The mini course `Graded bundles in geometry and mechanics'.

Talk 1: `Graded bundles'

Abstract: We start with showing that the multiplication by reals completely determines a smooth real vector bundle. Then we consider a general smooth actions on the monoid of multiplicative reals on smooth manifolds. In this way homogeneity structures are defined. The vector bundles are homogeneity structures which are regular in a certain sense. It can be shown that homogeneity structures are manifolds whose local coordinates have associated degrees taking values in non-negative integers, i.e. graded bundles are born. Canonical examples are the higher tangent bundles. We show also how to lift canonically homogeneity structures (graded bundle structures) to tangent and cotangent fibrations.

Talk 2: `Double structures and algebroids'

Abstract: We define double graded bundles (in general n-tuple graded bundles) in terms of homogeneous structures. Classical examples are double vector bundles obtained from lifts, especially TE and T*E for a vector bundle E.
We show the canonical isomorphism of double vetor bundles T*E* and T*E and define general algebroids (in particular, Lie algebroids) in terms of double vector bundle morphisms.

Talk 3: `Linearization of graded bundles and weighted structures'

Abstract: We consider weighted structures which are geometric structures with a compatible homogeneity structure, for instance weighted Lie groupoids and weighted Lie algebroids which are natural generalizations of VB-groupoids and VB-algebroids.
We introduce also the functor of linearization of graded bundles. Linearizing subsequently a undle of degree n we arrive at n-tuple vector bundle. Those n-tuple vector bundles can be characterized geometrically, so that we obtain an equivalence of categories.

Talk 4 `Tulczyjew triples and geometric mechanics on algebroids'

Abstract: Starting with the classical Tulczyjew triple involving TT*M, T*TM and T*T*M, we define the triple associated with a general algebroid involving TE*, T*E and T^*E^*. Using now Lagrangian and Hamiltonian functions we explain how to construct dynamics out of them, also in constrained cases, and Euler-Lagrange equations. We end up with mechanics on Lie algebroids with higher order Lagrangians.

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