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We thought we knew about circles

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Abstract.

Convex geometries are closure systems with the Anti-Exchange Axiom, which are ubiquitous in Discrete Mathematics, but also came up in infinite form in Universal Algebra. Important model of convex geometries is given by point configurations in R^n, together with convex hull operator. Since not all finite convex geometries can be represented that way, it was suggested by G. Czedli in 2013 to generalize representation by using balls instead of points. Moreover, he showed that it is enough to use circles in R^2 to represent all convex geometries of convex dimension 2. Since then, a flurry of papers appeared investigating representation of geometries by convex shapes, and by circles on the plane, in particular.

In 2020, in the middle of pandemic that cancelled many in-person projects for undergraduates, I participated in the PolyMath REU project that allowed more than 300 undergraduates from US and other countries to participate virtually in a number of projects conducted by big subgroups of students under mentor's supervision. 17 students in my group, within 10 weeks project June-August 2020, were able to catalogue representations by circles on the plane of 672 convex geometries on 5-element set. 7 students from this group continued to work for two more academic years to conclude that 38 geometries from this set cannot be represented by circles for a good reason: there are some properties of circle configurations on the plane that prohibit formation of some geometries.

In summer 2022, in New York Combinatorics REU, where I was in-person mentor of two students, we attempted to understand whether the result of Czedli of 2013 can be extended to geometries of convex dimension 3.

This talk is accessible to undergraduate students, especially to those who heard before about matroids, antimatroids or convex geometries, possibly, in the context of special set systems.