Willet's contact reduction

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A *contact manifold* is a pair (M, ξ) consisting of an odd-dimensional manifold M endowed with a one-codimensional completely non-integrable distribution ξ on M. If $\xi = \ker \eta$ for a differential one-form η on M, which is not unique, the pair (M, η) is called a *co-orientable contact manifold*. In this talk, I will present a Marsden–Weinstein reduction for co-oriented contact manifolds devised in C. Willett, Contact reduction, *Trans. Amer. Math. Soc.* **354** (2002) 4245–4260. Roughly speaking, this reduction uses a Lie group action of symmetries of η to obtain from it a new co-oriented manifold on a quotient of a submanifold of M in a manner that does not depend on the choice of η corresponding to a fixed ξ.

More in detail, I will first introduce some basic notions on co-oriented contact manifolds and briefly explain how they can be viewed as *symplectic* $\mathbb{R}^\times$*-principal bundles*. In particular, I will recall the definition of a contact Lie group action $\Phi : G \times M \to M$ and its associated momentum map $\mathbf{J}^\Phi_\eta : M \to \mathfrak{g}^*$ with respect to a contact manifold (M, η). Next, I will introduce the notion of an *orbifold* and a contact quotient $M_\mu := \mathbf{J}^{\Phi-1}_\eta(\mu)/K_\mu$ for $\mu \in \mathfrak{g}^*$ and some subgroup $K_\mu \subset G$ satisfying certain particular properties. Finally, I will present a co-oriented contact Marsden–Weinstein reduction theorem that ensures that the contact quotient is a contact orbifold. To understand the more complicated case of a co-oriented contact orbifold, I will examine a symplectic orbifold obtained through symplectic reduction. If time permits, I will discuss removing the singularity and the integrability assumptions in favour of the existence of a slice.