Generalized Gandy-Păun-Rozenberg machines
for tile systems and cellular automata

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The paper proposes an extension of membrane computing towards modelling those systems whose underlying topology evolves in a more complicated way than by membrane division and membrane creation.

We introduce a concept of a *generalized Gandy–Păun–Rozenberg machine*, briefly called a *generalized G–P–R machine*, which is aimed to be applied for modelling various systems of multidimensional tile-like compartments (cells) with common or overlapping parts (tile faces) of compartment boundaries by graph rewriting.

# Generalized G–P–R machines

The above systems of multidimensional tile-like compartments comprise the underlying tile systems of cellular automata (see anywhere for cellular automata on multidimensional grids), of the DNA based self-assembly systems, of the general self-assembly systems for certain purposes, the tile systems appearing in tile logic, and in geometrical or topological programming.

Configuration $\mathcal{C}$ of hexagonal tiles is transformed by simultaneous application of two graph rewriting rules $\mathcal{R}_1$, $\mathcal{R}_2$.

Explanatory example


$\mathcal{C}$:

$\mathcal{R}_1$: $x \rightarrow x/a \quad b$
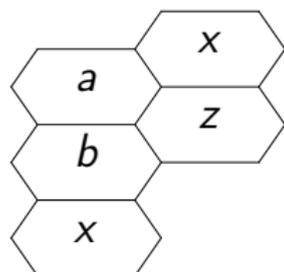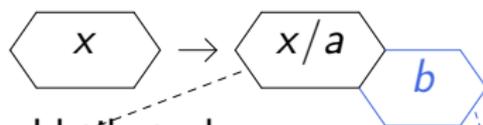
old tile to be relabelled

new tile to be introduced

$\mathcal{R}_2$: $z \rightarrow z/c \quad d$
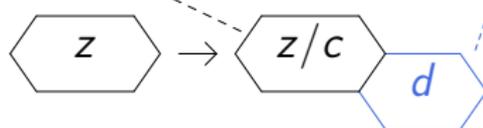
Explanatory example

The result of transformation
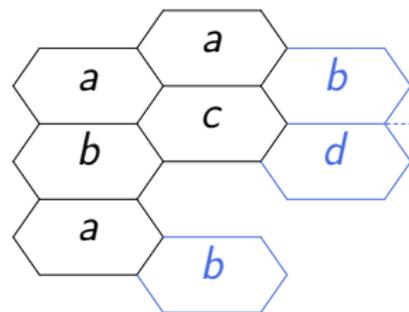
common face to keep the resulting configuration in a hexagonal grid

Explanatory example

The graph rewriting rules $\mathcal{R}_1, \mathcal{R}_2$ should be completed by an *auxiliary gluing rule*:
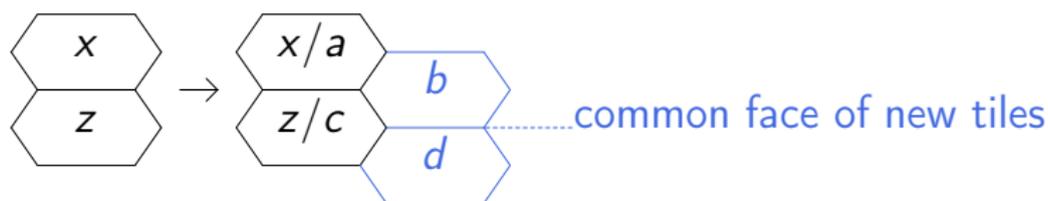
to respect common face of new tiles.

Definition

A generalized G–P–R machine is a system of transformation (processing) of finite labelled graphs by simultaneous application of graph rewriting rules with respect to auxiliary gluing rules, and its mathematical definition is given in categorical terms of diagrams and their colimits in categories of labelled graphs.

Looking forward

Looking forward, since self-assembly is an important attribute of life, the generalized G–P–R machines, being systems equivalent to Turing machines (via their representation by Gandy machines) and aimed to model self-assembly systems like, may serve for modelling computable approximations of life, whenever it is not computable.