followed by *a/an*.

EXCEPTIONS: This rule does not apply if *such* is preceded by a quantifier: *one such map; for every such map; some such difficulty.*

Wrong: *In ~~the Section 2~~*

Right: *In Section 2*

Comments: If a series of objects are numbered by positive integers, corresponding to ordinal numbers, no article is used: *in Section 2; on page 4; in row n.*

However, often the numbering/labelling is not as direct, and then *the* may apear; e.g. usually we write *Definition 2.1*, but you can say both *inequality* (2.1) and *the inequality* (2.1).

Wrong: *~~The~~ closed sets are Borel sets.*

Right: *Closed sets are Borel sets.*

Comments: *The* does not mean "all". If you talk about things in general, use no article.

Exceptions: The above example corresponds to an *inclusion* between certain (families of) sets. When you want to stress that some sets are *equal*, you can use 'the':

- In the plane, the open sets are precisely the unions of open discs.

Also, use *the* when talking about a set as a whole:

- *The linear operators on V can be identified with the matrix space M.*

Wrong: *The ~~number of the solutions~~ of* (1)*; the ~~set of the solutions~~ of* (1)

Right: *The number of solutions of* (1)*; the set of solutions of* (1)

Comments: On the other hand, you say e.g. *the union of the sets U_i.*

Singular or plural?

Wrong: *There ~~is a finite number~~ of elements such that...*

Right: *There are a finite number of elements such that...*

Comments: Here the quantifying expression *a finite number of* has the same meaning as *finitely many*, and it has the same syntax, i.e. it requires a plural verb.

Which tense to use?

Wrong: *In 2008 Fox ~~has shown~~ that...*

Right: *In 2008 Fox showed that...*

COMMENTS: If you are giving a date, it is understood that you are thinking about a definite moment in the past; you then have to use the *Simple Past* tense.

However, you can well say, without specifying the time: *Fox has shown that...*—Fox proved something in the past, but when talking about it, you are also thinking about the present: IT IS PROVED NOW, because he proved it (no matter when). In such circumstances, use the *Present Perfect* tense.

Syntax of verbs

WRONG: *~~Let F denotes~~ a function such that...*

RIGHT: *Let F denote a function such that...*

COMMENTS: *Let* is the imperative of the verb *to let* and has to be followed by an infinitive (without *to*). You can also say: *We let F denote a function...* or *Let us denote by F a function...*

WRONG: *This lemma ~~allows to prove~~ the theorem without using* (2).

RIGHT: *This lemma allows us to prove the theorem without using* (2).

COMMENTS: The verb *allow* requires an indirect object: you have to say WHOM the lemma allows to prove the theorem. If you do not want to say that it allows you ("us"), you can say: *This lemma allows one to prove the theorem*, that is, it allows you and the reader.

You can avoid adding *us/one* by using a noun or an ing-form:

- *This lemma allows proving the theorem without the use of* (2),

or the passive voice:

- *This lemma allows the theorem to be proved without using* (2).

The same problem concerns the verbs *enable* and *permit.* Here are examples of their correct use:

- *Repeated application of Lemma 2 enables us to write...*
- *Theorem 3 enables discontinuous derivations to be built.*
- *This will permit us to demonstrate that...*
- *Formula* (6) *permits transfer of the results in Section 2 to sums of i.i.d. variables.*

Another verb requiring an indirect object is *remind*:

WRONG: *The purpose of this section is to ~~remind some~~ results on...*

RIGHT: *The purpose of this section is to remind the reader of some results on...*

If you do not want to involve the reader, you can use *recall*:

- *The purpose of this section is to recall some results on...*

Wrong: *We should ~~avoid to use~~* (2) *here, because...*

Right: *We should avoid using* (2) *here, because...*

Comments: After some verbs you cannot use an infinitive; they have to be followed by an ing-form. These include *avoid*, but also *finish* and *suggest*:

- *After having finished proving* (2), *we shall return to...*
- *This suggests investigating the solutions of...*

Here are other similar examples:

Wrong: *Section 3 is ~~devoted to prove~~ this theorem.*

Right: *Section 3 is devoted to proving this theorem.*

Wrong: *The ~~possibility to obtain~~ a better bound*

Right: *The possibility of obtaining a better bound*

All that glitters is not gold

Wrong: *~~Every linear subspace of~~ V ~~is not~~ of the form* (3).

Right: *No linear subspace of* V *is of the form* (3).

Comments: Using *all* or *every* with a negative statement is risky, as shown by the proverb above, which says of course that NOT ALL that glitters is gold, although at first sight it may seem (to a non-native speaker of English) that something is being said about "all that glitters". Therefore, constructions with *no, none, never* etc. are preferable. If, however, you want to keep *all/every* and a negative statement at all costs, you can start the sentence with a clearly separated quantifier:

- *For every linear subspace* $V \subset X$, *the formula* (3) *does not hold for* V.

Every or *any*?

Wrong: *For ~~every two maps~~* f *and* g; *for ~~every positive integers~~* m *and* n

Right: *For any two maps* f *and* g; *for all positive integers* m *and* n

Comments: *Every* has to be followed by a singular noun: *for every map.*

Exceptions: The combination *every two* can appear when talking about frequency, e.g. *The government changes every two months.*

Not or *non-*?

Wrong: *A ~~not empty set~~*

Right: *A nonempty set*

Comments: If you want to negate an adjective that appears before a noun (*attributive position*), you have to use *non-*: *a nonempty set; a non-locally convex space; a non-Euclidean domain.* Using *not* is only possible when the adjective follows the verb *be* (*predicative position*): *This space is not Euclidean.*

First **or** ***at first*?**

Wrong: ~~*At first, we prove*~~ (2).

Right: *First, we prove* (2).

Comments: *At first* is used when you are talking about what happens in the early stages of an event, in contrast to what happens later: *It might seem at first that the noncompactness is not an obstacle.*

Use of prepositions

Wrong: *We can ~~join a with~~ b by a path* π.

Right: *We can join a to b by a path* π.

Wrong: *..., which ~~contradicts to~~ Theorem 2.*

Right: *..., which contradicts Theorem 2.*

Comments: The verb *contradict* is *transitive*: *to contradict something*, and not "to contradict to something" or "with something". If you want to use *to* at the end of an indirect proof, you can write: *..., contrary to Theorem 2.*

Wrong: ~~*Continuous in the*~~ *point* x

Right: *Continuous at the point* x

Comments: But of course a function can be *continuous in the set* A (or *on the set* A).

Wrong: ~~*Independent on*~~ x

Right: *Independent of* x

Comments: On the other hand, we have: *depending on* x*; independence of* $F(U, V)$ *from* V.

WRONG: *~~Disjoint with~~ X*

RIGHT: *Disjoint from X*

WRONG: *Then F ~~is equal B~~.*

RIGHT: *Then F is equal to B.*
Then F equals B.

WRONG: *We shall prove this ~~in the end of Section 3~~.*

RIGHT: *We shall prove this at the end of Section 3.*

COMMENTS: *In the end* means finally, as a result of the previous situation, e.g. *Thus in the end, after all these transformations, F will be homogeneous.*

WRONG: *The ~~coefficient by~~ x^3 in the expansion*

RIGHT: *The coefficient of x^3 in the expansion*

WRONG: *Then F is ~~greater or equal~~ to 3.*

RIGHT: *Then F is greater than or equal to 3.*

COMMENTS: The adjectives *greater* and *equal* require different prepositions: "greater to 3" does not make sense. There are many ways of avoiding this clumsy construction: *F is at least/most 3; F does not exceed 3; F has no more/fewer than 3 elements; F is of degree 3 or less/more.*

Walking in the street, the sun was shining

WRONG: *Setting $x = 0$, the assertion follows.*

RIGHT: *Setting $x = 0$ yields the assertion.*
Setting $x = 0$, we obtain the assertion.
If we set $x = 0$, the assertion follows.

COMMENTS: The subject of a clause containing a participle (here, *Setting*) should be the same as the subject of the main clause (*the assertion does not set anything; we do*). This error is called an *unattached participle* or *dangling participle.*

EXCEPTIONS: Unattached participles are normal in some expressions referring to the speaker's attitude, e.g.

- *Roughly speaking,... Considering the proof,... Assuming F is continuous,...*

Word order

WRONG: *~~Theorem 3 we shall~~ prove in Section 4.*

RIGHT: *We shall prove Theorem 3 in Section 4.*
Theorem 3 will be proved in Section 4.

COMMENTS: The subject normally precedes the direct object.

WRONG: *We can ~~prove easily Theorem 3~~ by applying* (2).

We will ~~prove in Section 4 Theorem 3~~.

RIGHT: *We can easily prove Theorem 3 by applying* (2).
We shall prove Theorem 3 in Section 4.

COMMENTS: In general, it is best not to put anything between the *verb* and the *direct object.*

WRONG: *A ~~bounded by 1 function~~*

RIGHT: *A function bounded by 1*

COMMENTS: If an expression qualifying a noun contains a preposition (here, *by*), the expression has to follow the noun.

EXCEPTIONS: This rule is violated by certain expressions which are felt as one word, e.g. *a global in time solution*, previously written with hyphens: *a global-in-time solution.* Generally, any sequence of words joined with hyphens can play the role of an adjective, e.g. *the you-know-which map*; but you can hardly use this stylistic device systematically in a mathematical paper.

WRONG: *The ~~two following~~ sets*

RIGHT: *The following two sets*

WRONG: *Let f be ~~such a function~~ that...*

RIGHT: *Let f be a function such that...*

WRONG: *We now list all the ~~involved functions~~.*

RIGHT: *We now list all the functions involved.*

COMMENTS: Past participles often appear after a noun, especially if they replace an identifying relative clause: *We now list all the functions that will be involved in our study.*

Other examples where the past participle must appear AFTER the noun: *the process described; the problem discussed/mentioned; the solution obtained/adopted.*

Exceptions: However, there are many past participles that can appear before nouns, e.g. *an involved explanation* = a complicated explanation (so the word *involved* changes its meaning according to its position); *the stated properties; the above-mentioned problem.*

Wrong word used

Wrong: *This proves the ~~thesis of our theorem~~.*

Right: *This proves the assertion/conclusion of our theorem.*

Comments: A *thesis* is either a dissertation, or an argument, theory etc. that can be accepted or rejected.

Wrong: *~~To this aim~~, we first consider...*

Right: *To this end, we first consider...*

Wrong: *We denote it ~~shortly~~ by* A_f.

Right: *We denote it briefly by* A_f.

Comments: *Shortly* = soon. For example: *A precise definition will be given shortly.*

Wrong: *We ~~expose examples~~ of maps such that...*

Right: *We present/give examples of maps such that...*

Comments: *Expose* = uncover, reveal; this word seldom appears in mathematical texts, as opposed to *exposition* = a comprehensive explanation (of a problem): *A detailed exposition is given in* [5].

Wrong: *The ~~function f verifies~~ equation* (1).

Right: *The function f satisfies equation* (1).

Comments: In English, *a function cannot verify anything; only a person can.* Note that this construction is normal in French: *la fonction f vérifie l'équation* (1).

Wrong: *The solution can be carried over to U ~~with the help~~ of the mapping F.*

Right: *The solution can be carried over to U with the aid/use of the mapping F.*

Comments: You can do something with *a person's* help.

WRONG: ~~*Contrary to*~~ [7]*, we do not assume the compactness of* X*.*

RIGHT: *In contrast to* [7]*, we do not assume the compactness of* X*.*

COMMENTS: The phrase *contrary to* is correctly used e.g. in indirect proofs: *Then* $f = 1$*, contrary to assumption.*

WRONG: *Then* $f = 1$*, ~~what completes the proof.~~*

RIGHT: *Then* $f = 1$*, which completes the proof.*

COMMENTS: On the other hand, you can say: *The function* f *is continuous; what is more, it is differentiable.* Where lies the difference between these two examples? *Which* refers to what precedes it in the sentence, while *what* refers to what follows it.

WRONG: *The function* F ~~*will be precised*~~ *in Section 2.*

RIGHT: *The function* F *will be made precise in Section 2.*
The function F *will be specified in Section 2.*

COMMENTS: It is tempting to translate the French *préciser* or Polish *sprecyzować* as "to precise"; unfortunately, there is no such word: *precise* is only an adjective.

Miscellanea

WRONG: *In this paper ~~we prove among others~~ that...*

RIGHT: *In this paper we prove among other things that...*

COMMENTS: *Among others* can only be used when it is clear what "others" you have in mind, e.g. *Our result generalizes Brown's theorem, among others.*

WRONG: *There exists a limit* $\lim_{x\to 0} F(x)$*.*

RIGHT: *The limit* $\lim_{x\to 0} F(x)$ *exists.*

WRONG: ~~*On the other side*~~*, F fails to have property P.*

RIGHT: *On the other hand, F fails to have property P.*

WRONG: ~~*As usually*~~*, we can rephrase this as a uniqueness theorem.*

RIGHT: *As usual, we can rephrase this as a uniqueness theorem.*

Punctuation

Wrong: *We ~~prove, that~~...*

Right: *We prove that...*

Wrong: *Let f be any ~~function, which satisfies~~ condition* (1).

Right: *Let f be any function which satisfies condition* (1).

Comments: Do not put a comma before a "defining *which*". On the other hand, put a comma before *which* if it starts a non-identifying clause; a typical example appears at the end of a proof: *Then $f = 1$, which completes the proof.*

Wrong: ~~*Let $n \in N$, then*~~*...*

Right: *Let $n \in N$. Then...*

Wrong: *This case has been thoroughly ~~studied, see~~* $[2, 3, 8]$.

Right: *This case has been thoroughly studied (see* $[2, 3, 8]$).
This case has been thoroughly studied; see $[2, 3, 8]$ *for more details and examples.*