Intermediate Factor Theorems: From Dynamics to Operator Algebras

Let X and Y be G-spaces, where G is a second-countable, locally compact group, and $G \curvearrowright X$ and Y by homeomorphisms. A very well-studied question from the dynamics is to understand all intermediate factors of the form

$$\begin{tikzcd} X \times Y \arrow[r] \arrow[rr, bend left, "\mathrm{Proj}_X"] & Z \arrow[r] & X. \end{tikzcd}$$

In particular, to find conditions on G, X, and Y such that every such intermediate factor splits as a product of the form $X \times Y'$, where there is a continuous G-map $Y \to Y'$.

Of course, we can also ask such a question in the measurable setup, where we replace topological compact spaces by measurable spaces.

$$\begin{tikzcd} (X,\mu) \times (Y,\nu) \arrow[r] \arrow[rr, bend left, "\mathrm{Proj}_X"] & (Z,\eta) \arrow[r] & (X,\mu) \end{tikzcd}$$

When is $(Z,\eta) \cong (X,\mu) \times (Y',\nu')$, where there is a G-map $(Y,\nu) \to (Y',\nu')$?

Such results, known as *Intermediate Factor Theorems*, have been at the heart of rigidity results, starting from Margulis to Nevo–Stuck–Zimmer to Bader–Shalom.

During this talk, we shall reformulate this problem in the C^* and von Neumann algebraic language and then give some new results in this direction. Parts of them are based on joint work with Yongle Jiang and Shuoxing Zhou, respectively.

No prior knowledge of any of these will be assumed during the talk.