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Struktury poissonowskie w modularnej teorii Tomity-Takesaki

praca wspólna z D.Beltitą

ABSTRACT

Przedstawimy struktury geometryczne: grupoidy i algebroidy Banacha-Liego oraz grupoidy poissonowskie związane w sposób kanoniczny z dowolną $W^*$-algebrą $\mathfrak{M}$ (algebrą von Neumanna).

Pokażemy również, że standardowa realizacja $(\mathfrak{M}, \mathcal{H}, J, \mathcal{P})$ $W^*$-algebry odpowiada naturalnej foliacji przestrzeni Hilberta $\mathcal{H}$ wyposażonej w bogatszą strukturę $\tilde{\mathcal{H}}$ rozmaitości Banacha. Opiszemy strukturę $(\tilde{\mathcal{H}}, \tilde{\omega}) \rightrightarrows \mathfrak{M}_*^+$ grupoidu presymplektycznego nad przestrzenią stanów normalnych $\mathfrak{M}_*^+$ algebry von Neumanna $\mathfrak{M}$. Pokażemy, że grupoid Banacha-Liego $(\tilde{\mathcal{H}}, \tilde{\omega}) \rightrightarrows \mathfrak{M}_*^+$ jest izomorficzny z grupoidem działania $\mathcal{U}(\mathfrak{M}) * \mathfrak{M}_*^+ \rightrightarrows \mathfrak{M}_*^+$, gdzie $\mathcal{U}(\mathfrak{M}) \rightrightarrows \mathcal{L}(\mathfrak{M})$ jest grupoidem Banacha-Liego częściowych izometrii nad kratą projekcji ortogonalnych $\mathcal{L}(\mathfrak{M})$.

Poisson geometrical aspects of the Tomita-Takesaki modular theory

joint work with D.Beltita

ABSTRACT

We investigate some genuine Poisson geometric objects in the modular theory of an arbitrary von Neumann algebra $\mathfrak{M}$. Specifically, for any standard form realization $(\mathfrak{M}, \mathcal{H}, J, \mathcal{P})$, we find a canonical foliation of the Hilbert space $\mathcal{H}$, whose leaves are Banach manifolds that are weakly immersed into $\mathcal{H}$, thereby endowing $\mathcal{H}$ with a richer Banach manifold structure to be denoted by $\widetilde{\mathcal{H}}$. We also find that $\widetilde{\mathcal{H}}$ has the structure of a Banach-Lie groupoid $\widetilde{\mathcal{H}} \rightrightarrows \mathfrak{M}_*^+$ which is isomorphic to the action groupoid $\mathcal{U}(\mathfrak{M}) * \mathfrak{M}_*^+ \rightrightarrows \mathfrak{M}_*^+$ defined by the natural action of the Banach-Lie groupoid of partial isometries $\mathcal{U}(\mathfrak{M}) \rightrightarrows \mathcal{L}(\mathfrak{M})$ on the positive cone in the predual $\mathfrak{M}_*^+$, where $\mathcal{L}(\mathfrak{M})$ is the projection lattice of $\mathfrak{M}$. There is also a presymplectic form $\widetilde{\omega} \in \Omega^2(\widetilde{\mathcal{H}})$ that comes fom the scalar product of $\mathcal{H}$ and is multiplicative in the usual sense of finite-dimensional Lie groupoid theory. We further show that the groupoid $(\widetilde{\mathcal{H}}, \widetilde{\omega}) \rightrightarrows \mathfrak{M}_*^+$ shares several other properties of finite-dimensional presymplectic groupoids and we investigate the Poisson manifold structures of its orbits as well as the leaf space the foliation defined by the degeneracy kernel of the presymplectic form $\widetilde{\omega}$.

## Literatura

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