Loosely Bernoulli nonhyperbolic ergodic measures

Abstract: Given a finite collection of matrices $A = \{A_1, \ldots, A_k\} \subset SL(2,\mathbb{R})$, the matrix cocycle generated by A is the dynamical system $F : SL(2,\mathbb{R}) \times \{1,\ldots,k\}^{\mathbb{Z}} \to SL(2,\mathbb{R}) \times \{1,\ldots,k\}^{\mathbb{Z}}$ defined by

$$F(B,\omega) = (A_{\omega_0}B, \sigma\omega).$$

For a given point $\omega \in \{1,\ldots,k\}^{\mathbb{Z}}$ we define the Lyapunov exponent of ω the following way:

$$\chi(\omega) = \lim_{n\to\infty} \frac{1}{n} \log ||\pi_1 \circ F^n(B)||,$$

where π_1 is the projection to the first coordinate. Clearly, the limit (if it exists) does not depend on the choice of B. By the Oseledets Theorem, one can define the Lyapunov exponent of any ergodic measure as the value the Lyapunov exponent takes at almost every point with respect to this measure.

The classical result of Furstenberg states that, except for a meager set of matrix cocycles, every Bernoulli measure has positive Lyapunov exponent (his result is actually much more general, I'm just presenting it in the simplest case), that is every Bernoulli measure is hyperbolic. It was further generalized by Vircer and by Goldsheid to the class of Markov measures.

I will present the result, joint with Katrin Gelfert and Lorenzo Diaz, in which we prove that the Furstenberg Theorem does not work for loosely Bernoulli measures. Namely, we prove that for an open class of $SL(2,\mathbb{R})$ matrix cocycles there exist loosely Bernoulli ergodic nonhyperbolic measures. Moreover, for those matrix cocycles the nonhyperbolic loosely Bernoulli ergodic measures are dense in the class of all nonhyperbolic ergodic measures (in the weak*+entropy topology), and their metric entropies take all possible values in $[0, h_0)$, where h_0 is the topological entropy of the set of points with Lyapunov exponent 0.