An anti-holomorphic dynamical system on a torus, with applications

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We count solutions of the equation

$$\zeta(z) + az + b\overline{z} = 0,$$

where ζ is the Weierstrass zeta-function of a lattice, and a, b are unique constants which make the LHS periodic with respect to this lattice. These solutions have two interesting interpretations: they correspond to a) critical points of the Green function on the torus, and b) they define metrics of constant curvature 1 on the torus with one conic singularity with angle 6π.

This equation also defines the simplest anti-holomorphic dynamical system, whose parameter plane consists of just two hyperbolic components separated by a smooth curve.