Maurycy Rzymowski
Uniwersytet Mikołaja Kopernika w Toruniu

Reflected BSDEs with two optional barriers on Brownian filtration

Let B be a standard d-dimensional Brownian motion and $\mathbb{F} = \{\mathcal{F}_t\}_{t\geq 0}$ be a standard augmentation of the filtration generated by B. We present results on Reflected Backward Stochastic Differential Equations (RBSDE for short) of the following form

$$\begin{cases} Y_t = \xi + \int_t^T f(r, Y_r, Z_r)\,dr + \int_r^T dR_r - \int_t^T Z_r\,dB_r, \quad t \in [0,T], \\ L_t \leq Y_t \leq U_t, \quad t \in [0,T], \quad (Y,Z,R) \in \mathit{Prog}(\mathbb{F}) \\ dR \ \mathit{is}\ \text{"}\mathit{minimal}\text{"}, \quad \int_0^T |Z_r|^2\,dr < \infty, \end{cases} \tag{A}$$

where ξ (terminal time) is an $\mathcal{F}_T$-measurable random variable, mapping $f : \Omega \times [0,T] \times \mathbb{R} \times \mathbb{R}^d \to \mathbb{R}$ (generator) is $\mathbb{F}$-progressively measurable with respect to the first two variables and L, U (barriers) are optional processes satisfying some separation condition.

It has been widely recognized that reflected BSDEs provide a useful framework for studying problems in many fields, such as financial mathematics, stochastic optimal control and partial differential equation (e.g. optimal stopping problem, Dynkin games, stopping and control games, switching problem, PDEs with singular data, homogenization, boundary problems, regularity problems, numerical schemes etc.).

The theory is well studied for cádlág barriers L, U. However, there is only a few papers concerning BSDEs with non-càdlàg barriers (see e.g. [1]–[6]). It is caused mostly by the following: the main component of the solution, process Y, does not have to be a càdlàg process, the minimality condition for dR (guaranteeing uniqueness) is complicated and unintuitive, and the basic proof technique of cádlág case, i.e. penalization method, does not apply to non-cádlág case.

We will present the results on the existence and uniqueness of a solution to the problem (A). We assume that generator f is only non-increasing and continuous with respect to y (without any growth condition), satisfies Lipschitz condition on z and data are in L^p for some $p \geq 1$. We also present some results concerning methods of approximation of the solution via modified penalization scheme, and present the link between solutions of RBSDEs with optional barriers and the value processes for generalized nonlinear Dynkin games.

The results were obtained in cooperation with Tomasz Klimsiak and Leszek Słomiński.

Bibliography

[1] Grigorova, M., Imkeller, P., Offen, E., Ouknine, Y., Quenez, Marie-Claire: Reflected BSDEs when the obstacle is not right-continuous and optimal stopping. *Ann. Appl. Probab.* **27** (2017) 3153–3188.

[2] Grigorova, M., Imkeller, P., Ouknine, Y., Quenez, M.C.: Doubly Reflected BSDEs and $\mathcal{E}^f$-Dynkin games: beyond the right-continuous case. *Electron. J. Probab.* **23** (2018) Paper No. 122, 38 pp.

[3] Grigorova, M., Imkeller, P., Ouknine, Y., Quenez, M.C.: Optimal stopping with f–expectations: the irregular case. *Stochastic Process. Appl.* **130** (3) (2020) 1258–1288.

[4] Klimsiak, T., Rzymowski, M., Słomiński, L.: Reflected BSDEs with regulated trajectories. *Stochastic Process Appl.* **129** (2019) 1153–1184.

[5] Klimsiak, T., Rzymowski, M., Słomiński, L.: Reflected backward stochastic differential equations with two optional barriers. *Bull. Sci. Math.* **158** (2020) 102820, 49 pp.

[6] Klimsiak, T., Rzymowski, M.: Reflected BSDEs with optional barriers on filtered spaces. *Electron. J. Probab.* **26** (2021) no. 91, 1–24.

[7] Marzougue, M., El Otmani, M.: BSDEs with right upper-semicontinuous reflecting obstacle and stochastic Lipschitz coefficient. *Random Oper. Stoch. Equ.* **27** (2019) 27–41.